Photonic statistics: math vs. mysticism

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ABSTRACT

Critical analysis is given for mystical aspects of the current understanding of interaction between charged particles: wave-particle duality and nonlocal entanglement. A possible statistical effect concerning distribution functions for coincidences between the output channels of beam splitters is described. If this effect is observed in beam splitter data, then significant evidence for photon splitting, i.e., against the notion that light is ultimately packaged in finite chunks, has been found. An argument is given for the invalidity of the meaning attached to tests of Bell inequalities. Additionally, a totally classical paradigm for the calculation of the customary expression for the "quantum" coincidence coefficient pertaining to the singlet state is described. It fully accounts for the results of experimental tests of Bell inequalities taken nowadays to prove the reality of entanglement and non-locality in quantum phenomena of, *inter alia,* light It also fully accounts for the results of experimental tests of Bell inequalities which are taken nowadays to prove the reality of entanglement and non-locality in quantum phenomena of, *inter alia,* light.

1. INTRODUCTION

The long standing dispute over the exact ontological nature of "light," i.e., is it fundamentally particulate or undulatory, has never really come to a close. Nowadays, artist inspired terminology and science-fiction styled narrative is commonplace in discussions on this issue. Among all imaginable options, however, one that is largely ignored is that both of these paradigms may be inappropriate for application to light.

The particulate paradigm, especially the modern "photon" avatar, can be seen to be a consequence of the ineluctable reality that light detection is accomplished using the photoelectric effect; that is, by observing an electron current that has been evoked by light, or in general impinging electromagnetic radiation. Insofar as electron currents are composed of identical, minimally sized entities, namely electrons, naturally the behavior of such currents facilitates the inference on the nature of whatever evoked it to the effect that radiation is also "chunk-wise" quantized.

Likewise, regarding the wave paradigm, any self consistent relationship (analytical function) describing the interaction of source and sink charges, when rendered mathematically, can be resolved in terms of Fourier components. Because the simplest Fourier analysis, especially for free space, is based on the simplest hyperbolic differential equation (wave equation), a natural tendency will be to imagine its Fourier components as if they were ontologically substantive, in spite of the well known fact that Fourier components are fictitious—only the total signal can be considered substantial. In fact, the total signal between charged particles is less wavelike than a time dependent Gaussian ($\sim r^{-2}$), restricted to light-cones.

Thus, the question: what alternative(s) remain? Historically, the main one considered is the so-called: "direct interaction" picture, also know as "interaction-at-a-distance" (IAAD). This paradigm takes it that, the elemental building block for the interaction of charged particles is an eternal link between each charged particle as source and as sink for all other charges; it is the double ended link that is basic, not the individual charges themselves. Of course, the historically original version of this paradigm, namely *instantaneous* interaction-at-a-distance, can be rejected empirically. But, IAAD *on light cones* remains viable, even while, perhaps, requiring certain revisions in Special Relativity as taught currently.

The point to this report is: 1.) to propose analysis leading to a possible experiment to test the quantum or packaged character of "photons;" and 2.) to publicize analysis criticizing the currently fashionable notion of "quantum entanglement" by providing a self consistent, classical alternative paradigm.

2. PHOTON PACKAGING

Arguably the most cited evidence for the integrity of photons is based on the behavior of light at beam splitters. The fact that at minimum power level there are virtually no coincidences in the detection of photo-electrons between the two output channels of beam splitters, has been credited to the fact that "photons cannot be split." This argument, however, is weak on its face. Even if a photon were to be split at a half-silvered mirror, it would result in two half-photon pulses, one for each channel, each of half intensity but with equal pulse length, that, according to current knowledge, elicits a photo-electron at an random moment within the evoking pulse length. From basic probability theory it follows that, as the window-width (time interval) defining an experimental coincidence (obviously in applications it cannot be infinitesimal) is narrowed, the probability of so defined "simultaneous" photo-electron emission in both channels diminishes. Thus, splitting or not, coincidence counts go down with decreasing window widths; this fact cannot address this issue in principle.

On the other hand, there could be another effect that does distinguish between those coincidences arising from "split photons" and those from accidental pulse overlaps. Such overlaps can occur, for example, when distinct pulses are generated at separate mezzo regions within the macro source (crystal). Each photo-electron emission in either channel has an "arrival-instant" determined by two stochastic processes, one at the source and another at the detector. But, those photons (pulses) split at the mirror should have identical pulse-head arrival times at the detectors whereas accidentals generally do not. This effect leads in principle to different statistical characteristics of the distribution function of detections in these two circumstances.

