A CRITIQUE OF MAXWELL-LORENTZ ELECTRODYNAMICS

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Recherces critiques sur les théories électrodynamique de CL. MAXWELL et de H.-A. LORENTZ
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1. INTRODUCTION

The development of the new ideas introduced by MAXWELL into the science of electricity, and theories derived therefrom, certainly constitutes one of the most interesting chapters in the history of science, especially for its psychological aspects. For those being habituated to the aesthetic value of clarity given to classical theories by mathematical physics, his new ideas, upsetting the established order, evoked intellectual repugnance and, at first sight, seemed to promote outlandish confusion. MAXWELL's first publication (1856): *On* FARADAY*'s Lines of Force;* took thirty years and the full authority of a HELMHOLTZ to gain purchase as a new theory, not acceptance, but just to be considered worthy of interest. Acceptance finally resulted after experiments by HERTZ and followers, demonstrated the identity of light and electromagnetic oscillations, which thereby, confirmed the general ideas of MAXWELL, broke the last barriers and made it legitimate 'physics.' The origin of the obscurities in MAXWELL's works derives, in large measure, from the fact that he unified two very different conceptions. On the one hand, one tended to explain electric interaction in terms of the properties of a medium (an explanation that lead MAXWELL to various accessory hypothesis, which, in spite of his efforts, was a diversion from the concerns of electrodynamics). On the other hand, he calls on a phenomenological explanation by means of partial differential equations, and on an hypothesis on electromagnetic energy pertaining to certain vectors that characterize the electric and magnetic state of a body. The second tact brought only difficulties.

MAXWELL's theory, as extended by HERTZ to moving bodies, is not in accord with certain optical experiments (aberration, FIZEAU, etc.), or with those by EICHENWALD on the action of dielectrics in motion. The new form given by LORENTZ to MAXWELL's theory, on the other hand, is in perfect accord with these experiments; moreover, in incorporating FECHNER's and WEBER's assumption, namely, that all electric current is convection current, i.e., due to electron flow, a hypothesis verified ever more often recently, considerably simplifies the equations. The atomic paradigm it supports gives a clear view of these phenomena. Finally, by considering aether as immobile and present even in the interior of atoms, it overcomes an indeterminate gap in MAXWELL's theory that had not been corrected theretofore. An indetermination resulting from aether motion, that also exists in HERTZ's theory, but the existence of which no experiment has thus far confirmed. Finally, the reciprocal interpenetrability and ubiquity of aether within matter explains how a body traversing aether experiences no resistance, and that the 'aether wind,' which FRESNEL and LORENTZ estimate for the earth's motion about the sun to be about 30k./sec., has never been seen, even by the most sensitive of experiments.

In sum, MAXWELL's theory has a simple formulation, and overcomes mathematical difficulties, LORENTZ has bridged the chasm that separates MAXWELL's theory and the classical theories founded on the notion of action-at-a-distance, and made precise the reciprocal relationship on the one hand between the equations of WEBER and CLAUSIUS, and on the other, between MAXWELL's and his own.

Moreover, the simplified theory provides another advantage, namely, that it permits more rigorous criticism of the principles on which it is founded. These principles are a diverse lot. They include, to begin, the experimental basis, which at first view seems to confirm it, but which, in fact doesn't do so without reproach, as it verifies some points and leaves others, of equal importance, in the shadows. Thus, the question remains open, which modifications of LORENTZ's equations can be made without actually coming in conflict with experiments?

In addition, one may ask: what is the real meaning of the vectors **E**, electric force, and **H**, magnetic force, which enter into its equations? And, how shall they be related to the empirical facts they should represent? Analogous questions have been posed, in mechanics, where no roundly accepted answer has been forthcoming. Also, by introduction of the notion of electromagnetic 'mass,' and by the impotence of theory to explain the mechanical properties of aether, modern physics is inclined to conceive, conversely, of an electromagnetic origin for the laws of mechanics; thus making out of electrodynamics the pivot of a novel paradigm of nature replacing the old mechanical conceptions. It is, therefore, particularly important that no cloud obscures the logical foundation of this vast, new, intellectual edifice.

