Sagnac: a laboratory for Special Relativity

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Abstract

The Sagnac experiment is reanalyzed from the point of view that attributes all time dilation and Lorentz-Fitzgerald contraction to being only "apparent" effects, i.e., they exist only in the eye, as it were, of an observer, not as material modifications of the observed object.

1 Introduction

All Einstein's writings preceding his 1905 paper on the electrodynamics of moving bodies,[1] and in virtually all papers for the following 20 years, can be contrasted with those of most other writers, as follows: His introduction sections are long, up to a third of the paper, and based on, or motivated by, some issue directly related to material circumstance or experimental results, and which are discussed on a very intuitive and usually simple level. Moreover, when describing calculations, it is absolutely clear that Einstein virtually never sets up a mathematical formulation of a problem only to subsequently turn off his physical intuition and "turn the crank" mathematically. He seems always to know what each term represents physically and has an idea of its relative magnitude. These characteristics pertain also to his 1905 paper introducing his rendition of what has become known as "Special Relativity," that is, they do up to the last two paragraphs of Section 4, wherein he introduces the "clock," or what thereafter was dubbed the "twin," paradox.

Until those two paragraphs every reference to a dilated time or contracted space interval is described as "apparent." Suddenly, at this point, however, he abandons this term, and his style of argumentation, and simply considers a time dilation not to be an "appearance" but an actual (ontic) modification of the observed object, in that instance, a clock. In other words, it is for him, suddenly in these two paragraphs for the first time, not an issue derived from arrival times of light signals at the eye of an observer whose motion with respect to the clock is responsible for dilation, but actually and factually a dilation of time itself as the clock experiences it before, presumably, any signal is physically emitted toward an observer, where, by cause of the relative motion and the time taken for the transmission to reach the observer, the arrival time intervals could differ from signal departure time intervals. He clearly takes it, that by cause of motion a traveling clock suffers time dilation which is accumulated physically and observable subsequently by comparison with a stationary clock without mediation of light signals.

While one can speculate nowadays how these two paragraphs were inserted—possibly as a result of influence by a second party, his wife, for example—in fact we cannot now know. What we do know, however, is that by 1918 his discomfort with the idea of ontic aberrations was still sufficient to cause him to rationalize it, but then with attribution to an entirely different hypothetical foundation, namely gravity, which entered into his reasoning only well after he found cause to propose the possibility of ontic time contractions for moving clocks in the first place.[2][1]

Nowadays it would seem to be appropriate, therefore, to explore the possibility, that at the point of these two paragraphs, Einstein went astray. It would be useful to determine if the alternate interpretation which denies ontic time dilations and space contraction can be carried out throughout all the applications of relativity theory consistently. Whatever the facts, they will not be known until this possibility is thoroughly and carefully examined; and, it is the intention herein to carry just such analysis another step forward. We say "another step," because this writer has already taken the view point in which time dilutions and space contractions are merely aberrations affecting perception, not modifications of moving

1. See also Ref. [3]

objects, either clocks, twins or whatever, and, with this as basis, found a very sober and sensible resolution for the "twin" paradox: the twins do not age factually differently, only their reports sent by means of electromagnetic signals to each other are in disagreement.[4, 5] This is effect is analogous to the common experience with tourists, who often return home before the post cards they sent underway have arrived.

In the next section the formalism most convenient for applying this view-point will be developed. Then in the following section this formalism will be applied to the Sagnac experiment. Finally, we shall draw some tentative conclusions.

2 Special Relativity as the structure of observation

A motivation or basis of the "epistemological," vice "ontic," interpretation of time dilation and space contraction can be found in the mathematical structure used in Special Relativity, namely the well known Lorentz-Minkowski pseudo-Riemann manifold. The currently common understanding of this structure is very strongly affected by its historical roots in studies to understand the electrodynamics of moving bodies. This situation, especially among physicists, obscures the fact that this structure can be developed as a logical or mathematical construct from abstract hypotheses without reference to the material world.

Such a program has been carried out by various mathematicians[6][7] and their results can be summarized thus:

Let us consider space as simply the capacity for extension. As philosophy this is regrettably vague; but our need, and intention, here is not to formulate a complete and closed formal logical structure, but just to find algorithms leading to unambiguous, self consistent and empirically verified calculations. Let us take this 'capacity for extension' simply as we experience it the first instance, i.e., as a three dimensional Euclidean vector space, the points of which, in what the mathematicians consider a "coordinate free" sense, are pure locations independent of any labeling system. Time, then, is taken as independent of this 'space,' essentially as what is usually denoted "absolute time."

Now let us imagine that there is a "Green's function" source possible at each of the spacial points; that is, there is an infinitesimal burst of light emitted at this point which subsequently spreads as a spherical shell into space at the speed of light. In what follows, these "Green's bursts" shall be imagined to occur only at "events," i.e., at points in space at a particular instant of absolute time, of interest, that is of utility for the analysis of a phenomenon of interest. It follows at once, of course, that given a "snap shot" of space containing multiple spheres, the relative time in an absolute sense of the source events is proportionally directly to the radii of the spheres. In this space with its Green's spheres the concept of velocity is meaningless because the events are all unrelated to each other. In order to define velocity, it is first necessary to relate the events to each other in reference to some persisting, stable item, a particle, say. This will be done below in connection with the analysis of particular phenomena.