Let the probability density for a single pulse (photon) generation in the source be given, for example, by

$$\rho_Q(T) = 1/L, \quad \{0 < T < L\}, \tag{1}$$

where T is the source-pulse length. The probability density for elicitation of a photo-electron in the detector according to standard theory is given by:

$$\rho_T(\tau) = \lambda e^{-\lambda\tau}, \quad \{0 < \tau\}. \tag{2}$$

Then the probability density of photo-electron emission in each channel will be the sum of these two stochastic processes for which the density is the convolution of the densities of these two subprocesses, i.e.:

$$\rho_{t,r}(t) = \int \rho_Q(\tau - t)\rho_T(t)\, d\tau. \tag{3}$$

Similarly, the probability density for the difference in emission time of the photo-electrons in the two channels of a beam splitter, $\delta t = t_t - t_r$, i.e., the time interval between photo-electron arrival instants in the transmitted and reflected channels, again is represented by the convolution of the respective individual densities:

$$\rho(\delta t) = \int \rho_t(\delta\tau - z)\rho_r(z)\, dz, . \tag{4}$$

Integrating this density from $-\infty$ to w (i.e., the window width):.

$$F(w) = \int_{-\infty}^{w} \rho(\delta t)\, d\delta t, \tag{5}$$

gives the distribution function with argument w, that is, the total number of coincidences detected that have a time-of-arrival difference between the output channels less than or equal to the window width, given the two processes with density functions ρ_Q and ρ_T. Distribution functions can be easier to extract from data than coincidence probability densities.

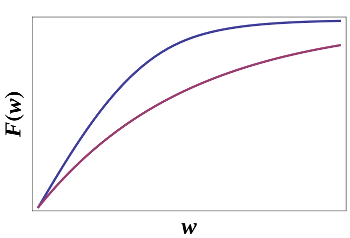

Figure 1. Distribution functions for split photons (upper curve) vs. overlaps as functions of w.

Computing and plotting such distributions for the options "splitting" vs. "no splitting." with numerically auspicious estimates results in Fig. 1. An "accidental" curve can always be deduced from experimental data by comparing displaced time-of-arrival sequences. If the non displaced sequence yields an identical curve, then evidence contra splitting has been found. (See Ref.[1] for greater detail.)

From this it is seen that, in principle a distinction between these options is observable. In any case, the general line of analysis may well lead to an experiment addressing the question of the true nature of the photon, that is, the fundamental character of the interaction between charged particles. (For further discussion on the general issue of the character of interaction among charged particles. see Ref .[2])

3. ENTANGLEMENT

Nowadays "entanglement" is considered a new found "resource" and the object of intense attention and research. The concept originated already over 75 years ago in an attempt to analyze the prevailing interpretation of Quantum Mechanics. In short, it is somehow a "quantum" parallel concept for statistical correlation. A problem remains open, however, in that the total theoretical interpretation structure involving entanglement cannot be closed free of contradiction. No matter how the arguments are laid out, if every aspect is taken into consideration at once, one or another Physics principle employed for the interpretation of Quantum Theory remains unsatisfied or nonsensical. (More below.)

Historically the resulting dissatisfaction led to the surmise that insofar as Quantum Theory appeared the least counterintuitive when understood in terms of Born's interpretation of a wave function, perhaps there exists an extended deterministic theory free of quantum weirdness. Bell then sought to find constraints an extended theory would have to satisfy so that its average would return Quantum Theory. In particular, he derived certain inequalities he argued must be satisfied by correlation coefficients both in the extended theory and in its average. Empirically, such inequalities are violated; so nowadays it is said this "proves" that Quantum Mechanics, and by extension: Nature itself, cannot respect the central hypothesis for their derivation, namely: "locality," (meaning interaction by 'local' contact or with delay limited by the speed of light). In other words, somehow photons can interrelate "faster than the speed of light," i.e., show "nonclassical correlations" or "entanglement." This feature clearly contradicts Special Relativity, although, in fact nothing faster than the speed of light has ever been observed; what is seen is just a violation of a Bell inequality, with significance depending on the rectitude of Bell's arguments.

