The singlet state and Bell-inequality tests

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ABSTRACT

It is observed that a critical aspect of tests of Bell-inequalities is the employ of entities considered to be in the singlet state. This state is known to require extra-logical consideration to render it compatible with the current most popular interpretation of Quantum Theory. We show that the critical structure of this state for the analysis of these tests can be spoofed by feasible, classical effects, that thus far have not been absolutely precluded. Finally, we present statistical analysis showing that selecting for valid pairs of correlated signals by reducing the time off-set or window-width defining acceptable coincidences, actually and perversely supports the spoof mechanism.

Keywords: nonlocality, singlet state, Bell-inequalities, Quantum Theory

1. PECULIARITIES OF THE SINGLET STATE

The singlet state for pairs of polarized photons

$$\Psi = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle \otimes |\rightarrow\rangle - |\rightarrow\rangle \otimes |\uparrow\rangle \right), \tag{1}$$

constitutes the crossroads for the difficulties interpreting Quantum Mechanics. This state as written is the sum (difference) of mutually exclusive possibilities; as such, if it is to pertain to to single ontic entities, it is a logical abomination. Moreover, as such a state has never been observed, it was found auspicious to introduce the so-called "projection hypothesis." According to this hypothesis, the singlet state is the representation of a ontic entity that is in fact ambiguous, but which upon observation is "projected" onto one of its options while annihilating the other. Such a process must lie outside the ambit of quantum theory because it is not described by the Schrödinger Equation.

This same difficulty arises again in connection with the Born interpretation of a quantum wave function, to wit: its modulus is an expression of the probability of presence as a function of space and time of the entity it represents; namely

$$\Psi^* \Psi = \rho(x,t), \tag{2}$$

where ρ can serve, as is customary in statistical theories, in the formula for, say, the average position $\langle x \rangle = \int x \rho \, dx$. Now, computing this probability using the singlet state gives

$$\Psi * \Psi = \frac{1}{2} \left(\langle \uparrow | \uparrow \rangle + \langle \rightarrow | \rightarrow \rangle + \langle \uparrow \otimes \rightarrow \rangle \langle \rightarrow \otimes \uparrow \rangle + \langle \rightarrow \otimes \uparrow \rangle \langle \uparrow \otimes \rightarrow \rangle \right). \tag{3}$$

Customarily, it is said, that the factors of the form $\langle \uparrow \otimes \rightarrow \rangle$ represent the inner product of orthogonal Hilbert vectors, and thus equal zero, thereby yielding the normalized sum of two ontic states: $\langle \uparrow | \uparrow \rangle /2 + \langle \rightarrow | \rightarrow \rangle /2$. In spite of the evident appeal of this tactic, it is defective in two regards. One, mathematically the indicated inner products are across a Cartesian product, which implies that the two Hilbert vectors belong to separate Hilbert spaces and therefore their inner product is undefined. [Of course, a mathematical identification of the two spaces is trivial; but, such an identification within a physics theory must have a physical justification. At present no such exists.] Secondly, the sum of two (Born) probabilities should be interpreted as the probability for an either/or situation. This would be inconsistent with the notion that the singlet state pertains to a unique, albeit, ambiguous single entity. Accepting the conventional viewpoint, on the other hand, implicitly supports Einstein's position *vís-a-vís* Bohr, namely: quantum theory pertains *not* to individual systems, but to ensembles of similar systems—still nowadays considered a heterodoxical opinion!

Nevertheless, the singlet state, as a calculational object, is so essential to obtaining correct, that is: empirically verified, results in spectroscopy and other areas of Physics, that it appears that it cannot simply be banished from theory, at least easily.