One finds among its basic assumptions the hypothesis that there exists an absolute system of coordinates; moreover, MICHELSON and MORLEY's experiment, as well as more recent and more precise versions, have revealed a formal contradiction to this theory, in so far as uniform translations, as in mechanics, seem to have no influence on concurrent optical or electromagnetic phenomena. LORENTZ, EINSTEIN, POINCARÉ and others have deduced from this the requirement to introduce a new hypothesis *without altering the fundamental equations.* They find it necessary therefore, to: a.) renounce the classical idea of universal time, thereby making simultaneity a relative concept, b.) invalidate the conception of the invariability of mass, c.) to suppress the idea of a rigid body, d.) to suppress the axioms of kinematics, and e.) the arithmetic addition of velocities, etc. This last point means, that if a radium atom emits two β-rays in opposite directions, each with velocity of 2.5×10^6km./sec., we can not say that the relative velocity of one ray with respect to the other is 5×10^6km./sec., ,rather, it is still: 2.5×10^6km./sec. Likewise, two simultaneous times for two events for some observer, need not be simultaneous for a second observer who is in motion with respect to the first. And it is a curiosity worthy of note, that a few years ago it was believed sufficient in order to refute a theory to show that only one or another of its deductions is false; nowadays however, MAXWELL's equations are considered so absolutely untouchable, that none of its consequences frightens anybody. Rather than conclude that these equations need be modified more or less seriously, it has been decided in stead to sacrifice kinematics, the notion of time, etc. After first having been ignored, even as a fruitful theory, more or less systematically for thirty years, we now take here the direct opposite extreme, and ask: do its equations really merit such excessive confidence?

My answer is generally negative, and I shall present here a resume of critiques of the theories of MAXWELL and LORENTZ in view of the relevant experiments. The details have been given elsewhere[1].

2. LORENTZ: ELECTRODYNAMICS

To begin, let us recall the fundamental equations of LORENTZ's formulation. Electric charges are fixed on ions considered as undeformable. Let **H** be the magnetic vector, **E** the electric vector, ρ be the charge density measured in electrostatic units, at the point x, y, z, at the instant t, where the coordinate system is that of the aether rest system, and v is the velocity of the electric carryng matter in the system x, y, z, t, and where c is the speed of light. The following equations obtain among these quantities:

$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + 4\pi\rho\frac{\mathbf{v}}{\mathbf{c}}, \tag{2.1}$$

and

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \tag{2.2}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \tag{2.3}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{2.4}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \tag{2.5}$$

The field so created by other charges in the aether exercises a vector force, $\mathbf{F}\rho dV$, on the charge element ρdV, where:

$$\mathbf{F} = \mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{H}. \tag{2.6}$$

In this theory, there is no magnetism: rather interaction by virtue of AMPÈRE's. currents.

Conditioned on certain hypothesis that we shall recall below, this system of equations can be integrated by introducing *retarded* potentials. One takes it, in effect, that any solution of Eqs. (2.1) through (2.5), where ρ and **v** are given, can be put in the form:

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \tag{2.7}$$

$$\mathbf{H} = \nabla \times \mathbf{A}, \tag{2.8}$$

where Φ is the scalar-, and **A** are the vector-potential, which in turn satisfy:

$$\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - \triangle\Phi = 4\pi\rho, \tag{2.9}$$

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \triangle\mathbf{A} = \frac{4\pi\rho\mathbf{v}}{\mathbf{c}}, \tag{2.10}$$

[1] RITZ, W., *Recherches critiques sur lÉlectrodynamic général* in *Œuvres,* 317-426 **XVIII** (1909). See also: *Du rî̂e de l'ether en Physique, Œuvers,* 447-461 **XX** (1909).

and

$$\nabla \cdot \mathbf{A} = -\frac{1}{c}\frac{\partial \Phi}{\partial t}. \tag{2.11}$$

The functions:

$$\Phi(\mathbf{x}, t) = \iiint \frac{[\rho']}{r} dV', \tag{2.12}$$

and

$$\mathbf{A} = \frac{1}{c}\iiint \frac{[\rho' \mathbf{v}]}{r} dV', \tag{2.13}$$

where $r^2 = \mathbf{x} \cdot \mathbf{x}$, are particular integrals of the system Eqs. (2.9), (2.10) and (2.11); they have the form of NEWTON's potentials with the difference that, in stead of taking the value of $\rho(\mathbf{x}', t)$ at the time t, one is to use the *past* time: $t - r/c$ where r is the distance between $\mathbf{x}$ and $\mathbf{x}'$, which we, following LORENTZ, shall indicate with the notation: $[\rho']$, $[\rho'\mathbf{v}']$, or generally:

$$[f] := f(\mathbf{x}, t - r/c).$$

The field is completely determined thereby, and introducing its values into Eqs. (2.6), (2.7) and (2.8), one obtains analogue expressions, that is, a triple integral over "retarded forces," which are, however, quite complicated, so that we shall not write them down, but which express the force exercised by a point charge on another unit charge by means of elementary interaction analogous to that considered in the old electrodynamics by GAUSS, except for the element of retardation. For two charges at a finite separation, given certain here unimportant conditions, one gets the following expression[2] for the force of the charge e' with velocity $\mathbf{v}'$ and acceleration $\mathbf{w}'$ on the charge e with velocity $\mathbf{v}$:

