

# “Quantum” beats in classical physics\*

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## Abstract

We present analysis of, and a model classical for, atomic decays that do (do not) exhibit what has been denoted “quantum beats.” The model shows that the appearance of beats is dependant on implicit, but not necessary extant, phase coherence of the excitation signals. The significance of the existence of this model for arguments on the tenability of Neoclassical Electromagnetic Theory as an alternate theory to QED is discussed.

**Keywords:** Quantum Electrodynamics, Semiclassical Theory, Neoclassical Theory, Quantum Beats

## 1 The legacy

Quite early in the development of Quantum Electrodynamics (QED), it was noticed that some of the phenomena described by QED could also be handled by semiclassical methods in which matter is quantised but radiation not. The most well known example is perhaps WELTON’s calculation of the LAMB shift based on considerations of the stochastic agitation of an electron’s orbit.[1] This line of development was then promoted by JAYNES and collaborators, who, to the dismay of QED enthusiasts, more or less continuously expanded the domain of success of what become known as “neoclassical theory” (NCT).[2] However, in the late 1970’s and somewhat beyond, as JAYNES’ career converged to its denouement, the NCT program was depreciated. There appears to have two main arguments to justify this attitude. One was the then new results from CLAUSER[3], and others, seemingly supporting BELL’s analysis purporting to show that no nonlocal theory, which the NCT certainly is, could ever fully replace Quantum Mechanics (QM), and with it QED. The other argument concerned the phenomenon of “quantum beats.” At that time, it was argued that only QM encompassed all peculiarities of this effect.[4] Current literature repeats this claim.[5]

Within a short time, however, JAYNES identified a fatal error in BELL’s analysis, although the persuasive effect of his criticism did not find wide resonance during the remainder of his life.[6] Nevertheless, his argument, unrefuted to date, must be considered solid and valid; moreover, it has been rediscovered by others, including, at least, PERDIJON[7] and this writer.[8] JAYNES’ point has been extended to a detailed simulation[9, 10], making the issue of correlations as considered by EINSTEIN, PODOLSKY and ROSEN (EPR) no longer a valid argument against the NCT, and so leaving only the issue of quantum beats standing.<sup>1</sup>

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<sup>1</sup>Subsequently hypothesised phenomena thought to require QM for theoretical encodification, e.g., teleportation, quantum computing, etc., are all applications of BELL’s analysis to EPR-type circumstances. As such, they all fall to JAYNES’ criticism.

It is the purpose of the note to propose analysis of this phenomena that undermines its use also as a counterexample to the NCT. To do so, first the basic notions found in the literature pertaining to quantum beats are reviewed. Then we shall propose a fully (semi)classical model that exhibits exactly those features commonly thought to require QM for a proper description. A simple and very elementary classical demonstration of a phenomenon with (or without) such beats (no longer restricted to QM) will be described. At last, we shall draw the obvious conclusion: This effect is not a counterexample to the aspiration of replacing QED with the NCT, such will have to be found elsewhere.

## 2 Quantum beats: the theory

Quantum beats are seen to occur where multiple excited states can decay to a single lower state. They do not occur where a single excited state decays to *multiple* lower energy states. The prototypical basic ingredients, in either case, are three atomic states. Since such atomic states are known to oscillate, let us take as a generic form for their wave function for each of the three states, denoted  $a, b, c$

$$|\Psi_j(r, t)\rangle = K_j e^{i\omega_j t} |\Psi_j(r)\rangle, \quad j = a, b, c. \quad (1)$$

Obviously, for a semiclassical analysis, the same quantised states can be used. (See: Ref. [11] for further details.)