With these imaginary constructs now, the essential tactic of applying the view point that all time dilutions and space contractions are apparitions, can be realized. It consists in answering the following question: how does an observer at some location in 3-space perceive the passage of a Green's sphere?

First note that an observer is exactly an "object" to which one can attribute a "velocity" in terms of relating a string of primary events We take it, that such an observer has, and can determine, a velocity with respect to the event source of the momentarily passing Green's sphere. An infinitesimally small patch of the expanding sphere at the point of the observer can be characterized by a wave-vector, rooted at the observer's position, pointing radially outward with respect to the shell and with an arbitrary frequency or wave number and associated to an amplitude inversely proportional to the sphere's radius. Such a wave vector can be though of as that for a Fourier component for real signals to be constructed for the particular application. For the observer, this wave vector represents an incoming signal, and, as such, is subject to exactly the aberration considered in the first paragraph of this section.

3 Analysis of the Sagnac experiment

In previous publications this writer has presented analysis of the famed "twin" paradox from the point of view in which it is maintained that time dilations and space contractions are only "apparent" effects. The prototypical trip considered in discussing this paradox is a realization of linear and constant motion. The simplicity of the associated complex of ideas invites the conclusion that the applied principles are advantageous for this problem only because of the simplicity, that they are not of general value. To determine if this is indeed so, more complicated circumstances must be treated.

For this purpose, the Sagnac Effect is ideal. It involves uniform circular motion, and as such, it represents the next higher level of complexity. Success analyzing this effect will advance the credibility of this approach.

The basic setup for this experiment is as follows. A light source and detector are mounted on a rotatable table such that a light beam from the source is split into two beams, which, by means of mirrors, are sent around the periphery of the table in both directions, after which they are brought together in the detector to observe the interference pattern. The experiment consists of comparing the interference pattern seen when the table is stationary with that seen when the table is rotating. This comparison is particularly interesting because at first blush it would seem that as both the source and detector maintain a constant geometrical relationship with each other, rotation as experienced and observed on the table, should have no effect. But in fact it does have an effect, and the question is: why? Our goal here is not to answer this question but only to determine whether mathematical analysis on the basis of the epistemological interpretation yields correct, that is empirically verified, results in the form of algorithms or calculations.

Now, let us analyze the signal at the detector in terms of the passage of "Green's spheres." For the sake of simplicity, let us suppose that there are, in addition to the source-detector, just two additional mirrors, so that the optical or light path consists of three legs. If the table is not rotating, then clearly the light "going" both ways will traverse the same length path and a particular interference pattern for this circumstance can be recorded. When the table rotates, however, the matter is complicated by the fact that during the time of flight of the light from the last mirror, the detector advances by cause of rotation, the length of the leg with respect to an "absolute" event space will be lengthened in the direction of rotation and shortened in the other case. The following figure (Fig. 1) shows the geometry of a single optical path link and establishes the notation that shall be used below.

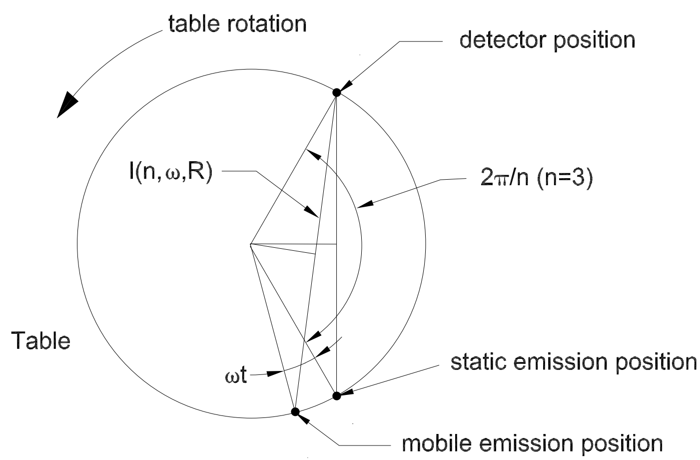

Figure 1.

When the table is rotating, the co-moving signal arriving at the detector did not originate from a point on the circle at an angle of $2\pi/n$ (where n is the number of legs comprising the total optical path length) radians clockwise backwards, rather at a point even further back by an amount sufficient to allow for the travel time of the pulse to get to the detector, a time interval which depends on the size of the radius and on the angular velocity of rotation of the table, ω. Given these parameters, we can write the equation:

$$l = 2\mathrm{R}\sin(2\pi/2n + \omega t/2), \tag{1}$$

where l is the optical path length of one leg, R is the radius of the table and t is the time taken by light to cover the extended leg. Now $t = l/c$, so that Eq. (1) becomes:

$$l = 2\mathrm{R}\sin(\pi/n + \omega l/2c). \tag{2}$$

The same reasoning with respect to the counter rotating ray gives:

$$l' = 2\text{Rsin}(\pi/n - \omega l'/2c). \tag{3}$$

The solutions to these equations can be use to compute the difference in optical path length, $\Delta(R,n,\omega)$, between the two paths as a function of R,n,ω , i.e. of the parameters of an experimental setup. As such, any displacement of an interference pattern will depend on Δ, and is the most immediately observable quantity characterizing this phenomena.