$$F_i = ee'\left\{K_i + \frac{1}{c}\left[(\mathbf{v} \cdot \mathbf{K})\left(\cos(rx_i) - v_r K_i\right]\right\}, \quad i = x, y, z; \tag{2.14}$$

where $\mathbf{K}$ is the electric force at $\mathbf{x}$, given by the expression:

$$K_i = -\frac{w_i'}{c^2 r\left(1 - \frac{v_r'}{c}\right)^2} + \frac{1 - \frac{v'^2}{c^2} + r\frac{w'r}{c^2}}{r^2\left(1 - \frac{v'}{c}\right)^2}\left[cos(rx_i) - \frac{v_i'}{c}\right], \quad i = x, y, z. \tag{2.15}$$

The distance r is that between e' and e taken at that past time, t', at which a light wave departing e' takes to reach e. The coordinates $\mathbf{x}'$ of e', $\mathbf{x}$ of e and their derivatives, the velocities and accelerations are all well determined functions of time, the instant of emission, t' is determined by the equation:

$$c^2(t - t')^2 = (\mathbf{x} - \mathbf{x}') \cdot (\mathbf{x} - \mathbf{x}'). \tag{2.16}$$

If the velocities are much less than the speed of light, and their changes are not too rapid (quasi-stationary states that is), in the majority of cases one considers in electrodynamics (with the exception of HERTZian oscillators and KAUFMANN's experiments with β-rays), one may expand a function, such as $f(t - r/c)$, with TAYLOR's formula:

$$f\left(t - \frac{r}{c}\right) = f(t) - \frac{r}{c}f'(t) + \frac{r^2}{2c^2}f^{(2)}(t) - \cdots,$$

[2]This expression was given by: SCHWARZSCHILD, K., *Gött. Nacht. Math.-Phys. Klause*, 126 (1903); see also: POINCARÉ, *Rendiconti del Circ. math. de Palermo*,**XXI,** 129 (1906); and: LANGEVIN, *Journal de Physique*, (1904).

and neglect terms with a factor $1/c^2$ or smaller. This gives an expression for the elementary action of e' on e in the form of action-at-a-distance:

(2.17)
$$F_i = ee' \left\{ \frac{\cos(rx_i)}{r^2} \left(1 + \frac{v'^2 - 3v_r'^2 - 2(\mathbf{v} \cdot \mathbf{v}')}{2c^2} \right) + \frac{v_i' v_r}{r^2 c^2} - \frac{w_i' + w_r' \cos(rx_i)}{2rc^2} \right\}, \quad i = x, y, z,$$

This formula is particularly auspicious for comparison with classical formulas.

The writer's criticisms of the LORENTZ formulation are based on the following considerations.

3. LORENTZ: A SPECIFIC CRITIQUE

To begin, as said above, LORENTZ considered by hypothesis only the particular integrals, Eqs. (2.12) and (2.13) of the system of partial differential equations, Eqs. (2.9), (2.10) and (2.11); but there are also other solutions. We note the fundamental importance of this restriction: In distinction to mechanical phenomena, electrodynamic phenomena are irreversible by cause of radiation. But the equations given by LORENTZ do not change under a change of sign for time; they are reversible. To the contrary, in retarded potentials and elementary interactions, Eq. (2.14), the positive and negative time directions play different roles. Still, *one has introduced a velocity which by hypothesis, is impossible to alter*, i.e., the velocity with which waves *extend* away from their source charge(s); this is the cause of irreversibility of electromagnetic phenomena. One can easily see that the system of Eqs. (2.9), (2.10) and (2.11) admits an infinity of integrals other than Eqs. (2.12) and (2.13) which also satisfy the continuity conditions and behave well at infinity; in effect the general solution contains two arbitrary functions. Among these solutions, there are also those corresponding to convergent waves; i.e., containing $t + r/c$, in stead of $t - r/c$, in Eqs. (2.12) and (2.13), i.e., which emerge from infinity and converge onto the point charge—just the reverse of retarded interaction. These converging waves are physically absurd, however; they imply the possibility of *perpetuum mobile*. That is, if in Eqs. (2.12) and (2.13) $t - r/c$ is changed to $t + r/c$, in other words, if the sign of c is changed, it is easy to verify that the sign of POYNTING's vector is also changed. In so far as the usual solution pertains to a source which *loses* energy to radiation (that is to say, it continues to animate other particles to nonuniform motion), the sign-changed version must correspond to a *gain* of energy, which is provided by the aether at infinity rather than other bodies, and is, therefore, presumably inexhaustible. Under these circumstances, a charge constitutes a system capable of *perpetuum mobile*. In other words, *the equations of* LORENTZ *and* MAXWELL *admit an infinity of solutions which satisfy all conditions imposed by the theory, but which contradict empirical experience.*

