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Is ‘entanglement’ always entangled?

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Abstract
Entanglement, including ‘quantum entanglement’, is a consequence of correlation between objects. When the objects are subunits of pairs which in turn are members of an ensemble described by a wavefunction, a correlation among the subunits induces the mysterious properties of ‘cat-states’. However, correlation between subsystems can be present in purely non-quantum sources, thereby entailing no unfathomable behaviour. Such entanglement arises whenever the so-called ‘qubit space’ is not afflicted with Heisenberg uncertainty. It turns out that all optical experimental realizations of the Einstein, Podolsky and Rosen (EPR) Gedanken experiment in fact do not suffer Heisenberg uncertainty. Examples will be analysed and non-quantum models for some of these described. The consequences for experiments that were to test EPRs contention in the form of Bell’s theorem are drawn: valid tests of EPR’s hypothesis have yet to be done.

Keywords: Entanglement, non-locality, EPR correlations, Bell’s theorem, quantum mechanics

1. Introduction

The above title needs ‘disentangling’. The quantum wavefunction of entangled, i.e. of correlated subsystems, cannot be written as the product of the wavefunctions for the subsystems. Likewise, the probability of correlated events cannot be written as the product of probabilities for two independent events. The latter fact is elementary and very well understood; it presents absolutely no mystery, but in contrast, the same fact is utterly impenetrable in quantum mechanics (QM).

What is the difference?

It arises from the following considerations. In probability theory, the probability for joint events is given in general by Bayes’ formula

$$P(a, b) = P(a)P(b|a) \tag{1}$$

where $P(b|a)$ is the conditional probability that the event b occurs given that event a has been seen [1]. When the two events are correlated, in other words are not statistically independent, then (1) cannot be written as the product of independent probabilities, $P(a)P(b)$, that is, they are ‘entangled’. This is a statement about the *knowledge* that the observer has about the *joint* events; it is an *epistemic* statement, and, as such, the dependence of $P(b|a)$ on a is devoid of communicative implications.

Now, in QM, according to the Born interpretation, the modulus squared of a wavefunction, i.e. $\psi^*(x)\psi(x)$, is the probability that the object to which it pertains will be found in the infinitesimal volume d^3x. This straightforward concept is complicated, however, by the peculiarity of QM, namely, a wavefunction is known empirically to diffract at boundaries just like water or electromagnetic waves and this seems to make sense only if wavefunctions have ontic substance. In turn, this appears to vest a causative relationship into conditional probabilities computed from wavefunctions for correlated events. That is, if wavefunctions are onta, then when a measurement collapses one member of a correlated pair, then the onta of the other member must likewise instantly collapse also, even if it is located at a space-like displacement—in contrast to the fundamental precept of special relativity that no physical interaction can transpire faster than the speed of light. On the other hand, were a wavefunction only a symbol for information, as are expressions in probability theory, such a collapse would not violate physics precepts. In short, entanglement$_{\mathrm{QM}}$ is somehow ontic, but entanglement$_{\mathrm{Prob}}$ is epistemic. In this light the title is: *Is (in the microscopic domain) entanglement$_{Prob}$ always entangled$_{QM}$?* The purpose of this paper is to argue that in virtually all of the crucial experimental tests of Bell’s theorem that the answer is: no!

Born’s interpretation of the wavefunction has led many, in particular Einstein, Podolsky and Rosen (EPR), to argue that the necessity for probabilistic concepts in QM arises because the theory is limited fundamentally by ignorance; i.e. that QM should be 'extendable,' at least in principle, so as to encompass the heretofore missing information, perhaps using 'hidden variables'. The tactic taken by EPR was to show that 'Heisenberg uncertainty' (HU) is not something novel, that is, that basic logic regarding correlated objects demands that the missing information be due to simple ignorance. This they did by considering the symmetrical disintegration of a stationary particle into twin daughters. For each separate daughter, the HU principle implies that both the position and momentum cannot be simultaneously known to arbitrary precision. Some go on to argue that this is so, because they in fact do not exist simultaneously. EPR countered, arguing (in the author's rendition) that in the case of such a disintegration one can measure the position of one daughter and the momentum of the other to arbitrary precision, and thereafter call on symmetry to specify to equal precision the momentum of the first and position of the second. What can be specified in principle to arbitrary precision, EPR argued, must be an 'element of reality' that enjoys the status of onta. In any case, EPR intended that their Gedanken experiment should expose the true character of HU, that in the end it could evidence ignorance, not something fundamentally new [2].