Although there are additional modifications of the interface pattern, which shall be considered below, a comparison of the value determined by self consistently solving these equations with the results of the conventional solution, addresses, to some degree, the question of the validity of the reasoning leading to these equations, as least in so far as failure to agree would negate this reasoning because the conventional results have been verified partially empirically.

Let us make a numerical comparison for a set of feasible parameters for the experiment as conceived and executed by Sagnac, i.e., $n=3$, $R=1$m and $\omega \simeq 6$ (or: $\upsilon \simeq 1$, i.e., one revolution per sec.). Following the calculation will be done using Maxima.[2] Eqs. (1) and (3) are transcendental equations for which there is no known analytic solution and which, because of the small size of υ/c, make certain approximations either questionable or introduce doubt regarding the usefulness for our purpose; in other words, we see as advantageous to solve these equations numerically using Maxima.

3.1 Optical path length variation

First we set the immutable parameter, c (speed of light) and, an apparatus parameter: R (the radius of the rotatable table, taken to be one meter for convenience):

```
(%i1) c:3*10^8$ R:1$
```

Here, as a technical matter, we have to set the search limits for the "find_root" routine used below:

```
(%i3) a:2*R*sin(%pi/n)$ b:2*R$ d:0$
```

Eqs. (1) and (3), are encoded, where the angle addition due to rotation of the table equals ωt, so that $l=ct$ or $t=l/c$. This is achieved by defining a function for which there is a prebuilt Maxima routine. This can done with the following statement: opl$(n,\upsilon)\triangleq$find_root(l- $2R\sin(\pi/2-2\pi\upsilon l/2c), l, a, b)$ a , b), where a and b are the limits between which the algorithm seeks the root of the expression within the parenthesis. The counter clockwise optical path length is analogously determined by opl$'(n,$ $\upsilon)\triangleq$find_root(l'- $2R$ $\sin(\pi/2-2\pi\upsilon l'/2c)$, l', a, b) c , d), where the limits c and d are chosen to fit the geometry. Finally the difference in the total optical path lengths, $y(n,\upsilon)$ is n times the difference of these two functions. In Maxima code:

```
(%i7) opl(n,nu):=find_root(l - 2*R*sin(%pi/n + l*2*%pi*nu/(2*c)),l,a,b);
      opll(n,nu):=find_root(ll - 2*R*sin(%pi/n - ll*2*%pi*nu/(2*c)),ll,d,a);
      y(n,nu):=n*(opl(n,nu) - opll(n,nu));
```

(%o6) $\text{opl}(n,\nu) := \text{find\_root}\left(l - 2R\sin\left(\frac{\pi}{n} + \frac{l\,2\pi\nu}{2c}\right), l, a, b\right)$

(%o7) $\text{opll}(n,\nu) := \text{find\_root}\left(\text{ll} - 2R\sin\left(\frac{\pi}{n} - \frac{\text{ll}\,2\pi\nu}{2c}\right), \text{ll}, d, a\right)$

(%o8) $y(n,\nu) := n\,(\text{opl}(n,\nu) - \text{opll}(n,\nu))$

2. http://maxima.sourceforge.net/docs.html

Since what is intended here is to compare this calculation with the customary one, we input the standard formula:

```
(%i10) w(n,nu):=4*R^2*2*%pi*nu*n*sin(%pi/n)*cos(%pi/n)/(c*(1-(R*2*%pi*nu/c)^2));
```

(%o9) $w(n,\nu) := \dfrac{4\,R^2\,2\,\pi\,\nu\,n\sin\left(\frac{\pi}{n}\right)\cos\left(\frac{\pi}{n}\right)}{c\left(1-\left(\frac{R\,2\,\pi\,\nu}{c}\right)^2\right)}$

```
(%i10)
```

Now, let us plot for various numbers of links, n, these two functions as functions of the velocity of the table rim, which, in terms of a factor, k, with the units of c and a dimensionless parameter for plotting, x, requires defining:

```
(%i11) k:c/(2*%pi)$
```

which leads to the following plot statement:

```
(%i12) plot2d([w(3,k*x),w(4,x*k),w(200,k*x), y(3,k*x),y(4,k*x),y(200,k*x)], [x,0,0.9],
       [xlabel, "Tangential table velocity in units of 'c'"],[ylabel, "Differential
       optical path length in meters"],
       [legend,"w(3)","w(4)","w(200)","y(3)","y(4)","y(200)"]  );
```