Now, for quantum analysis, the wave function of the system comprising the three atomic states and the two radiation states for the case of one lower state,  $a$ , and two upper states,  $b$  and  $c$ , (Commonly referred to as the “ $V$ ” configuration), would be

$$|\Psi_V\rangle = \sum_j k_j |\Psi_j, 0\rangle + k_1 |\Psi_a, \omega_\alpha\rangle + k_2 |\Psi_a, \omega_\beta\rangle, \quad (2)$$

where everything not dependant on time is collected in the  $k$ 's and  $\omega_\alpha = \omega_a - \omega_b$ , and  $\omega_\beta = \omega_a - \omega_c$ . If now the expectation of the of a beat signal is computed, we get:

$$\langle \Psi_V^* | E^*(\omega_\alpha) E(\omega_\beta) | \Psi_V \rangle \propto \langle \Psi_a | \Psi_a \rangle e^{i(\omega_\alpha - \omega_\beta)t}. \quad (3)$$

The same calculation for what is called the “lambda”,  $\Lambda$ , version, starts from:

$$|\Psi_\Lambda\rangle = \sum_j k_j |\Psi_j, 0\rangle + k_3 |\Psi_b, \omega_\alpha\rangle + k_4 |\Psi_c, \omega_\beta\rangle, \quad (4)$$

from which the beat signal for this case would be given by:

$$\langle \Psi_\Lambda^* | E^*(\omega_\alpha) E(\omega_\beta) | \Psi_\Lambda \rangle \propto \langle \Psi_b | \Psi_c \rangle e^{i(\omega_\alpha - \omega_\beta)t}. \quad (5)$$

The crucial difference between them is, that  $\langle \Psi_a | \Psi_a \rangle = 1$ , whereas  $\langle \Psi_b | \Psi_c \rangle = 0$ , by virtue of the orthogonality of eigenfunctions of the SCHRÖDINGER equation. These calculations are the justification for the assertion that QM “forbids” the appearance of ‘quantum beats’ for the lambda configuration.

By way of contrast, in the NCT, the usual calculation found in the literature, proceeds as follows: The transition fields of either case, take the form:

$$E = k_1 e^{i\omega_\alpha t} + k_2 e^{i\omega_\beta t}, \quad (6)$$

for which the intensity would be:

$$E^2(t) = k_1^2 + k_2^2 + 2|k_1 k_2| e^{i(\omega_\alpha - \omega_\beta)t}, \quad (7)$$

when expressed using the ‘analytic signal formalism,’ which gives just the modulation or envelope. Since this form is valid for either variant, both  $V$  and  $\Lambda$ , it is widely asserted that the NCT *always* predicts beats.

### 3 An experiment

A typical experiment exhibiting quantum beats is configured as follows: An atomic beam of atoms with the appropriate configuration of states is extracted through collimator from an oven. As the beam emerges from the oven, it is blasted with a narrow excitation beam in a direction transverse to the beam. Then observations are made with a detector that views the beam, again from the transverse direction through collimators, but now at ever increasing distances from the opening of the oven. The signal so seen is then the intensity of the decay signal as emitted by the beam atoms as a function of distance from the oven opening or, in other words, the point of excitation, and therefore, since the atoms move at a constant velocity, at time elapsed since excitation, of the beam atoms . [4]

Accomplished experiments show that only in the  $V$  configuration do beats occur, thereby empirically verifying the quantum calculation and justifying the assertion that for the  $\Lambda$  variant, “quantum mechanics forbids the formation of beats” whereas the NCT ‘requires’ them.

### 4 The loophole

The just repeated claim depends on the validity of a negative statement, to wit: there does not exist classical phenomena under the circumstances of the experiment for which the beats could wash out. Such negative, non existence assertions are notoriously often false. Indeed, that is the case here; the inevitability of the appearance of beats in the classical analysis depends on the covert hypothesis, that the  $N$  atoms visible in the detector’s window at once have been excited in phase. This is evident in Eq. (7), which is written as if there is a just two signals under consideration, an approximation that is acceptable only if all the atoms were excited in phase, if not, then an additional average must be made, and must be made manifest in Eq. (7), as follows:

$$E^2(t) = \frac{1}{N} \sum_n^N (k_1^2 + k_2^2 + 2|k_1 k_2| e^{i(\omega_\alpha - \omega_\beta + \phi_n)t}). \quad (8)$$

Obviously, such an average can cause the beat term to wash out, unless  $\phi_i = \phi_0 \text{ mod}(2\pi)$  (i.e., some particular fixed value) for sufficiently many values of ‘ $n$ .’