For the purposes of an experimental realization of EPR's Gedanken experiment, however, the difficulties in finding a suitable particle source of the sort envisioned, are daunting. Thus, Bohm proposed a change of venue; instead of momentum–position, he suggested using the (anti)correlated spin-$\frac{1}{2}$ states of daughter particles derived from the disintegration of a boson [3]. His motivation, apparently, was that it should be easier to construct an appropriate source, and easier to measure the dichotomic values of the daughters. Ultimately, this proposal too, turned out to be impractical, but the algebraically isomorphic situation with polarized 'photons' from a cascade transition, or from parametric down conversion is workable, and several such experiments have been done [4].

2. Entanglement$_{\mathrm{QM}}$ versus entanglement$_{\mathrm{Prob}}$

The fundamental premise of this paper is that Bohm's transfer of venue introduced a major error. It is the following: the space of the variables for either spin or polarization, in contrast to phase space where EPR formulated their Gedanken experiment, is not afflicted with HU. There is no HU in the plane of the spin or polarization vector. Neither $\{E_x, E_y\}$ nor $\{\sigma_x, \sigma_y\}$ are Hamiltonian canonically conjugate variables; their creation and annihilation operators commute. Anti-commutation of spin operators arises here for the same reason it does for angular momentum operators in classical mechanics, i.e. for geometrical vice dynamical reasons. Thus, while they do share some of the characteristics of the variables of phase space, they do not share the one relevant for the argument of EPR. Where there is no HU, there is no QM, all is ruled by classical physics.

This fact has a number of immediate consequences, the most salient of which is that probabilities of these variables do not exhibit the quantum phenomena that ultimately demands that QM probabilities have an ontic character. This means, in particular, that conditional probabilities of these variables do not imply causality. Thus, Bell's argument, that because there is to be no *causal* relationship between the two detection events, the probability relationship between them cannot take the form

$$P(a, b) = \int P(a|b, \lambda))P(b|\lambda)P(\lambda)\,\mathrm{d}\lambda \tag{2}$$

which, in turn, implies that (1) must read

$$P(a, b|\lambda) = P(a|\lambda)P(b|\lambda) \tag{3}$$

which does not follow for this class of experiments, because, in fact there need be no causative link between these variables. In other words, Bell's encoding of 'locality' with respect to these variables is not justified in these circumstances [5]. A conditional probability involving a state of polarization as a 'condition' is an epistemic statement about the state of knowledge, not an ontic statement about EPR's 'elements of reality'. A correlation here indicates no more than that both daughters share a common cause in the intersection of their past light cones. In short, statements about joint probabilities between such states do not imply the existence of superluminal causal relationships; the non-factorizability of their wavefunction is no more problematic than that of probabilities of correlated events. Insisting nonetheless, on the validity of (3), therefore, is equivalent to precluding all correlations.

Moreover, independent of Bohm's displacement, whatever is correlated cannot be encoded by 'hidden variables', λ, without also being 'visible' in the instrument readings a and b, because if it is not so visible, it cannot be a matter of concern to observers. Bell's reasoning would have been standard statistical analysis had it pertained to the case in which a new 'λ-meter' could be introduced into the experiment; instead, it is an effort to infer the existence of hidden λ, using only the a- and b-meters. That is to say, as a statistics proposition, (3) is dubious on its own merits. See [6] for further incisive and original analysis of this very point[1].

3. Non-quantum models of EPR-Bohm (EPR-B) experiments

In view of the facts developed above, which imply that experiments exploiting polarization that are intended to test EPR (or Bell inequalities), in so far as they are not cast in a space suffering HU, should be modelable classically. This is indeed the case, and the most common types of EPR-Bohm (EPR-B) experiments are presented below. These include those based on polarization and a second category in which orthogonality of the signals is achieved by other means, usually as pulses with a phase offset. This latter category includes the 'Franson'-, 'Ghosh and Mandel'- and 'Suarez–Gisin'-type experiments.