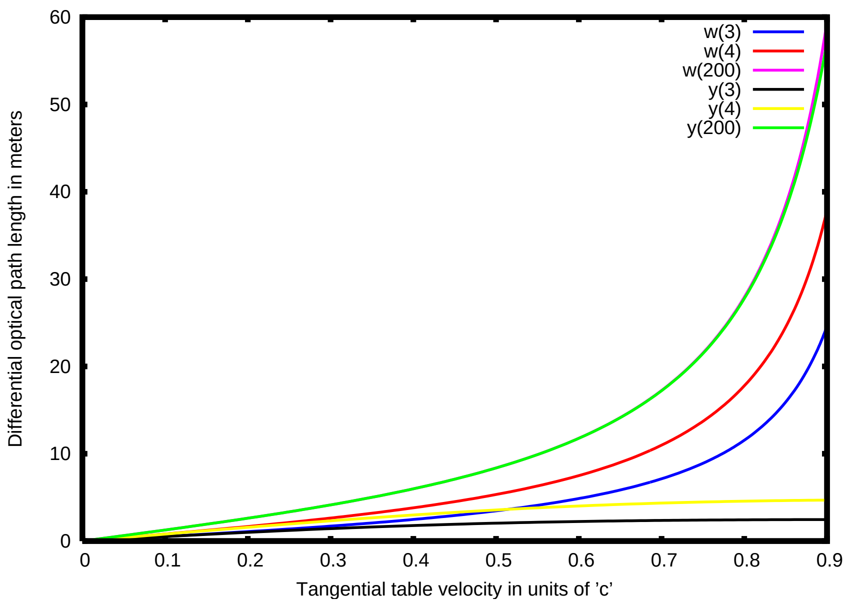

Figure 2. These curves reveal that the new paradigm seems to lead to a finite optical path length difference even when the table's rim speed equals the speed of light. This difference is most pronounced for setups involving a small number of links. For a setup involving a large number of links (here, 200, which closely approximates the customary curved mirror), the results appear to diverge only at speeds near that of light. Of note is, that in low speed, practical regimes, the results are essentially identical, which does not admit rejecting the new paradigm.

The coincidence of the curves in Figure 2 for the two paradigms involving 200 mirrors up to $0.9c$, leaves some ambiguity regarding what transpires near the speed of light, i.e., does the curve for $y(200)$ diverge? To help resolve this issue, let us plot the above curves between $0.9c$ and 0.99c.

```
(%i13) plot2d([w(3,k*x),w(4,x*k),w(200,k*x), y(3,k*x),y(4,k*x),y(200,k*x)],
       [x,0.9,0.99], [xlabel, "Tangential table velocity in units of 'c'"],[ylabel,
       "Differential optical path length in meters"],
       [legend,"w(3)","w(4)","w(200)","y(3)","y(4)","y(200)"],
       [gnuplot_term,ps],[gnuplot_out_file, "Fig3.eps"]);
```

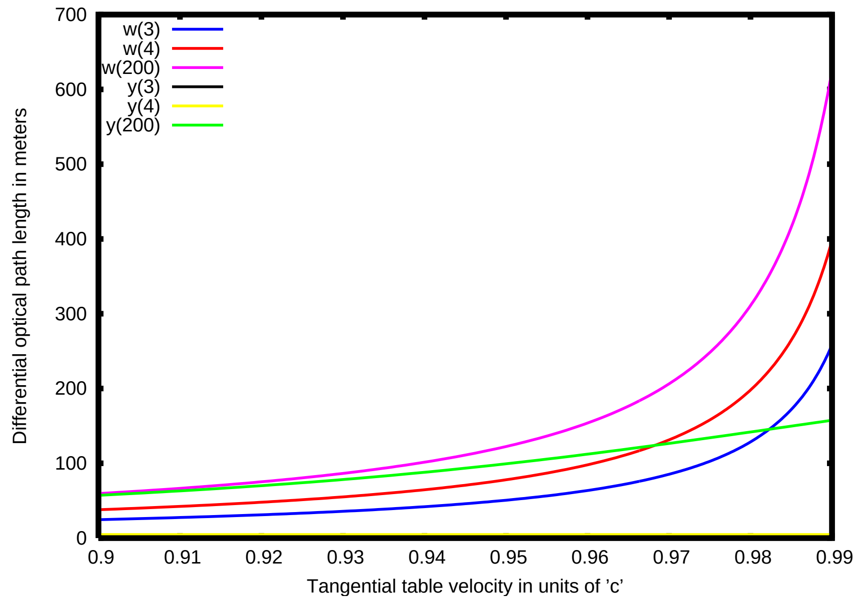

Figure 3. The curves here seem to show that $y(200)$, as do the curves $y(3)$ and $y(4)$, intercepts a finite value when the rim speed equals that of light.

One aspect of the difference between these two paradigms or view points of special interest is the possibly of experimentally distinguishing them. As this is the least impractical at the slow end of the curves, let us plot them for 0 to $0.025c$.

```
(%i14) plot2d([w(3,k*x),w(4,x*k),w(200,k*x), y(3,k*x),y(4,k*x),y(200,k*x)], [x,0,0.2],
       [xlabel, "Tangential table velocity in units of 'c'"],[ylabel, "Differential
       optical path length in meters"],
       [legend,"w(3)","w(4)","w(200)","y(3)","y(4)","y(200)"],
       [gnuplot_term,ps],[gnuplot_out_file, "Fig4.eps"]);
```

```
gnuplot: illegal width value .borderWidth:5
(%o11)
```