It is certainly necessary, therfore, to add additional hypotheses to the theory, be they to the initial state, or to the boundary conditions at infinity, which exclude generally and completely all solutions except Eqs. (2.12) and (2.13). But, this seems impossible to do without undermining the basis of the theory itself. I have shown (*loc. cit.*, p. 166), that the only admissible and sufficient condition is that Eqs. (2.12) and (2.13) are acceptable as an initial state at time $t = t_0$ and at the consecutive instant $t_0 + dt$. All other hypothesis proposed thus far, in particular those of POINCARÉ, ABRAHAM and others, that the fields vanish at large distance at the instant t_0 are inadmissible, in so far as then at times $t < t_0$ *convergent waves* would be acceptable. But if the validity of Eqs. (2.12) and (2.13) are restricted to the instants t_0 and $t_0 + dt$, this imposes a condition with no meaning in terms of MAXWELL's ideas. This concerns an essential aspect of his doctrine which does not consider elementary interactions and the origin of fields, and that it doesn't concern itself with more than the immediate point. One sees that it is nothing but a means to eliminate the physically impossible solutions to his equations. Thus it follows, that one should adopt *a priori* the form of these retarded potentials leading to elementary interactions, like those of classical theories, and then verify that they satisfy the equations. Thus, these elementary interactions can completely replace the partial differential equations, while the opposite in not true. *These partial differential equations are thus inadequate to encompass the laws of propagation of the action of electricity and illumination.*

4. FIELDS: A GENERAL CRITIQUE

But, if retarded potentials are accepted, then what significance is to be given the vectors **E** and **H** which seem to play such an essential role in the theory? I say, that these vectors can be eliminated completely, and that, they play a role only as mathematical assistance in certain special cases.[3] Indeed, without knowing the significance of **E** and **H**, one can integrate the equations by means of Eqs. (2.12) and (2.13) simply by inserting them into (2.6), (2.7) and (2.8) to obtain **F** (i.e., the mechanical force exercised on a unit charge) expressed as the sum of elementary interactions originating from other charges. Moreover, **F** itself can be eliminated, as the state of motion of a charge, or system of charges, is by hypothesis (whether or not they reside on real masses), determined by D'ALEMBERT's principle:

$$\sum\left(m_j\frac{d^2\mathbf{x}}{dt^2}-\mathbf{F}-\mathbf{P}\right)\cdot\delta\mathbf{x}=0, \tag{4.1}$$

where **F** represents forces arising from elementary interactions, and where **P** results from other, non electric forces. Thus, Eq. (4.1) concerns only the objective state of motion of these objects; fields in aether play no role in it at all. In any case, to determine the state of a field at a point, one must insert a charge at that point. It would be otherwise if **E** and (or) **H** were to modify aether or set it in motion, as supposed by MAXWELL. In that case, it might be possible to utilize interference effects of light, without putting a charge at the point of interest, to reveal the effects of such alterations. Numerous clever experiments with this aim have given, however, only negative results. The hypothesis regarding these supposed aether motions has led to no mechanical explanation of electrodynamics. LORENTZ, and with him many others, have been forced, therefore, to concoct an abstraction.

We see therefore, that from the point of view of the facts, that the notions of electric and magnetic fields, and their partial differential equations with continuity conditions, are insufficient. We see that to determine the solutions, only elementary interactions, or more precisely, Eq. (4.1), is fully adequate, which is not true for LORENTZ's theory. The former has from the start the advantage of containing nothing but space-time relations, and certain invariant constants called the 'charges.' The concept of force can be completely eliminated.

Moreover, as SCHWARZSCHILD showed (*loc. cit.*), elementary interaction links up quite directly with classical physics. Also, CLAUSIUS has pointed out an equation that expresses, with the hypothesis of action-at-a-distance, the action of one charge on another (this formula is the analogue of the celebrated WEBER formula, but based on considerations involving absolute motion), to which it is only necessary to add the time-of-flight or 'law of propagation' to get LORENTZ's formulation. If one imagines that he has heard of

[3]Ordinarily one defines **E** as a mechanical force exercised on a unit charge at a point, where this charge is taken to be at rest with respect to the aether. But, it is not known how to apply this condition; this definition, therefore, must be rejected. In reality, one observes only **F**, and then deduces **E** and **H** using Eq. (2.6); these two vectors are defined only by their equations, whatever point of view one takes.

this notion already from GAUSS and RIEMANN, he might be astonished to realize how, in this regard, science in its linear development passes along twisted routes through logical thickets and then returns so close to its point of departure.