3.1. 'Clauser–Aspect'-type experiments

In these experiments the source is a vapour, typically of mercury or calcium, in which a cascade transition is excited by either an electron beam or an intense radiation beam of fixed

[1] I thank Barry Schwartz for this reference.

orientation. Each stage of the cascade results in emission of radiation (a 'photon') that is polarized orthogonally to that of the other stage. In so far as the sum of the emissions can carry off no net angular momentum, the separate emissions are antisymmetric in space. The intensity of the emission is maintained sufficiently low so that at any instant the likelihood is that radiation from only one atom is visible. Photodetectors are placed at opposite sides of the source, each behind a polarizer with a given setting. The experiment consists of measuring the coincidence count rate as a function of the polarizer settings [7, 8].

A model consists of simply rendering the source and polarizers mathematically, and a computation of the coincidence rate. Photodetectors are assumed to convert continuous radiation into an electron current at random times with Poisson distribution but in proportion to the intensity of the radiation. The coincidence count rate is taken to be proportional to the fourth-order coherence function evaluated at the detectors.

The source is assumed to emit a double signal for which individual signal components are anticorrelated and, because of the fixed orientation of the excitation source, confined to the vertical and horizontal polarisation modes; i.e.

$$S_1 = \left[\cos\left(n\frac{\pi}{2}\right), \sin\left(n\frac{\pi}{2}\right)\right]$$
$$S_2 = \left[\sin\left(n\frac{\pi}{2}\right), -\cos\left(n\frac{\pi}{2}\right)\right] \tag{4}$$

where n takes on the values 0 and 1 with an even, random distribution. The transition matrix for a polarizer is given by

$$P(\theta) = \begin{bmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) \\ \sin(\theta)\cos(\theta) & \sin^2(\theta) \end{bmatrix} \tag{5}$$

so the fields entering the photodetectors are given by

$$E_1 = P(\theta_1)S_1$$
$$E_2 = P(\theta_2)S_2. \tag{6}$$

Coincidence detections among N photodetectors (here $N = 2$) are proportional to the single-time, multiple location second-order cross correlation [9], i.e.

$$P(\theta_1, \theta_2, \ldots \theta_N) = \frac{< \prod_{n=1}^{N} E^*(r_n, \theta) \prod_{n=N}^{1} E(r_n, \theta) >}{\prod_{n=1}^{N} < E_n^*(r_n) E_n(r_n) >} \tag{7}$$

where $< \cdot >$ indicates an average over the ensemble. It is easy to show that the denominator consists of constant factors of the form $(\cos^2(a) + \sin^2(a))$ so that it equals 1. The final result of the above is

$$P(\theta_1, \theta_2) = \tfrac{1}{2}\sin^2(\theta_1 - \theta_2) \quad \text{or} \quad \tfrac{1}{2}\cos^2(\theta_1 - \theta_2) \tag{8}$$

where the first variant pertains to correlations between symmetric detectors and the latter asymmetric, in the usual way. This is immediately recognized as the so-called 'quantum' result for the coincidence rates, which yield correlations that violate Bell inequalities. (Of course, it is also Malus' law, thereby being in total accord with the premise of this paper.)

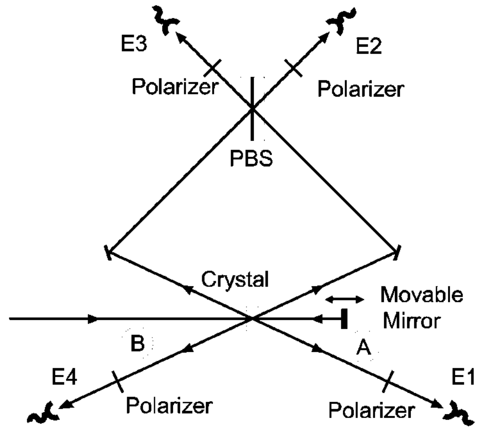

Figure 1. Schematic of the experimental set-up for the measurement of four-photon GHZ correlations. A pulse of laser light passes a nonlinear crystal twice to produce two entangled photon pairs via parametric down conversion. Coincidences between all four detectors are used to study the nature of quantum entanglement.