Additionally, the notion of nonlocal processes in quantum theory arose originally with the conception of "wave packet collapse" or "projection" upon measurement. This speculative conception was introduced to rationalize the fact that measurements usually yield specific values, even when they pertain in principle to wave functions which are finite over an extended domain. Applied to systems of two or more particles, instantaneous collapse is presumed to occur for the component wave packets of all formerly interacting and subsequently correlated subcomponents, even when space-like separated, as measurement is made on just one of them.

Now, it turns out that, on the one hand correlations and on the other wave packet character enter into the interpretation of experiments considered to prove the nonlocal character of "entanglement" in virtually independent ways. That is, derivations of Bell inequalities utilize a hypothetical assertion regarding encoding locality that is essentially unrelated to that feature of wave collapse that seems to be relevant to experiments employing paired photons with correlated polarization. Let us consider each separately.

3.1 Bell Inequalities

For present purposes all polemics and previous interpretations are ignored; herein the significance of mathematical symbols used by Bell and others shall be determined solely by their mathematical use. This is justified insofar as the meta logic of the derivation of a Bell inequality is as follows: Because Quantum Mechanics has certain features suggesting a possible statistical interpretation, it is taken as an hypothesis that there exists an extended theory involving additional variables (traditionally denoted with λ, and signifying either a single variable or set of various parameters of various types), such that when the λ(s) are averaged over or integrated out, the (extended) meta theory projects back to or reduces to standard quantum theory. The extended theory was hoped to be free of the difficulties interpreting Quantum Mechanics, if not in fact fully in concordance with classical (relativistic) mechanics. This means that all considerations made at the meta level with these additional variables λ, are not to have any problematic properties peculiar to quantum theory; all structure there is to be (hypothetically, of course) purely conventional. Thus, at this level, all statements and formulas must be formulated and interpreted in accordance with pre-quantum relativistic and statistical mechanics.

The essential fundamental hypothetical input for derivation of a Bell inequality pertaining to correlated photon pairs (as employed in EPR experiments) is the assertion that the relation of the recovered quantum correlation function, $K(a, b)$, to is meta sources, is:

$$K(a, b) = \int A(a|\lambda)B(b|\lambda)\rho(\lambda)\, d\lambda, \tag{6}$$

where $A(a|\lambda)$ and $B(b|\lambda)$ are the meta probabilities of a detection made on Alice's and Bob's detectors when they are set to a and b respectively. Just here Bell made a crucial step: he asserted that to encode locality, $A(a|\lambda)$ cannot depend on b, nor $B(b|\lambda)$ on a. This he supported with the argument that locality would preclude the choice of measurement parameter at the location of $A(a|\lambda)$ having instantaneous influence on the result of measurements at the location of $B(b|\lambda)$. Although at first this sounds logical; does it really make sense?

To begin, note that the measurement parameters a and b have a different role than do the λ. The latter specify properties of the photons whereas the former properties of measurement apparatuses. Thus, only the latter can have significance in a mechanics of the generation and propagation of the photons, which reasonably would be constrained somehow by the principles of relativity. This means that, the symbols A and B, which stand for ratios (detections/trials) when the measurement apparatuses are set to detect photons passing polarizing filters with their axes set to the angles a and b respectively, will be augmented only when detections are made under conditions specified by these arguments, all other detections shall be just ignored or attributed to a different ratio.

Moreover, of course, a detection at station A with setting a can happen only when the photon encountering it has appropriate values of λ. Let that specific variable in the set λ with the appropriate value pertaining to its polarization state be: λ_a; likewise for station B requiring its own value λ_b, where these particular $\lambda_{a,b}$ pertain, obviously, to the *photons'* polarization states, not the polarizer filter at the detection station. At this point it can be said with certainty that, because the objects of intended study are just densities of detection events of photon pairs having *correlated* polarization states, the coincidence probabilities (of the sort appearing in the integrand of Eq. (6)) for pairs susceptible to detection given the detector settings at the instant before detection, is to be written as:

$$A(\lambda_a|\lambda_b)B(\lambda_b). \tag{7}$$

That is, at least one of the factors, here A was chosen arbitrarily, *must* be a *conditional probability,* conditioned on the presence of a photon vulnerable to detection at the companion station. This is necessary because the pulses (photons) of interest are *correlated* in polarization by intentional, advance stipulation, thus mandating the use of conditional probabilities. But note, however, as all λ pertain just to the pulses, nothing precludes these $\lambda_{a,b}$ from being determined by or parameterizing a time evolution involving a common cause in the overlap of the past light cones of both measurement stations; in other words, the photons could well be generated in an absolutely "local" process. Nonlocality is *not* an inevitability.