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2. BELL-INEQUALITIES: THEORY AND THE SINGLET STATE

As is now widely known, Bell developed some inequalities pertaining to data taken in certain so-called "Einstein-Podolsky-Rosen" type experiments intended to test Einstein's view that Quantum Mechanics is an incomplete theory; i.e., pertains not to individual systems, but to their ensemble. Bell's analysis leads to the conclusion, that a certain inequality, which under ideal conditions is of the form:

$$|3\cos(\theta) - \cos(3\theta)| \leq 1, \tag{4}$$

(where θ is the angular discrepancy between polarizers filtering the left and right wings of correlated photon pairs generated in the singlet state), should be respected by all local-realistic (classical) states, i.e., if this inequality is empirically violated, then Nature is inexorably quantum in character. So much is discussed thoroughly in the literature (See: Ref. ([1]) for a particularly succinct and clear discussion.) .

It is evident from the derivation of Eq. (4), that $\cos(\theta)$ is the functional form of the correlation coefficient for coincidence measurements resulting from pairs described by the singlet state. Such a coefficient is defined in general by the ratio:

$$Cor = \frac{N_{++} + N_{--} - N_{+-} - N_{-+}}{N_{++} + N_{--} + N_{+-} + N_{-+}}, \tag{5}$$

where the N_{**} are detection counts (the correlated channels are indicated by like signs and anti-correlated channels for unlike signs); in short, this is the ratio of the difference between the coincidence probabilities for correlated and anti-correlated pairs normalized by the total number of pairs.

According to the Born rule, each N would be calculated using the formula, for example:

$$N_{++} = \langle V| \uparrow \otimes \rightarrow |H\rangle\,, \tag{6}$$

where $\uparrow \otimes \rightarrow$ denotes a filter (operator) selecting the appropriate combination of signals right and left. The functional for of these individual coincidence probabilities is $\pm\sin^2(\theta)$ for correlated and $\mp\cos^2(\theta)$ anti-correlated [signs depend on chosen conventions].

Now, just here in the story, something remarkable occurs, namely when a coincidence probability is calculated for the singlet state using Eq. (6), in stead of obtaining just a coincidence probability the result jumps directly to a correlation coefficient! This is a consequence of the fact that the expectation for the difference of the coincidence probabilities *for that state*, coincides with the correlation coefficient with respect to the various signals that impinge on the particular polarizer settings, because the singlet state is composed of mutually exclusive options.

This situation explains the conformity of a simple quantum expectation with a correlation coefficient.

Experiments to test Bell-inequalities in principle need to determine the four particular coincidence probabilities, N_{**}, for four combinations of angular settings, two on each side, of the polarizers, when the source emits pairs of signals (ostensibly) described by the singlet state. The source is selected from among those known not to emit signals other than correlated pairs but ambiguous with respect to which pair is generated at each individual emission event. These sources are known not produce individual signals, which would be uncorrelated with any other signal.

A serious challenge in executing Bell-inequality tests lies in the fact, that to calculate the correlation coefficient the total number of coincidences also must be known. Because of inefficient detectors, accurately determining the total number of generated pairs cannot be done. Various tricks pioneered by Clauser, however, have led to alternate schemes in which certain combinations of measurable quantities still allow a determination of the values taken by the function in Eq. (4).

3. EXPERIMENTS: CONCEPTUAL PROBLEMS

The point of a test of Bell-inequalities is to determine whether the phenomena involved in the experiment conform to the conclusion of what has become known as "Bell's Theorem." In rawest form, this conclusion is: if Eq. (4) is violated, then Quantum Mechanics cannot be reformulated or covered by a local, realistic theory involving additional variables.

This conclusion would be preempted, however, if it can be shown that there is a perfectly classical account of the phenomena observed in tests of Bell-inequalities. It is this writer's contention, that exactly that can be done. That is, there exists a purely classical model of these phenomena, or better put, the critical structure involved, which is purportedly that of the singlet state. To protect the current popular conclusion, therefore, requires proponents of Bell's opinion to show that the classical inputs to the model described below, i.e. the classical effects, can be absolutely excluded from done experiments, that they can be shown to be irrelevant.