3.2. *'GHZ' experiments*

A number of proposed experiments involving more than two particles, many stimulated by analysis of Greenburger, Horne and Zeilinger (GHZ), are expected to reveal QM features with particular alacrity [10]. One of the most recent, which has the great virtue of being experimentally realizable, is that performed by Pan *et al* [11]. (See figure 1.) Two independent signal pairs are created by down-conversion in a crystal pumped by a pulsed laser. The laser pulse passes through the crystal creating one pair, which is then reflected off a movable mirror to repass through the crystal in the opposite direction creating a second pair. One signal from each pair is fed directly through polarizers to photodetectors (signals A_1 and B_1). The other signals from each pair, A_2 and B_2, are directed to opposite faces of a polarizing beamsplitter (PBS), (e.g., a beamsplitter which reflects vertically and transmits horizontally polarized signals) after which the signals are passed through adjustable polarizers into photodetectors. The path lengths of signals 2 and 3 are adjusted to compensate for the time delay in the creation of the pairs. By moving the mirror, the compensation can be negated to permit studying the coincidence dependence on the degree of interference caused by simultaneous 'cross-talk' between channels 2 and 3.

The principle results reported in [11] are the following. Of all the 16 possible regimes setting: $\theta_i = 0$ or $\pi/2$, only $\{0, \pi/2, \pi/2, 0\}$ and $\{\pi/2, 0, 0, \pi/2\}$ yield a (substantial) four-fold coincidence count, C; the regime $\{\pi/4, \pi/4, \pi/4, \pi/4\}$ occurs with an intensity $C/4$ and the regime $\{\pi/4, \pi/4, \pi/4, -\pi/4\}$ with zero intensity. Further, both of the latter regimes yield an intensity of $C/8$ when the time between pair creation is so large that that there is no 'cross-talk' between channels 2 and 3. (Actually, the data deviate from these values by small amounts attributable to noise.)

Equation (7) was implemented as follows: the crystal is assumed to emit a double signal for which individual signal components are anticorrelated and confined to the vertical and

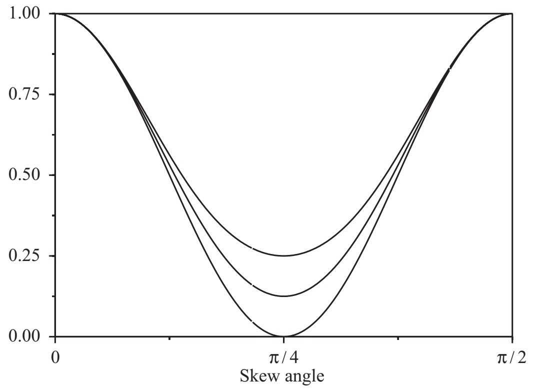

Figure 2. The upper curve shows the effect on the intensity of four-fold coincidences of skewing (rotating) all polarizers through a given angle in units of π-radians starting from the state $\{\pi/2, 0, 0, \pi/2\}$. The lower curve shows the same effect when one of the polarizers is rotated in the opposite direction. The middle curve shows the effect of either of these skewing schemes when the timing is such that the crossover signals do not arrive simultaneously with the reflected signals. Note that the values at $\pi/4$ coincide with those observed. This diagram differs from figure 4 in [11] in that it shows the split of these regimes as a function of polarizer skew for fixed delay rather than as a function of delay for fixed skew.

horizontal polarization modes; i.e.

$$\begin{aligned} A_1 &= \left[\cos\left(n\frac{\pi}{2}\right), \sin\left(n\frac{\pi}{2}\right)\right] \\ A_2 &= \left[\sin\left(n\frac{\pi}{2}\right), -\cos\left(n\frac{\pi}{2}\right)\right] \\ B_1 &= \left[\cos\left(m\frac{\pi}{2}\right), \sin\left(m\frac{\pi}{2}\right)\right] \\ B_2 &= \left[\sin\left(m\frac{\pi}{2}\right), -\cos\left(m\frac{\pi}{2}\right)\right] \end{aligned} \tag{9}$$

where n and m take the values 0 and 1 with a flat random distribution. The PBS is modelled using the transition matrix for a polarizer, (5), where $\theta = \pi/2$ accounts for a reflection and $\theta = 0$ a transmission. Thus the final field impinging on each of the four detectors is

$$\begin{aligned} E_1 &= P(\theta_1)A_1 \\ E_2 &= P(\theta_2)(P(0)B_2 - P(\pi/2)A_2) \\ E_3 &= P(\theta_3)(P(0)A_2 - P(\pi/2)B_2) \\ E_4 &= P(\theta_4)B_2 \end{aligned} \tag{10}$$

which, using (7), does not result in a simple expression. However, it can be easily numerically computed to obtain the same results as reported by Pan *et al*, or extended to other regimes, such as that shown in figure 2.