But, by following these twisted routes, it has gained competence. For example, it has come to conceive of light also as an electromagnetic phenomenon, and this conception has modified optics fundamentally. All that has been said above about electromagnetic phenomena, also pertains to optics. Aether and the partial differential equations are considered artifacts; in reality, what is observed, is nothing but elementary interactions between the atoms of the source and those of the eye or photographic plate. All optical phenomena derive from the principle of superposition.

Aether, which seemed to play such an essential role in the theory, is robbed of its domain, and step by step it has been reduced in significance to that of just being an absolute coordinate system, that is, one independent of any ordinary matter, a system with respect to which one measures the velocity of waves and electrons. It must be emphasized, that experiments never reveal this mathematical phantom, and that contrary to LORENTZ's formulas, absolute motion seems never to play a role in physics.

Let us mention, in passing, other objections to which the notion of aether gives occasion, that are generally admitted by modern physics, e.g.: the distribution and motion of rest energy, to a large extent arbitrary; however, there are multiple *simple* solutions to this problem (*loc. cit.*, 172-179). Moreover, in suppressing the motion of the aether, one also suppresses the principle of action and reaction—although, there are other paradigms that can be found for propagation of waves permitting the salvage of this principle, as we shall see below. Finally, the notion of field can not be applied to gravitation (*loc. cit.*, 179), as MAXWELL himself remarked, as aether would be in an unstable state by cause of the negative energy of gravitation. Thus, the notion of field can not constitute a general basis capable of replacing mechanics.[4]

Anticipating comments below, note, that Eqs. (2.14) and (2.15) for elementary interactions of point charges capture the essence of LORENTZ's theory, and involve an absolute velocity, be it explicitely, or be it in the law of propagation, Eq. (2.16). In so far as to date only relative velocities play a role in experiments, it is *a priori* clear that it should be possible, without contradiction with empirical evidence, to make significant modifications to LORENTZ's formulas concerning velocities; that is to say, this should be possible because these formulas are, to a large degree, *hypothetical.* In order to specify more precisely what these changes should be, to start let us consider quasistationary phenomena for which Eq. (4.1) pertains. So far, no electromagnetic effect depending on the velocity of a closed or nearly closed circuit of charge carriers, or where certain velocities are negligible with respect to others, has been observed. Experiments by ROWLAND and EICHENWALD, etc. on induction by cause of motion of cathode rays, fall into this category. One finds then for Eq. (2.17):

a) Terms of the order $\mathbf{v}'^2$ or $\mathbf{v}_\mathbf{r}'^2$, introduced by the series expansion of $f(t-r/c)$, which have little influence or effect;

[4] I shall not consider here the difficulties arising from the notion of an elastic aether, neither shall I show how superficial the analogy between MAXWELL's equations or those of optics and equations for an elastic solid, is; an analogy that has given MAXWELL and others vain hope. One can no longer doubt that there shall be a mechanical explanation for electric interaction. The writer has considered this question in the work cited above and also draws attention to: POINCARÉ, H., *Électricité et Optique*, Chap. 4 (Paris, 1901).

b) terms of the form:

$$\frac{ee'}{r^2c^2}\{-[\mathbf{v}\cdot\mathbf{v}']\cos(rx_j)+v'_jv_r\} = f_j,$$

that can be replaced by

$$\frac{ee'\cos(rx_j)}{r^2c^2}\{-[\mathbf{v}\cdot\mathbf{v}']\cos(rx_j)+3v'_rv_r\} = f_{1j},$$

(which corresponds to AMPÈRE's formula for the interaction of current elements), and more generally, one may add to these terms the differences $A(\mathbf{f}-\mathbf{f}_1)$ where A is an *arbitrary* constant, without affecting the agreement with experiment. Finally, one can complete these expressions for the terms in $\mathbf{v}'^2, \mathbf{v}^2$, etc. so that they contain only *relative* velocities and such that action and reaction are equal, which is not the case for $\mathbf{f}$. Let:

$$\mathbf{v}-\mathbf{v}' := \mathbf{u}, \quad \mathbf{u}\cdot\mathbf{u} = u^2,$$

and let k be an arbitrary constant, then the most general expression for the electrodynamic terms containing only relative velocities must be:

$$f_{2,j} = \frac{\cos(rx_j)}{4r^2c^2}\{(3-k)u^2-3(1-k)u^2r\} - \frac{(k+1)u_ju_r}{2c^2r^2}.$$