On the other hand, insofar as the λ's are hidden from the experimenter, although pertaining to ontological facts, they are in this application also by hypothetical precondition unobservable and therefore unavailable for analysis by mortals. Overcoming this impediment is the reason for employing measurement devices; the manifest purpose of which is to expose such "hidden" facts to observation. This all implies, per the ways and means of measurement in general, that the hidden variables correlate somehow (ideally deterministically) with readable parameters of the measuring devices; i.e., symbolically in this case: $a \simeq \lambda_a$ and $b \simeq \lambda_b$, so that, under the chosen circumstance, the above coincidence probabilities can also be rendered as:

$$A(a|b, \bar{\lambda})B(b|\bar{\lambda}), \tag{8}$$

where now $\bar{\lambda}$ denotes the set of hidden variables with $\lambda_{a,b}$ put in abeyance as their role has been assumed by a and b. Note that one of these factors, again, is a *conditional probability* in the variables a and b (without which, all EPR experiments would be pointless!). Within calculations the role of a and b are just identifiers of the relevant type of detections. They have nothing to say about how the values actually possessed by the photon were generated or transmitted to the stations A and B; they do no more than indicate which subset of all possible detections (measurements, or experiments) of pairs engendered independent of all measurement devices and settings, have been chosen for study. The complete background dynamics of the generation and transmission of those photons of interest are specified by entirely other variables—in Bell's analysis, denoted by λ('s).

With this form, namely Eq. (8), for underlying coincident probabilities (specifically including the conditional probabilities missing in Bell-type formulations) derivations of Bell inequalities are stymied. When carried out formally without regard to this technicality, the results can pertain then only to uncorrelated pairs.

Another way of looking at this relationship is to take into account that, after the polarization filters at the measuring station, the counts constituting A and B are augmented only when $\lambda_a \simeq a$ and $\lambda_b \simeq b$, (photons for which this is not so have been blocked) so that Eq. (7) for the purposes of an observer may be written in the form of Eq. (8). Alternately, by keeping exact account of the values of the various terms in the integrands in a derivation of Bell inequalities it can be seen that, some of its terms are actually always identically zero so that a derivation with phantom zeros removed gives:

$$K(a, b) + K(a', b') \leq 2, \tag{9}$$

a tautology.[3,4]

The final conclusion of this line of argument is that, Bell's encoding of "locality" inadvertently actually encoded statistical independence or non correlation, contrary to his initial intention. Bell inequalities simply, then, do not mean what is being read into them nowadays.

3.2 The "quantum" correlation

Perhaps the most experimentally convenient form of a Bell inequality for experiments is the so-called CHSH version, namely:

$$K(0, \pi/8) + K(0, -\pi/8) + K(\pi/2, \pi/8) - K(\pi/2, -\pi/8) \leq 2. \tag{10}$$

Maximum violation can be obtained by substituting the correlation function for the singlet state, namely:

$$K(\theta_1, \theta_2) = -\cos(2(\theta_1 - \theta_2)), \tag{11}$$

to get $2\sqrt{2} \leq 2$, a clear violation. Note that the left side is simply an arithmetical consequence of Eqs. (10) and (11) computed for correlated quantum states; whereas the right side derives from Bell's argument for uncorrelated states, albeit mostly unrecognized as such. From this the customary conclusion is that, Quantum Mechanics violates this Bell inequality, which proves that "non-locality" is an intrinsic feature of Quantum Theory with this state. Again, all this can be made to sound very convincing; but, once more: it is really valid and free of all contradiction?

First note that, the correlated quantum state used here is the singlet state:

$$\psi(A, B) = \frac{1}{\sqrt{2}}(\langle\uparrow_A | \langle\rightarrow_B | - | \rightarrow_A\rangle | \uparrow_B\rangle), \tag{12}$$

which, contrary to a widely held opinion, cannot be sensibly interpreted according to the Born-rule, namely:

$$\psi^*\psi = \text{Probability density of presence.} \tag{13}$$

This follows immediately from computation, to wit:

$$\psi^*\psi = \frac{1}{2}\left[\langle\uparrow_A | \uparrow_A\rangle \langle\rightarrow_B | \rightarrow_B\rangle + \langle\rightarrow_A | \rightarrow_A\rangle \langle\uparrow_B | \uparrow_B\rangle\right] - \langle\uparrow_A | \langle\rightarrow_B | \rightarrow_A\rangle | \uparrow_B\rangle . \tag{14}$$

The first two terms are sensible and present no problems. They are probabilities for one or the other component of the singlet state.