The central and critical physics assumption underlying the here proposed classical model is that at all intensity levels the source produces overlapping, correlated pairs, not isolated, individual pairs.* This in turn leads in the analysis to false identification of what are presumed to be correlated pairs. Legitimate (correlated) pairs can be thought of as comprised of two pulses, each of length l, sent into separate channels. Each pulse, according to conventional photo-electron theory, will elicit a photo electron at a random instant within the pulse length l. Thus, the evoked signals consisting of the photo-electrons flowing in detection circuits, will have random time off-sets, even when generated by legitimate, correlated pairs. Such time off-sets open the possibility that the observed time off-set between two uncorrelated detections will be shorter than the off-set between two from a given correlated pair. The final consequence will be then, that the ensemble of detected pairs contains two sets: one of legitimate correlated pairs, the other of illegitimately randomly matched pairs.

The following chart summarizes the possible detections for an arbitrarily chosen angular detector orientations for various signals.

	V-H	H-V	V/H-H/V	v-h	h-v	h-h	v-v
$\uparrow_+ - \nearrow_+$	1-c	0-s	1/0-c/s	1-c	0-s	0-c	1-s
$\uparrow_+ - \nearrow_-$	1-s	0-c	1/0-s/c	1-s	0-c	0-s	1-c
$\uparrow_- - \nearrow_+$	0-c	1-s	0/1-c/s	0-c	1-s	1-c	0-s
$\uparrow_- - \nearrow_-$	0-s	1-c	0/1-s/c	0-s	1-c	1-s	0-c

On this chart, V, H stand for vertical and horizontally polarized signals in correlated pairs, v, h for unpolarized signals. The arrows in the first column symbolize the detector orientations, the subscripts indicate which of the four measurements is concerned. In the remainder of the lower four rows $1, 0$ indicate a detection (yes or no) and c, s stand for a signal with magnitude reduced by a factor of $\cos^2(\theta)$ or $\sin^2(\theta)$, i.e., by Malus' Law. Symbols to the left of a dash pertain to the left wing of the experiment, etc.

The second and third columns give the detections expected from an ensemble of either/or anti-correlated pairs. The fourth column stands for the consequence of the cross terms from an ontic singlet state. The 7th and 8th columns gives the expected detections from the cross-like type terms that arise from randomly paired uncorrelated signals of the sort envisioned to occur if the source produces *overlapping*, correlated pairs (as we hypothesize). This chart illustrates the fact that reasonable arguments can be made that the cross terms from uncorrelated illegitimate pairs mimic or spoof what can be expected from the cross terms resulting from computing an expectation with the singlet state. In terms of the chart that is: the fourth column is the sum of the 7th and 8th. [Of course, quantum mechanical processes, being extraordinary, as physics may not parallel classical analogues, as for example: projection by measurement is unknown to classical physics; but in this application, the algebraic manipulations are parallel.]

If both legitimate and illegitimate pairs contribute to the total detected ensemble, then there will be a surfeit of correlated detections. Were all correlated detections just from the illegitimate pairs, then there would be balance in the numbers of correlated and anti-correlated detections; but, it seems, that legitimate pairs contribute extra correlated counts, thereby distorting the correlation coefficient, Eq. (5).

4. BELL-INEQUALITY TESTS: DATA ANALYSIS

This situation brings up the question of whether there exists a means of filtering the total data set so as to remove excess correlated counts. At present, for all experiments data taken was considered to have been improved by reducing the window-width within which detection pairs were identified as legitimate correlated pairs. This was done on the presumption, that spurious, presumably uncorrelated, detections should tend to have greater time off-sets.

*This writer's previous contribution to this conference series, ([2]), also was based on this conception of the source. In that report, however, a faulty selection mechanism was proposed; herein Section 4. is intended to remedy that inadequacy. This issue does not affect the remainder of that report.

Herein this presumption is rejected. The basis for doing so is an idiosyncratic statistical effect, which, upon reducing the window, preferentially eliminates legitimate coincidences, so that as the window approaches zero, the correlation coefficient approaches that theoretically expected for an ideal ensemble comprised of pairs described by the singlet state—but actually constituted exclusively from illegitimate pairs!