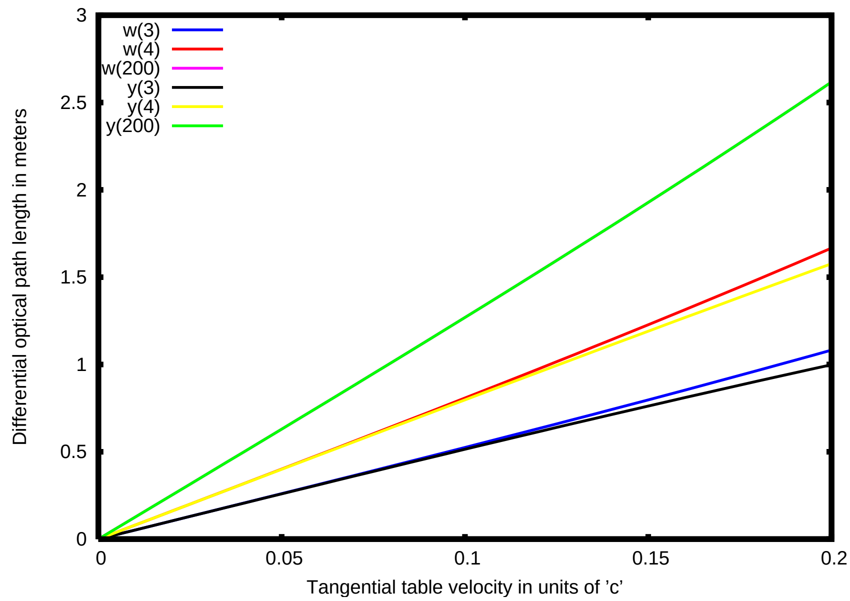

Figure 4.

From this graph is appears that only with velocities greater than about $0.1c$ for a table with $R=1$ meter radius. This corresponds to

$$v(\text{rpm}) = 60 \times 0.1\, c/(2\ \pi) \tag{4}$$

```
(%i18) rpm:(60*0.1*c/(2*%pi))$ float(%)
```

(%o21) $2.864788975654116 \times 10^{+8}$

That is nearly 300 million rpm, i.e., out of range of any practical experiment; unless there is a cosmological effect involving this magnitude of rotation, the difference cannot be observationally distinguished on this basis.

3.2 Doppler phenomena

Each intercept of a light ray by a mirror or the detector, in addition to the lengthening or shortening of its optical path length as analyzed above, it will suffer aberration and a Doppler shift. According to the paradigm proposed herein, a ray, as it arrives at the location of a mirror or detector, *is,* (here this "is" is taken to mean: epistemically, i.e., the way it "really is.") just as it arises at the location of the mirror or detector. Nevertheless, the mirror or detector does not 'perceive' the epistemic object, rather it registers the object as it is modified by a Lorentz transformation in which the velocity parameter is the velocity of the mirror or detector. This is, of course, just the way aberration of star light or the Doppler shift works. The source of the light, the star for example, is not in any way affected by the motion of an observer, and therefore signals it emits are unaffected, but the perception of them by the receptor is 'aborrated' or shifted, but only *for the observer at the location of perception.* These effects can be compared with simple geometric perspective, and might be called "space-time perspective."

For the sake of completeness, these effects have been computed for the experimental setup considered above, i.e., a Sagnac experiment on a table with a one meter radius.

The first quantity that must be determined is the intercept angle at the mirror. For the co-rotating signal, let us label this angle α_i where i denotes the step in the sequence of reflections with the final detection; i.e., at each reflection the ontic ray (wave vector) is that from the previous intercept so that the angular aberration, in the usual way, is given by

$$\cos(\alpha) = \frac{\cos(\alpha') - v/c}{1 - (v/c)\cos(\alpha')}. \tag{5}$$

Where it is understood that the 'perceived' angle is α, given that the ontic angle is α'. The latter is the argument of Eqs. (1) or (2) above, namely

$$\alpha(\omega) = \alpha(v/R) = \alpha(v) = \frac{\pi}{n} + \frac{l\pi v}{c}. \tag{6}$$

```
(%i22) alpha[i](n,nu):=%pi/n + opl(n,nu)*%pi*nu/c
```

(%o12) $\alpha_i(n,\nu) := \frac{\pi}{n} + \frac{\mathrm{opl}(n,\nu)\,\pi\,\nu}{c}$

Likewise for the counter-rotating signal:

```
(%i23) delta[i](n,nu):=%pi/n - opll(n,nu)*%pi*nu/c
```

(%o13) $\delta_i(n,\nu) := \frac{\pi}{n} - \frac{\mathrm{opll}(n,\nu)\,\pi\,\nu}{c}$

Now, this Doppler shift will occur at each reflection and at detection in view of the motion of the mirrors and the detector. Thus, as a multiple shift for a signal with frequency Ω, one at each of n inception events, is given by:

$$\Omega'_+(n,v) = \left\{\frac{\sqrt{1-(2\pi v R/c)^2}}{1+(2\pi v R/c)\cos(\alpha_i(n,v))}\right\}^n \Omega.$$

And for the counter-rotating signal:

$$\Omega'_-(n,v) = \left\{\frac{\sqrt{1-(2\pi v R/c)^2}}{1+(2\pi v R/c)\cos(\delta_i(n,v))}\right\}^n \Omega.$$

Let us denote the quantities in brackets as $f_{+,-}(n,v)$, for which their Maxima encoding is:

```
(%i36) f[p](n,nu):=((1-(2*%pi*nu/c)^2)^(1/2)/(1+(2*%pi*nu/c)*cos(alpha[i](n,nu))))^n
```

(%o14) $f_p(n,\nu) := \left(\frac{\left(1-\left(\frac{2\,\pi\,\nu}{c}\right)^2\right)^{\frac{1}{2}}}{1+\frac{2\,\pi\,\nu}{c}\cos\left(\alpha_i(n,\nu)\right)}\right)^n$

```
(%i43) f[n](n,nu):=((1-(2*%pi*nu/c)^2)^(1/2)/(1+(2*%pi*nu/c)*cos(delta[i](n,nu))))^n
```