Customarily, the third term is dismissed with the argument that, for example, $\langle\uparrow_A | \rightarrow_A\rangle \equiv 0$. But, while this appears to be the inner product of orthogonal Hilbert space vectors, in fact, however, it is only the *symbolic* inner product of vectors from two distinct Hilbert Spaces. They will be orthogonal only if there is *physical* justification for the two metrics being identical (being set equal). It is actually a consequence of locality that the metrics are not "identical," (rather just structural "duplicates") because they are space-like separated, that is, there is no physical interaction between these vectors to be represented by a common metric, rendering their inner product undefined, both mathematically and within the physical (Born) interpretation.

This general issue arises in an additional way. If the mentioned inner product *is* taken to be identically zero, then the question arises: when does it do so? The Born expectation is rotationally invariant only when the third term is nonzero. In other words; if the third term is identically zero always, then rotational invariance is lost! Its existence, however, has been empirically verified. Why, then, should interpretation be done only *after* checking invariance properties?

Similarly, without the third term the crucial expression for the a quantum correlations coefficient, Eq. (11), becomes:

$$K(\theta_a, \theta_b) = \cos(2\theta_a)\cos(2\theta_b). \tag{15}$$

This expression is not only not rotationally invariant (specifically, it is not invariant under the transformations $\theta_{a,b} \rightarrow \theta_{a,b} + \text{const.}$ simultaneously—in an experiment this corresponds to a rotation of the source about its propagation axis) but also does not lead to that quantum correlation essential for Bell's conclusion.

In short, exclusively the quantum correlation function resulting from the input of the singlet state type accommodates the violations of Bell inequalities. This state (and its analogues) is ontologically ambiguous. It is utterly unclear what an ontic object comprising components with mutually exclusive properties can be. On the other hand, if it is considered an abstraction representing an ensemble of items (in other words: if Quantum Mechanics is incomplete) such that it has certain statistical properties representing those of the ensemble as a whole even when it pertains to no individual member, the conflict vanishes. (But so do completeness, non-locality and entanglement! See Einstein's thoughts on this from long ago in Ref.[5])

3.3 Simulated experiments

The ambiguities in the structure described above might be removed once and for all with a numerical simulation of a data point-by-data point of the experimental realizations of Bell-inequality tests, based on verified fundamental principles of Physics. A central feature of such a stimulation would have to be the photo detection law, namely

$$I(x,t) \propto E(x,t)^2, \tag{16}$$

which specifies that the intensity of a photo-current (or probability of photo-electron emission) is proportional to the *square* of the electric field magnitude inducing it. For photons, the E-field is held to be proportional to its wave function, Eq. (4). Where this law does not pertain because all the factors of E are to the first power, e.g., in the third term of Eq. (14), there is simply no currently known physics explanation for the interaction of electric fields with photo-detectors.

Above it is argued that, the derivation of a Bell inequality is carried out at a mega level in which there is to be no quantum phenomena (or least no non-locality or irreality). In fact it can be seen from the algebraic and analytic manipulations used there that, no non-commutativity or factors of Planck's constant played a role in deriving Bell inequalities. The only connection to anything from Quantum Mechanics occurs, from the outside as it were, when the correlation coefficient calculated with theoretical quantum proscriptions from the singlet state is inserted into the story. Furthermore, the singlet state itself is a uniquely "quantum" entity; it finds use in no other Physics theory. Given these facts, it seems reasonable that, the real issue encumbering complete simulation of EPR experiments lies less with the structure of a Bell inequality than with the interference term arising from either trying to interpret the singlet state (compute $\psi^*\psi$) or fit it into a physical model for photo-current generation (correlate E^2 with a probability or photon detection rate). Once one has a Bell inequality in hand, whatever its meaning, then when an expression for the correlation coefficient is selected, all else follows by arithmetic. Thus, given any physically relevant expression for the correlation, it should be possible to numerically simulate the experiment, even while the consequence of a violation of the inequality, as argued above, is incorrectly attributed.