One might suppose, however, that a circulating electron current would engender only magnetic fields. One realizes this, if one recalls that the action of a magnetic field is not observed when it is due to a closed circuit, there is then, in any case, a magnetic potential proportional to the solid angle under which the current e' is seen (as we do not consider the linear case). Now, the surface of a polygonal figure traced on a sphere is expressed as the sum of the angles that are formed, each with the following one, by the sides of the polygon. For a continuous curve, its angles must differentiate the share of the angle of the continuous spherical curve, and this expresses by means of the radius of the curve of C and from the direction with respect to the radius. The molecular hypotheses permits expressing this curve, be it by the acceleration of the electron, be it by electron non-symmetry, by its rotation. One so obtains the entirely new decomposition of the action of a closed current as elementary interactions, which are considered variable for every current element closed or not, and which by integration along the current in all cases constitutes the magnetic potential. The magnetic field is also created by current elements likewise determined, and the force exercised on one charge e in motion is, as in LORENTZ's theory, $(e/c)(\mathbf{v}\times\mathbf{H})$, where v is the relative velocity with respect to the element.

In sum, regarding terms dependant on velocity, we are not now better informed that we were during the times of WEBER and HELMHOLTZ.

There is also the term:

$$\varphi_j = \frac{ee'}{2rc^2}[w'_j + w'_r\cos(rx_j)], \quad j = x,y,z,$$

which is dependant on acceleration. One can say that all empirical experience, all we know of electric oscillations, illumination and induction in open and closed circuits, is provided *uniquely* by this term. It can be decomposed into two others:

1) the first is: $ee'[-w'_j + w'_r\cos(rx_j)]/rc^2 = \Psi_j$, which is, taking terms of $1/c$, nothing other than the term:

$$\frac{-w'_j + w'_r\cos(rx_j)}{c^2r\left(1-\frac{v'_r}{c}\right)^2},$$

from Eq. (2.15), which plays the role of the FRESNEL vector in optics. This is the one from which all oscillatory phenomena at large distance from the source depend. This term plays no role in induction in closed circuits as it may be written:

$$-\frac{d}{d\mathbf{x}}\frac{w'_r}{c^2},$$

so that in an integration of the force

$$\vec{\Psi}\cdot d\mathbf{x},$$

around a closed circuit, it makes no contribution.

2) $ee'[w'_j - 3w'_r\cos(rx_j)]/(2c^2r) = \chi_j$, which comes entirely from the series expansion, and by cause of the finite speed of propagation, and which determines induction phenomena in closed circuits and electric forces in the immediate vicinity of HERTZian oscillators (with the electrostatic term $ee'\cos(rx_j)/r^2$, of which the form is in no doubt).

But this impels the remark that: *the propagation law at the wave center of an emitting ion at the instant* τ *stays constantly driven in rectilinear and uniform motion at a velocity equal to that of the ion at the instant* τ*, and also gives per the electrostatic term, the term* χ_j*, which result can be generalized.* One can not conclude that this center will remain at rest, as in the theory of an immobile aether.

The reaction of a charge system to itself, when there is acceleration, that is to say the expression of its electrodynamic mass for low velocities, depends exclusively on φ_x; the existence of such a reaction should not, therefore, be doubted; it is absolutely independent of all incertitude regarding relative or absolute motion in the electrodynamic terms, and of the law of propagation.

Let us return now to Eqs. (2.14) and (2.15); *one can, in them, lay out all the* $\mathbf{v}'$ without aid of any empirical data, be it from optics, be it from electrodynamics, even modified in some reasonable way, just that the term $-v'_x/(cr^2)$ must remained unchanged. Only this term, first order with respect to the speed of light, plays no role from the start in optics or for HERTZian oscillators, no term of its type remains in Eq. (4.1). Other laws, not involving considerations based on absolute coordinates, render it useless.

When terms in Eq. (2.14) linear in $\mathbf{v}$ (electric force properly speaking), contain the factor $1/c^2$ and play no role in quasistationary phenomena, we have seen how their form remains undetermined.