First, consider the coincidence probability between two uncorrelated detections, one each from separate correlated pairs. It is just the product of their individual, absolute probability densities, which can be calculated from the average frequency of pair generation, f. First note that f is the numerical inverse of the time interval for which the expectation of elicitation of one photo-electron in a detector (a "detection") equals 1. Because the search for the first hit is not confined to a window, its probability density is f^{-1}. The (accidental) partner hits are uncorrelated with respect to the first seen hit, which is the instant at which the window is, so to speak, opened. There is no *a priori* reason that the frequencies, f_n, for distinct signal pairs (different n), arising presumably in different locations within the source crystal under various geometrical and electrodynamic conditions, must be absolutely identical. Thus the total probability of one or the other illegitimate alternative pairing is just the sum of the probability densities of the possibilities integrated over the window, symbolically

$$\int_0^{w_0} f^{-1}\left(\sum_n f_n^{-1}\right) dt = k w_0, \tag{7}$$

where the sum is over all the possible (overlapping) disparate pairs.

A calculation of the probability of a legitimate coincidences is more complex, because the events are not statistically independent, and a joint probability density is no longer just the product of two absolute probability densities. The essential question now is: what is the average likelihood of seeing a legitimate partner hit, given that the first hit has occurred at t_0, if observation is restricted to within the window of width "w_0" opened at t_0? This probability involves a *conditional probability*, because the second pulse is correlated with the first pulse.

Given that the hit which opened the window occurred on the left, say, at t_0, it follows that its legitimate partner must be found on the right within the remaining pulse length, that is within the time interval $l - t_0$, where l is the pulse length. Thus, the conditional probability for this partner hit, is $1/(l - t_0)$. Now, given this probability density and initial hit, the accumulated probability within the window, w_0, will equal $w_0/(l - t_0)$ for every occurrence having this initial instant for the first hit.

However, when the window width w_0 exceeds $l - t_0$, then this condition probability must be 1, which means that this conditional probability comprises two segments:

$$\int_0^{w_0} \rho(t_0, w)dw = \rho(t_0|w_0) = \begin{cases} \frac{w_0}{l-t_0}, & 0 < t_0 < l - w_0 \\ 1 & l - w_0 \geq t_0 \geq l \end{cases} \tag{8}$$

This is a two-dimensional density, dependent on the instant of the first hit and the then opened window width, w_0, and which is variable over a triangular region in which the window width is insufficient to cover the remaining pulse length. We are most interested in the variation of the cumulative probability of this expression as a function of the window width, w. It is the integral of $\rho(t_0|w_0)$ over all times, t_0 of the first hit from 0 to $l - t_0$, or to the point at which the accumulated probability equals one, i.e., all partner hits have been found: Now, it turns out, that for any given fixed window width, w_0, all coincidences will have been registered when $l - t_0 = w_0$. Thus, for computing the accumulated probability of seeing a coincidence as a function of w_0 can be obtained by the integral:

$$\int_0^{l-t_0} \rho(t_0|w_0)\, dt_0 = \int_0^{w_0} \rho(t_0|w_0)\, dt_0 = -w_0 \ln(l - w_0). \tag{9}$$

This is the accumulated probability of encountering legitimate partners by filtering a data stream by searching in a window of width w_0 in the partner channel opened at the instant when a hit is seen in either channel.

The relative effects of reducing the window width are illustrated quantitatively in Figure 1. The quantity of illegitimate coincidences from uncorrelated pairs diminishes linearly with w_0, (where we consider the weakest case, i.e., $k = 1$ in Eq. (7). On the other hand, the number of legitimate coincidences from correlated pairs diminishes more rapidly with *decreasing* w_0. As a consequence, reducing w_0, with the aim of purging illegitimate coincidences, actually relatively achieves just the opposite, but then spoofs the detection patterns expected from analysis using the singlet state.