(%o15) $f_n(n,\nu) := \left(\frac{\left(1-\left(\frac{2\,\pi\,\nu}{c}\right)^2\right)^{\frac{1}{2}}}{1+\frac{2\,\pi\,\nu}{c}\cos\left(\delta_i(n,\nu)\right)}\right)^n$

```
(%i16) plot2d([f[p](3,k*x),f[p](4,k*x),f[p](200,k*x),f[n](3,k*x)+.01*sin(100*x),
       f[n](4,k*x)+.01*sin(100*x), f[n](200,k*x)+.01*sin(100*x)],[x,0,0.9],
       [xlabel, "Tangential table velocity in units of 'c'"],[ylabel, "Doppler
       factor"],
       [legend,"f[p](3)","f[p](4)","f[p](200)","f[n](3)","f[n](4)","f[n](200)"],
       [gnuplot_term,ps],[gnuplot_out_file, "Fig5a.eps"])
```

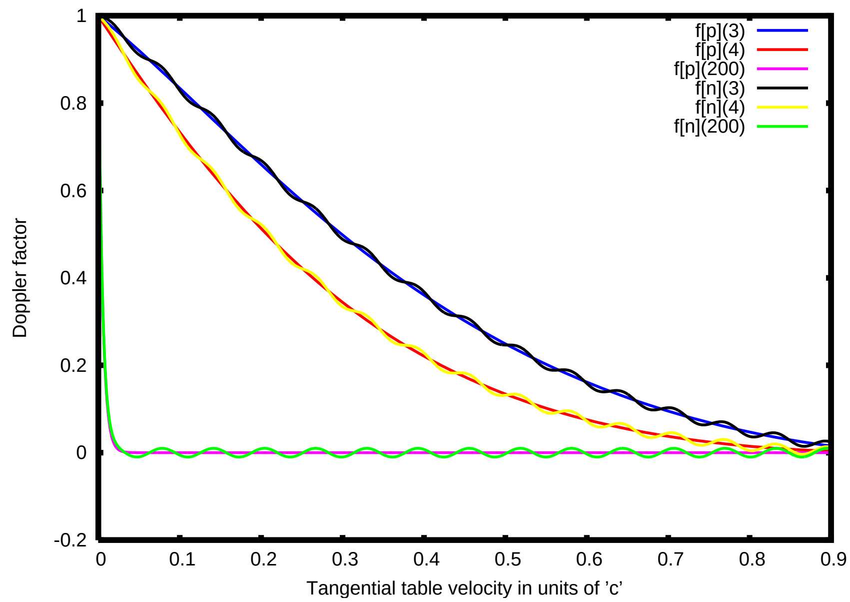

Figure 5. Plots of the Doppler factors for the co-rotating "[p]" and counter-rotating "[n]" for signals in a Sagnac experiment. (The warble imposed on the "[n]" curves is an artifact to render them visible.)

The Doppler factors are of opposite polarity; i.e., those for the co-rotating signals cause decreases whereas those for the counter-rotating signals cause increases in frequency. In the process of superimposing the two signals the effect will be to generate modulation on the interference pattern with a wave length equal to f/c. This modulation effect for the interference pattern seen in Sagnac experiments appears not to have been discussed in the literature, nor observed. Insofar as most realizations of Sagnac experiments have been of the 4-link variety, corresponding to the middle pair of curves, it appears that the modulation wave length could well be observable, as the "f"-factors at low revolution rate are nevertheless near 1.

Above it was taken that the "f" factor is the same at each mirror. This can be defended with the argument that the signal under consideration is a Green's pulse with only "instantaneous" structure. However, the actual signals are pulses extended in time and therefore, seemingly, requiring that consideration be taken of the extended vector character of the pulse as reflected in the wave vector to which the f factor is applies. This could be taken to mean that the transformed angle, i.e.,

$$\cos(\Theta) = \frac{\cos(\Theta') - (v/c)}{1 - (v/c)\cos(\Theta')}, \tag{7}$$

at a mirror becomes a property of the pulse as it proceeds to the next mirror. In turn, this would mean that the next computation for the Doppler factor must use, not the initial angle, but the transformed variant. This option can be calculated as follows. First, some initializations

```
(%i1)dpfac[0](omega):=1;  theta[0](n,omega):=alpha[i](n,omega*2*%pi)
```

(%o16) $\mathrm{dpfac}_0(\omega) := 1$
(%o17) $\vartheta_0(n,\omega) := \alpha_i(n, \omega\, 2\,\pi)$

Then the nested algorithm to compute the 'advancing' f factor:

```
(%i  dopplerp(n,omega):=block(
                for j:1 thru (n) do
          block(theta[j](n,omega):=(cos(theta[j-1](n,omega))-(R*omega/c))/
                     (1-(R*omega/c)*cos(theta[j-1](n,omega))),
          dpfac[j](omega):=(1-(R*omega/c)^2)^(1/2)/
                     (1+(R*omega/c)*cos(theta[j](n,omega)))*dpfac[j-1](omega) ),
          return(dpfac[n](omega)
                )                  );
```