Thus far, arguably, it has not been found possible to simulate these *experiments* including *all* the stipulations derived from analysis of Bell inequalities *and* with fully credible and transparent models for all the physical processes involved. Various technicalities have been brought to the analysis in connection with this failure, most of them are also discussed in terms of some “loophole” in the experimental realization. Currently, one of the more exploited is the so-called “detection loophole.” The central consideration for this loophole is the efficiency of detectors—practically always less than perfect—leading to cases in which a detection is made at one station but not the other (sometimes neither). By juggling the efficiency factors heuristically, a violation of a Bell inequality can be achieved even while the input signals do not include contributions from the interference term in Eq. (14), thus being made to appear both “local and real.” Most experimenters expect to close this loophole eventually, thereby proving empirically that Quantum Mechanics implies that something in Nature is “nonlocal.” Should the detection loophole mechanism turn out to be a viable physical effect for this purpose, its existence, apparently, would not accommodate rotational invariance, however.

Another possible tactic to rationalize the interference term in Eq. (14) is to search for as yet spurious classical signals that mimic the mathematical consequence of the interference term but have a form compatible with the photo detection law. An example of such additional signals are the usual components of the singlet state rotated by $\pi/4$. Adding these signals to the first two terms of Eq. (14) leads to the correlation coefficient:

$$K(\theta_a,\,\theta_b) = \frac{1}{2}\cos(2(\theta_a - \theta_b)); \tag{17}$$

that is one half of the quantum correlation coefficient for the singlet state. It does have the somewhat difficult to obtain feature of being rotationally invariant. This result is directly obtained by defining the two source signals as two dimensional vectors in the plane of polarization:

$$S_1(n) = [n, n-1];\ S_2 = [n-1, n], \quad n = 0, 1, \tag{18}$$

multiplied by the modified $2D$ rotational matrix:

$$P(\theta, k) = \begin{bmatrix} \cos(\theta + k\frac{\pi}{4}) & \sin(\theta + k\frac{\pi}{4}) \\ -\sin(\theta + k\frac{\pi}{4}) & \cos(\theta + k\frac{\pi}{4}) \end{bmatrix}, \tag{19}$$

to get the 8 electric fields, $E(\theta, n, k)_j = P(\theta, k)S_j(n)$, impinging on the detectors. With these the sum of even products of squares:

$$\mathfrak{E} = \sum_{n=0}^{1}\sum_{j=1}^{2}\sum_{k=0}^{1}\left[E_b(\theta_b, n, k)_j\ E_a(\theta_a, n, k)_j\right]^2, \tag{20}$$

and the corresponding term for the odd products:

$$\mathfrak{O} = \sum_{n=0}^{1}\sum_{j=1}^{2}\sum_{k=0}^{1}\left[E_b(\theta_b, n, k)_j\ E_a(\theta_a, n, k)_{(3-j)}\right]^2, \tag{21}$$

inserted into one expression for the polarization correlation coefficient, namely:

$$K(\theta_a, \theta_b) = \frac{\mathfrak{E} - \mathfrak{O}}{\mathfrak{E} + \mathfrak{O}}, \tag{22}$$

gives:

$$K(\theta_a,\,\theta_b) = \frac{1}{2}\cos(2(\theta_a - \theta_b)). \tag{23}$$

This result was obtained also by simulation, which leads to the conviction that it is unlikely that just this modification together with a so far overlooked factor of 2 can account for the results reported from experiments. Nevertheless, it is an encouraging result, and so it is tempting to speculate that possibly still other input states could be mixed in somehow to overcome the factor of 1/2.

In this regard, a possibility was suggest by S. S. Mizrahi and M. H. Y. Moussa[6] who showed that, a classical ensemble of electromagnetic pulses correlated in polarization but with random bias rotations about the propagation direction, has a correlation coefficient equal to that for the singlet state, namely: $K(\theta_a, \theta_b) = \cos(2(\theta_a - \theta_b + \delta))$, where δ is the fixed angle between the signals sent to Bob and Alice (for anticorrelated in polarization states, such as those comprising the singlet state, $\delta = \pi/2$).