The last point, however, entails two restrictions, with respect to terms higher than second order. Light pressure corresponds to one of these terms, which is dependant on both acceleration and velocity; but its form remains undetermined. Further, KAUFMANN's experiments on β-rays from radium confirm the ensemble of terms in Eq. (2.14). Unfortunately, no conclusion can be drawn from that, as one can rebut it, be it with WEBER's formula, CLAUSIUS' or RIEMANN's formulas, or finally, from $f_{2,x}$, a infinite complex of terms dependant on powers higher than two and pairs of velocities divided by the corresponding powers of c, terms which play no role unless velocity is close to c, that is, e.g., in KAUFMANN's experiments. Each of these theories can, with an auspicious series expansion, satisfy experiments (see: *loc. cit.*, 189-197, 260-270), which shows, that even if there exists electromagnetic inertial reaction as, in fact, has been seen, the variability of this reaction with velocity, on the other hand, is surely hypothetical; it could not be deduced from KAUFMANN's experiments, except by adopting *a priori* LORENTZ's hypothesis on absolute motion and the forces f_x. This theory of the *variablity of electromagnetic mass rests, therfore, on the weakest points of* LORENTZ*'s theory. One can explain just as well, perhaps better, all the observations by auspicious modifications in the expressions for the force in terms dependant on velocity, in the same way that relative motion was introduced.*

It is scarcely useful to add that the little we know about molecular forces does not permit us to assert that the known laws of electricity are valid at all distances, however small. In reality, it is always the laws for point charges that we have to deal with, and there is no evidence for restrictions on their domain of applicability.

Finally, we have to enforce in theory that which experience teaches about the laws of propagation, in other words, express in the equations that emission is at the instant t', while action takes place and at the instant, t. In LORENTZ's theory, the wave emitted by an electron in uniform motion at the instant t' remains at all latter times in the form of a sphere for which the center remains at the emission point, and thereafter does not participate in the electron's motion. This hypothetical concept introduces, therefore, the concept of absolute motion, and, if one assumes that future experiments will reveal no more evidence for such motion than revealed so far, it will be necessary to reject it and to consider light motion as purely relative and dependant on the motion of the body producing it—to be achieved by means of renouncing, along with LORENTZ and EINSTEIN, both kinematics and the notion of time. The principle of *relative motion*, in its *classical* form, requires that a.) waves emitted by a system in uniform motion, shielded from material external influence, move with the system, in the manner such that the center of each spherical wave continues to coincide with the electron which emitted it, and that b.) the radial velocity is universal constant equal to c. When the electron's motion is arbitrary, the principle of relativity then would no longer determine the speed with which it depends on the wave center, as it would always be this constant speed (if not, there would be instantaneous action-at-a-distance between the wave and its source particle). Under this hypothesis, it will no longer be possible, it is true, to preserve the image of an "aether" or "waves in an elastic body" for such a law of propagation; but, if we wish to preserve such nevertheless, and with it the partial differential equations, it would be necessary to add a new hypothesis, namely that implied by the LORENTZ-EINSTEIN transformations, which, actually, profoundly change the conditions of the problem—for which the image of the "aether" or "elastic body" are rendered entirely inapplicable. Moreover, light propagation in LORENTZ's and EINSTEIN's views, actually does not comport itself consistent with a mechanical image at all. On the contrary, the propagation law we have announced, above, corresponds simply to the image of particles emitted in every direction with the same radial speed, which then continue in uniform motion; it approaches, therefore, in this respect, the emission law of NEWTON. I have shown (*loc. cit.,* Part II) that if one supposes that this law is valid for what the motion of an electron would be, and takes it that these ficticious particles act on electric charges with which they come in contact, one has no difficulty to construct an infinity of electrodynamic theories in perfect accord with empirical evidence, without concerning oneself with the optics of moving bodies. Herewith, experiments, interpreted in terms of the atomic conception of electricity which we have just adopted, give this unambiguous and simple result[5]: until a light ray puts the ions of an arbitrary body into oscillation, the centers of these waves do not move with the speed of the body (as our hypothesis would have it), but with the speed of the source of the light. Or, that which the principle of action and reaction would forecast. En effect, this principle can be read, per our hypothesis, to imply that the action of our ficticious particles (which serve only to provide an image) on the ions does not correspond to any reaction of the ions back on the source particles. It is necessary, as in LORENTZ's theory, to attribute to the ray energy, or a *quantity* of directed motion, which is more naturally done if one considers this energy projected, than if it is considered

[5]One easily verifies this theorem closely following LORENTZ's demonstration in: *Versuch einer Theorie der elektr. u. opt. Vorgänge in bewegten Körper,* (Leiden, 1985).

propagated; and if the initial speed of these fictitious particles emitted by an ion is determined by the principle of *conservation of momentum*, or the principle of reaction. That is, in the case of optics, one would take it that all ray energy is provided by the source, and that screens or optical devices provide no contribution; it is natural, therefore, to think that the principle of reaction, which may be precisely stated, will have the effect that the speed of the ficticious particles reemitted by the screen, etc., are uniquely determined by that of their original source.