And excess of correlated pairs can be taken into account in Eq. (5) quantitatively as follows:

$$Cor(\theta, k) = \frac{(1+k)\sin^2(\theta) - \cos^2(\theta)}{(1+k)\sin^2(\theta) + \cos^2(\theta)}. \tag{10}$$

Further, from Eq. (9) k can be expressed as a function of window-width, w_0, enabling determining the maximum violation of Eq. (4) as a function of the window-width (See: Figure 2). In principle, for a given, well characterized source, this variation should be empirically verifiable.

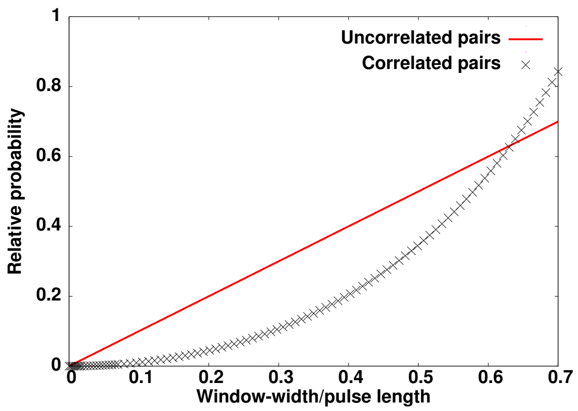

Figure 1. Sample curves showing the relative rapidity with which the number of pairs of each type is reduced as the window-with approaches zero.

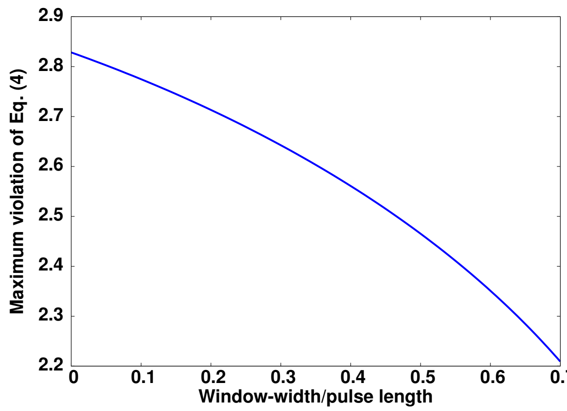

Figure 2. An illustration of the geometry of the curve at which the maximum violation of Eq. (4) approaches $2\sqrt{2}$ as the window-width approaches zero.

5. CONCLUSIONS

Bell's analysis is understood nowadays to have proved, that Quantum Theory is ineluctably either non-local or irreal. Besides apparently supporting a sever break with deep philosophical principles, these qualities put Quantum Theory in direct conflict with relativity. Thus, one is reasonably moved to examine its structure for the logical lacuna at the source of such difficulties. Given that the imputed structure of the singlet state, as a representation for complete ontic entities, requires something as blatantly *ad hoc* as the "Projection Hypothesis" to render it consistent with common observation, it seemed to this writer to be involved somehow with the suspected lacuna surmised above.

The analysis presented above is an assembly of considerations taken from various (largely unpublished) studies by this writer to fathom the inner structure of experimental tests of Bell-inequalities (See: web page). Separately, this writer has also critically analyzed Bell's own theoretical analysis of the phenomena in EPR experiments. In this general effort he re-found technical errors in the derivation of Bell-inequalities (evidently first noted by Edwin Jaynes), that render them irrelevant when applied to correlated events—contrary to Bell's intended purpose. In this report, where the consequence of Bell's fundamental error would invalidate Eq. (4), however, for the sake of argument, no use was made this fact; the current significance for Quantum Theory given to Eq. (4) was accepted as valid. As mathematics it is valid in any case. Explaining the data taken in Bell-inequalities tests in terms of classical effects, along with disputing the contention, that Quantum Theory is ineluctably nonlocal, then also constitutes independent support for Einstein's view of Quantum Theory: it is incomplete,

In the end, this analysis, if free of its own lacuna, constitutes a new "loophole" in the logic of tests of Bell-inequalities. It is as potent as are its hypothetical inputs feasible physically and factually. Its negation would require conclusive rejection of the relevance of these inputs for done experiments.

REFERENCES

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