This result they got starting from the fundamental definition for the correlation coefficient, often denoted Pearson's r-coefficient*, namely

$$P(\theta_a, \theta_b) = \frac{\frac{1}{N}\sum_i^N I(\theta_a)I(\theta_b) - \frac{1}{N}\sum_i^N I(\theta_a)\frac{1}{N}\sum_i^N I(\theta_b)}{\left[\left(\frac{1}{N}\sum_i^N I(\theta_a)^2 - \left(\frac{1}{N}\sum_i^N I(\theta_a)\right)^2\right)\left(\frac{1}{N}\sum_i^N I(\theta_b)^2 - \left(\frac{1}{N}\sum_i^N I(\theta_b)\right)^2\right)\right]^{(1/2)}}, \tag{24}$$

where $I(\theta_a, \chi_a) = I_0 \cos^2(\theta_a - \chi_a)$, i.e., Malus' Law. For convenience, it is advantageous to convert the variables I in this expression to a normalized, zero-mean form, e.g.,

$$A(\theta_j) = \frac{I_j - < I_j >}{< I_j >} = \cos(2(\theta_j - \chi_j)), \tag{25}$$

after which, replacing sums with integrals, Eq. (15) becomes:

$$K(\theta_a - \theta_b + \delta) = \frac{\frac{1}{\pi}\int_0^\pi \cos(2(\theta_a - \chi))\cos(2(\theta_b - \chi + \delta))\,d\chi}{\left[\left(\frac{1}{\pi}\int_0^\pi \cos^2(2(\theta_a - \chi))d\chi\right)\left(\frac{1}{\pi}\int_0^\pi \cos^2(2(\theta_b - \chi + \delta))d\chi\right)\right]^{(1/2)}} = \cos(2(\theta_a - \theta_b + \delta)), \tag{26}$$

which gives precisely the so-called "quantum result" obtained for the singlet state—and verified by direct point-by-point simulation without "quantum" type input.

4. CONCLUSIONS

The physical picture given immediately above of entanglement experiments, its mathematical rendition and its interpretation fit together without contradiction or artificial hypothetical input (e.g., a "projection hypothesis"). This constitutes a strong argument that, in fact the singlet state is just a convenient proxy for an ensemble, that is, it has some of the same characterizing statistical parameters of the ensemble, but does not represent any distinct ontic entity in the ensemble or otherwise. This constitutes significant rationalization of the total paradigm for the interaction of charged particles.†

Whether photons can be regarded as "unsplittable," pending credible experimental examination, can be taken still as an open question. Given the need for rococo embellishments on the current physical interpretation of photons, however, disproof of the claim that they cannot be split would be edifying.

In the larger picture, the analysis presented above for these two issues tends to support the view point that, interaction between electric charged particles is mediated neither by ontic waves nor by particulate photons. Furthermore, neither the wave nor the photon formulations lead to well posed, coupled equations of motion for the interaction of two (or more) charged particles, as does the direct interaction on light cones formulation.[2]

REFERENCES

[1] Kracklauer, A. F., "On the fundamental nature of the electromagnetic interaction," to be published in the *Proceedings, Vigier VII & British Computer Society Conference*, London, 15 Aug. 2012.

[2] Kracklauer, A. F. & P. T., "Electrodynamics: Action-at-no-distance?," in [Has the Last Word Been Said on Classical Electrodynamics?] Chubykalo, A. et al. (eds.), Rinton Press, Princeton, 118-132, 2004.

*Eq. (22) equals this expression under some symmetry conditions.

†Customarily this interaction is denoted "electromagnetic" as if it involved two aspects. However, magnetic effects can be seen just as a means of taking full account of the consequences of propagation time of the electric interaction (in other words: that the speed of light is finite). Setting $c \to \infty$, for example, annihilates magnetic fields in Maxwell's equations.

[3] Kracklauer, A. F., "Non-locality, Bell's Ansatz and Probability," *Optics and Spectroscopy* **103** (3) 466-475 (2007);

[4] Kracklauer, A. F., "On the nature of information erasing," *J. Mod. Optics*, **54**(16-17) 2365-2371 (2007).

[5] Einstein, A., "Quanten-Mechanik und Wirklichkeit," *Dialectica,* **2** 320-326 (1949).

[6] Mizrahi, S. S. and Moussa, M.H.Y., "Einstein-Podolsky-Rosen Correlation for Light Polarization," *Int. J. Mod. Phys.* **7** (5) 1321-1330 (1993).