Evidently, so far as a general theory directly based on such new views is absent, there will be place to study this issue in all its aspects, in particular, not to allow oneself to overlook the necessity of a new kinematics and dynamics, just as the LORENTZ-EINSTEIN hypotheses have led to.[6] But it is important to know, that *nothing* so far obliges us to consider the last hypotheses as correct, not to mention probable. And I believe that it will be regrettable for Physics if it does not find, in order to represent the laws of electrodynamics, simpler methods than those based on admitting from the start absolute coordinates, by writing a system of eleven equations, of which nine are partial differential equations, for which, after integration, by means of additional hypotheses one must reject impossible solutions or select possible solutions; and thereby complicate the already long procedure so obtained, with transformations destructive of the principles of kinematics, and, therefore, for which the explicit purpose is to preclude the consequences of the absolute coordinates misguidedly introduced in the first place. Finally, these are not the only reasons this theory displeases me. The equality of the units of the speed of light, one says, are explained in MAXWELL's and LORENTZ's theories. The complications of the first type render a clear view of the manner in which this result follows, difficult. But let us consider LORENTZ's equations. The speed c there figures into this issue in various ways, and it is not difficult to see that when c is held constant in the partial differential equations, but changes in Eq. (2.6) for the force from $1/c$ to $1/c'$, where $c' \neq c$, one does not modify the speed of propagation, nor units, nor energy[7], but rather the relationship of the electrostatic to the electrodynamic unit is changed; and the theory so explicated contains nothing that we made comprehensible because the coefficient of the term $\mathbf{v} \times \mathbf{H}$ is precisely equal to $1/c$. One chooses then, because observation demands it, exactly as it is in formulas from WEBER and CLAUSIUS, etc. This is nothing but that required by application of HAMILTON's principle, in a special form, where one finds the coefficient $1/c$ *a priori*. Only the principle, that which is used by LORENTZ, is clearly different than the principle in the ordinary sense, the variations are precisely those used elsewhere for fluids, for example; moreover, as SCHWARZSCHILD showed, there are different ways to use this principle. One of them determines directly the elementary forces, without considerations involving fields, a viewpoint to which we give preference in this work; The LAGRANGian function has the same form (near to propagation) that CLAUSIUS gave it:

$$\iint \frac{dEdE'}{r}\left[1-\frac{\mathbf{v}\cdot\mathbf{v}'}{c^2}\right],$$

[6] One must from the start note, that the LORENTZ-EINSTEIN theory is not, in part, a stage of the program of D'ALEMBERT's principle; or more generally, classical dynamics of systems is incompatible with their program, as EINSTEIN remarked; but, nothing else has replaced these fundamental principles. They persist, on the contrary, in the author's hypotheses.

[7] Terms with factors of $1/c'$ actually contribute no work. It is necessary from the start to recall that $\mathbf{H}$, as it is said, is defined by the theory itself; if it is defined *a priori* as the force (expressed in gauss) that it exercises on a pole of a permanent magnet, the coefficient of $\rho\mathbf{v}$, derives, in turn, from an empirical coefficient and remains unchanged.

where dE, dE' are charge elements, and where $\mathbf{v}$, $\mathbf{v}'$ must be taken at suitable instants.

By changing $1/c^2$ to $1/c'^2$, the above formula no longer conforms with observations, as the units cease to be c, but if the principle of least action continues to apply and the speed of propagation remains equal to c; then it is the partial differential equations which are no longer satisfied.

5. Conclusions

In summary, one sees that this remarkable relation does not result from LORENTZ's theory, however indirect, as much as from a determination of coefficients, as with WEBER and CLAUSIUS, to which one does not add the relativity condition to the principle of least action; in text books, ABRAHAM's for example, and even in LORENTZ's memoire mentioned above where he presents his theory, this principle is not mentioned and apparently considered secondary.

GAUSS, in a celebrated letter to WEBER indicated that without doubt, the electrodynamic terms result from the finite value of the speed of propagation, ensconced in a well chosen law, and developed in a series, as one has seen above, introducing effectively speed and accelerations with coefficients depending on c. The relation between the scale of units and the speed of light already has immediate significance. According to MAXWELL, the electrodynamic terms depend on the vector potential; this is, once again, a profound insight from GAUSS (it is important to reinforce this, as the opposite has been asserted), not in fact realized by MAXWELL and LORENTZ, but to which the future of electrodynamics may well belong.

Translated by A. F. KRACKLAUER,