The splitting or separation of the curves at a skew angle of $\pi/4$ as shown in figure 2 is interpreted (see [11]) as evidence of the quantum character of these four-fold coincidences. In fact it is even taken to exhibit 'teleportation'. Of course, the existence of a classical model for these coincidences, in particular the splitting, puts sharp doubt on that claim. As Bell inequalities for four-fold experiments have not been derived (presumably they would have 16 terms and be quite ungainly), it is not convenient to try to discriminate between quantum and classical behaviour on that basis (ignoring for the moment, the issue of the validity of (2)). In any case, because the model described above is at the more basic logical level of classical physics than the 'no-go theorems', they cannot impugn its validity. The model is 'up stream', as it were, and constitutes a counter-example to such no-go theorems[2].

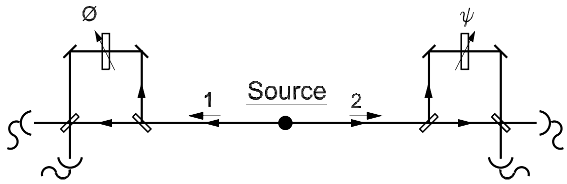

Figure 3. In a 'Franson' type experiment two identical pulses are directed through two interferometers, each comprised of a short path and a long path in which there is an additional adjustable phase shifter. By using fast coincidence comparison detectors, coincidences between pulses that traversed unequal paths can be excluded. The resulting interference is a function of the adjustable phase shifters.

3.3. 'Franson' experiments

Experiments of this type exploit phase shifts between pulses in the form of time offsets to define the orthogonal states played by the two states of polarization in the set-ups described above [13]. The original 'Franson' experiment measures the correlation between two detectors positioned after interferometers which divide identical incoming pulses, such that half take a short route and half take a long route which includes an adjustable delay. (See figure 3.)

There are two direct ways of modelling this set-up classically. One would be to write out terms for the long- and short-route pulses that had a time-separated modulation or time-limited coherence. Such separated pulses, when multiplied together and integrated, give zero, because regions where they are finite do not overlap, thereby fulfilling the definition of orthogonality in a Hilbert space sense. This approach has the disadvantage of leading to ungainly expressions. A much simpler tactic is to assign the signals in the long and short paths to orthogonal dimensions of a vector space; the resulting calculations are then transparent and devoid of irrelevant, gratuitous complexity. For example

$$\begin{aligned} E_l &= [\exp(-\mathrm{i}(kx - \omega t) + \phi), \exp(-\mathrm{i}(kx - \omega t))]/2^{3/2} \\ E_r &= [\exp(-\mathrm{i}(kx - \omega t) + \psi), \exp(-\mathrm{i}(kx - \omega t))]/2^{3/2} \end{aligned} \tag{11}$$

where ϕ and ψ are the extra phase shifts introduced in the long paths; one factor of $1/\sqrt{2}$ is the normalization, and two more are due to the beamsplitters. Then, using (7), with the convention that the tensor product be replaced by a vector inner product; i.e.

$$P(\phi, \psi) = \frac{(E_r^* \cdot E_l^*)(E_l \cdot E_r)}{(E_r^* \cdot E_r)(E_l^* \cdot E_l)} \tag{12}$$

[2] Although not necessary for the argument herein, we nonetheless hold that Kochen–Specker-type theorems which do not involve inequalities and that seemingly cover multi-fold GHZ coincidences are afflicted with assumptions that do not conform to the physics in EPR-B experiments. See [12].