To illustrate this effect, a simulation was made to illustrate that there exists a regime for which the lack of phase coherence results in the beats being washed out. See Fig. (1)

Such a simulation is very useful to study the effects of scaling various physical effects such as those that in this case lead to washing out beats. A necessary ingredient, however, is that the assumptions going into the simulation faithfully model actual physical effects. For the purposes of this note, the actual model simulated may not correspond to any realistic experimental situation occurring in nature; however, the mere existence of any effect whatsoever that can cause ‘wash out’, is sufficient to support the conclusion drawn herein, namely: there do exist mathematically sufficient effects that can replicate all results from experiments, thereby evading the customary conclusion that no classical effect could account for the lack of beats. Without specific analysis of done experiments, however, this is a limited technical point.

The cogency of the conventional, implicit hypothesis, that the stimulation signals for the ‘lambda’ transitions are phase coherent, becomes the fundamental issue for the overarching argument. There is a degree of intuitive acceptability to the notion that decay signals between different target states in the “ $\Lambda$ ” regime can be incoherent. In the “ $V$ ” regime the excitation is affected by a single broad band stimulus, whereas in the “ $\Lambda$ ” regime the decays to different target states can have different different lifetimes with independent, even if structurally identical, initiation circumstances.

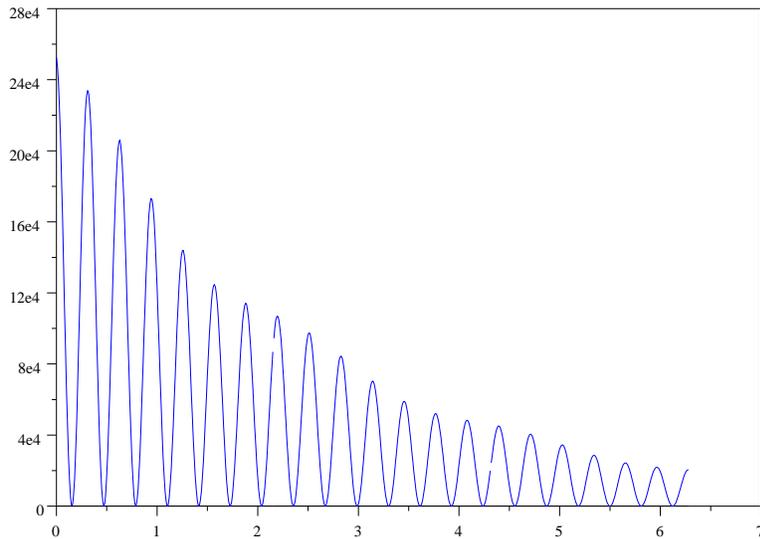


Figure 1: Results of a simulation of regime showing (incomplete) wash-out of beats. The situation analysed involved 500 radiators for each frequency for which one type had identical phases whereas the other had random phases. The exponential decay was put in by hand and simply represents the depletion of radiators with increasing distance from the source.

In other words, it is in the end (actually in the beginning) the combined structure of the team of excitation signals that determines the ultimate availability of the coherently excited atoms to decay so as to exhibit beats, or not.