(%o18) $\mathrm{dopplerp}(n,\omega) := \mathbf{block}\left(\mathbf{for}\ j\ \mathbf{thru}\ n\ \mathbf{do}\ \mathbf{block}\left(\vartheta_j(n,\omega) := \frac{\cos(\vartheta_{j-1}(n,\omega)) - \frac{R\,\omega}{c}}{1 - \frac{R\,\omega}{c}\cos(\vartheta_{j-1}(n,\omega))}, \mathrm{dpfac}_j(\omega) := \frac{\left(1 - \left(\frac{R\,\omega}{c}\right)^2\right)^{\frac{1}{2}}}{1 + \frac{R\,\omega}{c}\cos(\vartheta_j(n,\omega))}\mathrm{dpfac}_{j-1}(\omega)\right), \mathrm{mreturn}(\mathrm{dpfac}_n(\omega))\right)$

```
(%i25) plot2d([f[p](3,k*x),dopplerp(3,k*x/(2*%pi))],[x,0,0.1],[legend,"Constant Doppler
       factor", "Compound Doppler factor"],[xlabel,"Rim speed in units of
       'c'"],[ylabel,"Doppler factor"],[gnuplot_term,ps],[gnuplot_out_file,"Fig6.eps"])
```

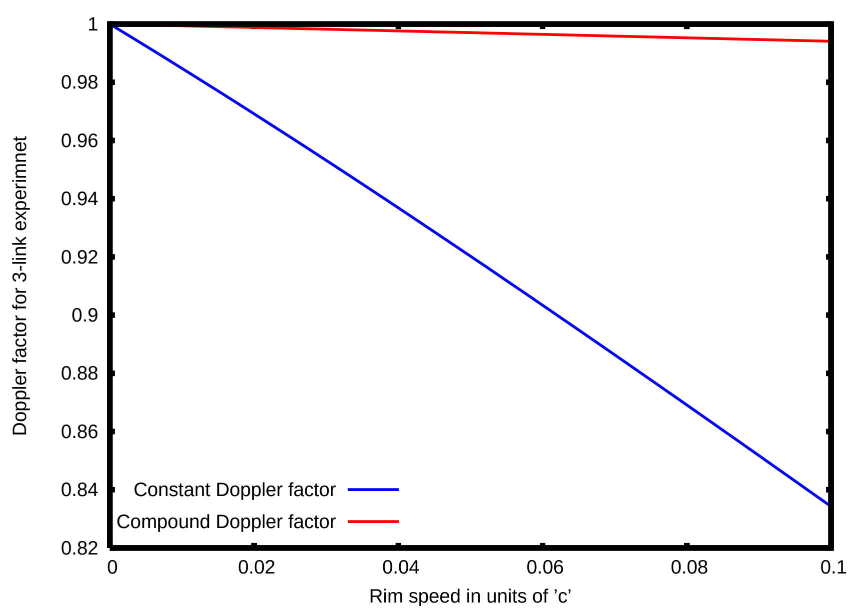

Figure 6. A comparison of the Doppler factors computed as the simple product (blue) and as a compound in which for each reflection the input angle is the aborrated output of the previous reflection.

The comparison of the two ways of computing the Doppler factor shows that compounding should lead to much lower modulation frequencies.

Unfortunately, as the Doppler effect pertains to both the customary and the new paradigms equally; it cannot be used to experimentally distinguish between them.

4 Discussion and conclusions

The paradigm based on the notion that relativistic effects following from the Lorentz-Minkowski have no "real" or epistemic existence, but are only "apparent" or artifacts of perception. These effects are analogous to geometric perspective and might be called "space-time perspective" effects. In previous work this writer has used this paradigm to analyze the renowned "twin paradox."[4, 5] The conclusion of this analysis is that the disparity in apparent aging arises only in the reports sent to and fro between the twins, but that after the twins re-meet and compare their personal clocks, i.e., their proper times, they see that they are exactly equal. Naturally, this implies some reinterpretation of the symbols and calculations as presented in the standard text books. This turns out to involve no more that a different permutation of interpretations, or assignments of meaning to symbols, already in use. In sum, one can say, that this analysis tactic is advantageous applied to uniform linear motion.

The next level of complication is, naturally, uniform rotational motion. As such motion is essential in the famed Sagnac Experiment, it provides the ideal platform for its study of such motion in association with accomplished experimental work. The calculations in the preceding sections show that this new paradigm, while in principle different from the standard analysis, as is made evident by the variance at the extreme, albeit experimentally inaccessible, rotation regime entailing velocities near that of light, cannot be excluded from consideration on the basis of observations made in accessible, low rotation rate experiments.

The next question is: does this paradigm offer abstract advantages? Does it facilitate intuitively acceptable analysis of the abstract "paradoxes"? There are at least three such paradoxical issues related the Sagnac Experiment considered in the literature: the "Ehrenfest Paradox," the "discontinuity issue" (for which Selleri is among the most articulate proponents), and the use of this effect as a demonstration of the nonintegrability of the differential of proper time, $d\tau$. We argue that this paradigm provides convincing rebuttals or clarifications for all three.

As is very well known, Ehrenfest first noticed that applying the Lorentz contraction factor to the rim of a rotating disk implies that the circumference must contract while the radius remains constant, this in violation of all experience and Euclidean geometry.[8] From the vantage of the new paradigm, this issue does not arise in the fist instance, as all contractions are considered just optical effects "within the eye" of the observer, as it were. It is common experience that objects *are* different than they *appear*. Common perspective provides a surfeit of examples. This perspective aberration, however, is no physiological effect, even inanimate "eyes" such as mirrors and detectors are subject the essentially kinematic-geometric effects, in the end resulting from the consequences of the finite speed of light as the medium for interacting among particles of matter.