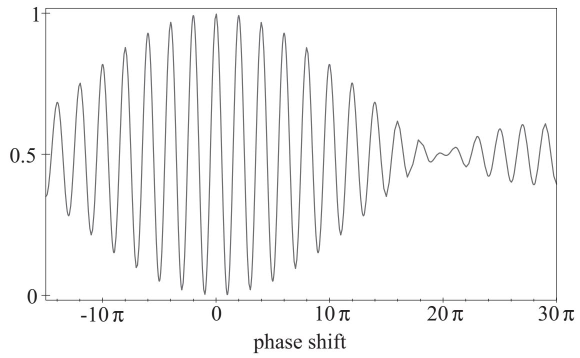

Figure 4. Plot of the relative interference intensity pattern as a function of phase shift (in units of π) in one arm in a Brendel-type experiment. This curve closely matches that observed by Brendel *et al* in an experiment in which the total spread was 10% of the pulse carrier frequency; as a result, the modulation curve node occurs at approximately 20π, as was observed.

(to algebraically enforce the orthogonality between pulses in the calculations that phase shifts enforce in the experiment) directly gives the observed correlation as a function of the phase shifts:

$$P(\phi, \psi) = \tfrac{1}{8}(1 \pm \cos(\phi - \psi)) \tag{13}$$

which exhibits the oscillations with 100% visibility characteristic of idealised versions of these experiments. (The plus sign in (13) holds for symmetric detector paths, the minus for asymmetric.)

'Ghosh–Mandel'-type experiments are a variation of the 'Franson' version in which the phase shift is achieved by path-length differences instead of time offsets; otherwise, the formulae are structurally identical [14].

3.4. *'Brendel' experiments*

In the above experiment the radiation source was taken to be ideal, that is, it produced two signals of exactly the same frequency with no dispersion. In some experiments, such as described in [15], the source used was a nonlinear crystal generating two correlated but not necessarily identical pulses, which satisfy 'phase matching conditions' so that if one signal in frequency is above the mean by s (spread), the other is down in frequency by the same amount. This leads to an additional phase shift at the detectors which is also proportional to those already there; i.e. $s\phi$ and $-s\psi$, so that

$$\begin{aligned} E_r &= [\exp(-\mathrm{i}(kx - \omega t) + \psi(1+s)), \exp(-\mathrm{i}(kx - \omega t)] \\ E_l &= [\exp(-\mathrm{i}(kx - \omega t) + \phi(1-s)), \exp(-\mathrm{i}(kx - \omega t)]. \end{aligned} \tag{14}$$

Since the value of s is different for each pulse (photon) pair, the resulting signal is an average over the relevant values of s:

$$\frac{1}{2s}\int_{-s}^{s} P(\phi, \psi, s)\,\mathrm{d}s \tag{15}$$

where $P(\phi, \varphi, s)$ is computed as for 'Franson' experiments. The final result closely matches that observed by Brendel *et al*. (See figure 4.)

3.5. *'Suarez–Gisin' experiments*

In experiments of this type, one of the detectors is set in motion relative to the other. By doing so with appropriately chosen parameters, it is possible to arrange the situation such that each detector precedes the other in its own frame [16, 17]. Thus, not only is the 'collapse' of the wavepacket 'nonlocal', it occurs such that there is also 'retrocausality'. In the model proposed herein, however, this complication (paradox) cannot arise in the first instance. All the properties of each pulse are determined completely at the common point at which the signals are generated. Properties measured at one detector in no way determine those at other detectors, regardless of the order in which an observer receives reports of the results from various detectors, and regardless of what conditional probabilities the observer might write to describe their hypothetical or real knowledge.

4. Conclusion

The model or explanation of the experiments described above is fully classical. It uses no special property peculiar to QM. The two states in these experiments (polarization or phase-displaced pulses) are not canonically conjugate dynamical variables; they do not, therefore, exhibit HU, and the model does not bring any in. The essential formulae are a straightforward application of second-order (in intensity) coherence theory, which is really just a generalization of wave interference. That this model faithfully describes the outcomes of these experiments, in addition to being a counter-example to claims that these experiments cannot be clarified using non-quantum physics, is a demonstration that they are not relevant to EPR's argumentation, and therefore, to date no such experiment could have established that non-locality has a role to play in the explanation of the natural world. It shows that there is no justification for ascribing an ontic meaning to conditional probabilities in the circumstances of these experiments, which, in turn, undermines the rationale for Bell's encoding of non-locality. When his encoding is withdrawn, no Bell inequality can be extracted. A manifestation of this fact is that the coincidence probabilities used in all these models violate Bell inequalities, which, of course they should so do, because the derivation of the inequalities depends on the unintentional implicit assumption, contrary to fact, that there is no correlation [6].