## 5 A demonstration

To demonstrate this effect in an incontestably classical setting, we suspended equal numbers of two pendulums from a bar. See Fig. (2). Then the bar itself was suspended at its ends on short flexible pendulum-like cords. The purpose of suspending the whole bar is to provide a coupling between the two sets of pendulums. Now, when all the pendulums, including both sets, are excited in phase by launching them together, the resulting oscillation, in particular of the bar from which the pendulums are suspended, exhibit beats. On the other hand, if the pendulums are excited out of phase, each at an arbitrary time with respect to all others, no beats are seen. The latter case is as plausible as the former in classical physics, so the argument that classical physics always mandates beats in the  $\Lambda$  regime, is based clearly on fully correct but incomplete considerations, that just happen to obtain in the experiments that were done to research this phenomenon.

## 6 Conclusions

The existence of a plausible semiclassical model of the experiment in which the beats are not seen for the ‘lambda’ configuration speaks to more than just the issue of to the viability of the NCT. It is this: The

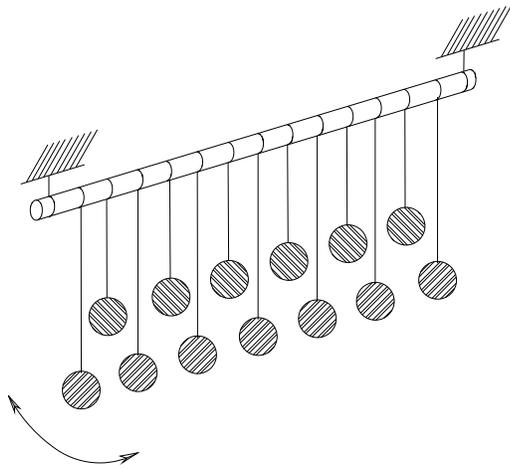


Figure 2: A classical analogue of experiments demonstrating “quantum beats”: consider a system comprising two sets of approximately equal numbers of pendulums, where all members of each set have the same frequency. Both sets are suspended from a rod which itself is suspended from fixed attachment points. This arrangement provides a means of coupling among all the pendulums in both sets. When all the pendulums in both sets are launched together, a distinct beat is observed. On the other hand, when there is no coherence in launching time, the beats wash out. This simple classical system exhibits all the key features of experiments held to demonstrate “quantum beats”.

model shows that the the difference is strictly whether or not the excitation signals are phase coherent. The fact that the algorithmic and notational rules embedded in the QM formalism imply that the stimulus is phase coherent, could well have meaning in analysis of the interpretation of QM. In this regard a critical question is: why is this phase coherence an intrinsic implicit feature of QM? What does this mean for the identity of wave functions? Or, what does it imply about underlying “hidden” phenomena that might be used eventually to rationalise the interpretation of QM; how is it that the Born rule for investing meaning into wave functions, in particular of photons, incorporates essentially the ‘analytic signal formalism’?

## Dedication

This note, written in the centennial year of Einstein’s *annum mirabile*, is dedicated to his honour for his judgement on the validity of ‘local realism.’

## References

- [1] Welton T A 1948 *Phys. Rev.* **17** 1157
- [2] Jaynes E T 1978 *Coherence and Quantum Optics IV* eds L Mandel and E Wolf (New York: Plenum Press) 495
- [3] Clauser J F 1976 *Phys. Rev. Lett.* **36** 1223

- [4] Scully M O 1980 *Foundations of Radiation Theory and Quantum Electrodynamics* ed A O Barut (New York: Plenum Press) 45
- [5] Silverman M P 1994 *More Than One Mystery* (New York: Springer-Verlag) 100
- [6] Jaynes E T 1989 *Maximum Entropy and Bayesian Methods* ed J Skilling (Dordrecht: Kluwer Academic Publishers) 1
- [7] Perdijon J 1991 *Ann. Fond. L. de Broglie* **16** 281
- [8] Kracklauer A F 2000 *Ann. Fond. L. de Broglie* **25** **193**
- [9] Kracklauer A F 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** S544
- [10] Kracklauer A F 2005 *Foundations of Probability and Physics-3* ed A Khrennikov (Melville: AIP Conf. Proc.) 219
- [11] Scully M O and Zubairy M S *Quantum Optics* (Cambridge: Cambridge University Press, 1997)