The "discontinuity issue" arises in the context of standard analysis of the Sagnac setup[9], in which it is said that from the point of view of an observer on the disk and moving with it, the velocity of the co-rotating signal is $c + R\omega$ and that for the counter rotation signal $c - R\omega$. If now, it is imagined that the radius of the disk is extended without limit, but ω diminished such that the product $R\omega$ remains constant, then a point is reached at which the local frame is arbitrarily close to an inertial Lorentz frame, in which the speed of light depends on its direction — contrary to the central fundamental precept of Special Relativity! The only reconciliation for this circumstance is hypothesized a "discontinuous" leap from two speeds to a unique speed for light at exactly the boundary to infinity (whatever that would be).[9] This issue also does not arise in the context of the new paradigm; the underlying structure is based on the proposition that the speed of light is constant in all direction for all sources, nevertheless the resulting calculations based on this assumption do not lead to contradiction with extant observations.

The final issue is an application of the structure described in the preceding paragraph. It it based on the assertion: "...the proper time $d\tau$ is not an integrable quantity. Different world lines that start from one event and meet again at another event, may therefore be associated with different lapses of time between these two events. ... a very similar phenomena which is much more easily accessible to experiment can serve to demonstrate the same point. ... Sagnac ... "[10] Once again, this assertion does not pertain to the new paradigm (and in fact can be seen therefrom not to pertain to the standard one either). By starting at the detector and tracing the optical paths back through the mirrors to sources, it is evident that the Green's pulses that meet in the detector originated as separate locations. This is obvious from the fact that the optical path lengths are different, implying along with the rotational motion, that the sources were both at different positions and at different times. In short, the Sagnac effect is *not* evidence for the nonintegrabiltiy of proper time. This conclusion is by no means novel, Sachs[11] (and before him, for example, Dingle[12]) has long argued that the internal consistency of both relativity theories, but especially General Relativity, require the integrability of proper time.

Finally, some philosophical remarks. The basic construct of the new paradigm is an "absolute" 3-d position space. It is taken to be "absolute" in the sense that events can be and must be somewhere that is unique and fixed, virtually by definition. Of course, they can be given coordinates relative to an infinite number of origins, but the events themselves, in the sense of "coordinate free" mathematical analysis, is unique. Furthermore, the time of each event is a unique instance, what ever its label. The only issue is, how do the instances of disparate events related one to another. Or, in other words, does there exist "absolute time"? The answer, given that absolute time is a concept, is, that it exists as a concept if it can be defined free of contradiction. The following may suffice, in principle. Select arbitrarily an event for which the equation of the Green's shell is

$$E(r,t) = \frac{\delta(r - \mathrm{ct})e}{r^2}. \tag{8}$$

Now, let the time at other events be calibrated by the passage of the Green's shell from this event by setting t_0 of the standard event as the origin for time and then assigning the relative time at other events to be given by:

$$t \triangleq \frac{1}{c}\sqrt{\frac{e}{E}}, \tag{9}$$

that is, proportional to the (measurable) amplitude of the passing Green's shell from the standard event. This definition suffers from no obvious inconsistency. It is founded on the basics of electrodynamics, however; but this seems to be a minimal and natural premise, given that the known world is in fact interrelated by just this electromagnetic interaction. Perhaps, it should not be surprising that 'time,' a manifestation of change, i.e., a consequence of interaction, is intimately involves light in the formulation of its interrelationships, that is, its synchronization.

Historically, the proposal that alternative understanding of the effects conventionally described by Special Relativity can be achieved by a paradigm based on a Euclidean 3-space and absolute time is not uncommon.[3] In one sense it is the view point taken by Lorentz. Frequently the properties here attributed to the 3-space and time are captured by some notion of an aether. If, it is taken that such an aether is somewhat abstract, devoid of a 'physical' essence like air or water, then there is perhaps a parallel. The pedigree of an idea in the end is unimportant, the issue really is the self consistency of the mathematical formulation to which it leads. Relativity theory, at least as it is discussed in the literature, is obscure on one essential point, namely, that, for each ray there are two times of vital relevance always: the instant of emission and the subsequent instant of reception. Those instants of the former sort relate to the epistemic essence of observed objects; whereas, those of the second, to the impression made by the source "in the eye of the observer." It is both common experience, and a well understood source of physical effects (Dopper shift and aberration), that objects appear different from the way they are in fact. Obviously, because the connection between these instants are connected by light which travels at finite speed, time intervals at the source can be perceived at the observer differently, in particular if the observer and source are in relative motion. Traditional analysis seems not to take this complication explicitly into consideration always. Confusion, then, is particularly acute with respect to "simultaneity;" apparent non simultaneity of received signals does not imply, for example, that the source events were non simultaneous.

The paradigm suggested herein was founded on the hope that it can facilitate elucidating these issues.

3. See, for example, Ref. [13]

Also of note, is that a fundamental element of the new paradigm is that the speed of light is fixed once and for all in the absolute space of position, a premise that seems to be operationally fully equivalent to the similar sounding premise of conventional Special Relativity.

A final question concerns whether this paradigm can be applied consistently to all applications heretofore thought to require Special Relativity, and, whether it can be melded smoothly into an equivalent version of General Relativity.

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