There are, of course, two arenas where HU is in evidence: phase space and 'quadrature space'. In principle, a test of EPR's contentions formulated in these arenas could show different results—at least in so far as it would not rely on Bell inequalities.

To a large extent, the model proposed herein is 'obvious'. It might be asked: why then has it not been proposed long ago? The answer involves issues resulting from the perceived need to maintain an ontic ambiguity with respect to the identity of wavefunctions until the moment of measurement, at which time this identity ambiguity is resolved by a 'collapse'. This need results from the tactic of describing particle beams with wavefunctions in order to account for their wave-like diffraction. That is, the wave-like navigation of particle beams in combination with their incontestable particle-like registration in detectors, has been explained, or at least encoded, calling on 'dualism', 'wave-collapse' and so on[3]. The experiments described herein, however, employ optical phenomena for which there is no need to invoke a particulate character. Wave beams diffract naturally. And, particulateness in photodetectors can be, indeed must be, attributed to the fact that they, because of the discrete nature of electrons, convert continuous radiation into a digitized photocurrent. The conceptual contraptions of 'duality' and 'collapse' are just not needed to explain the behaviour of radiation beams, even correlated sub-beams. There is no reason why these experiments could not be carried out in spectral regions in which it is possible to track the time development of electromagnetic fields, thereby avoiding the peculiarities of photodetectors. In fact, for simple 'Clauser–Aspect'-type set-ups, this has been done [19]. The results conform with ours and show that classical optics is not taxed to clarify EPR-B correlations.

Note: An electronic file with MAPLE routines for the above is available upon request.

References

[1] Feller W 1950 *An Introduction to Probability Theory and its Applications* (New York: Wiley) ch 5

[2] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[3] Bohm D 1951 *Quantum Theory* (New York: Prentice-Hall)

[4] Afriat A and Selleri F 1999 *The Einstein, Podolsky and Rosen Paradox* (New York: Plenum)

[5] Bell J S 1987 *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press)

[6] Jaynes E T 1989 *Maximum Entropy and Baysian Methods* ed J Skilling (Dordrecht: Kluwer) pp 1–29 (http://bayes.wustl.edu./etj/articles/cmysteries.ps)

[7] Clauser J F and Horne M A 1974 *Phys. Rev.* **D10** 526

[8] Aspect A, Grangier P and Roger G 1981 *Phys. Rev. Lett.* **47** 725
Aspect A, Grangier P and Roger G 1982 *Phys. Rev. Lett.* **48** 91

[9] Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press) p 447

[10] Greenburger D M, Horne M and Zeilinger A 1989 *Bell's Theorem, Quantum Theory and Conceptions of the Universe* ed M Kafatos (Dordrecht: Kluwer) p 73

[11] Pan J-W, Daniell M, Gasparoni S, Weihs G and Zeilinger A 2001 *Phys. Rev. Lett.* **86** 4435

[12] Kracklauer A F 2000 *Ann. Fond. L. de Broglie* **25** 193

[13] Franson J D 1989 *Phys. Rev. Lett.* **62** 2205

[14] Ghosh R and Mandel L 1987 *Phys. Rev. Lett.* **59** 1903

[15] Brendel J, Mohler E and Martienssen W 1992 *Europhys. Lett.* **20** 575

[16] Gisin N, Scarani V, Tittle W and Zbinden H 2000 *Preprint* quant-ph/0009055

[17] Suarez A 1997 *Phys. Lett.* A **236** 383

[18] Kracklauer A F 1999 *Found. Phys. Lett.* **12** 441

[19] Evdokimov N V, Klyshko D N, Komolov V P and Yarochkin V A 1996 *Physics–Uspekhi* **39** 83

[3] See [18] for a more extensive discussion and local realistic model of particle beam dualistic behaviour.