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# Bell's inequalities and EPR-B experiments: are they disjoint?

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**Abstract.** It is shown that practical constraints of a fundamental nature preclude the application of Bell Inequalities to data from do-able EPR-B experiments. Thus, the violation of these inequalities by data from such experiments is without significance for the questions posed by EPR or BELL. Further, other implicit, misguided assumptions in analysis of this issue are discussed and a counterexample to conventional opinion in the form of a local realistic model and simulation of EPR-B experiments is displayed.

## MOTIVATION AND BACKGROUND

Covert hypotheses are the bane of logical reasoning. Bell's theorem provides a stark example. In fairness to BELL, he never stated what has become known as a 'theorem'. What he did do, however, is state very clearly what has the form of a conclusion of a theorem:

> In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measurement device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant.[1, p. 20]

This statement, as it stands, may be technically incontestable. This is so thanks to the phrase: "parameters are added to quantum mechanics". Strictly taken, this statement requires that an alternate explanation remain faithful to all the baggage that comes with quantum mechanics (QM), including VON NEUMANN's measurement theory (vNMT), with superposition of mutually exclusive outcomes and the concept of 'wave collapse'. That is, although such quantum features may be where the source of the limitation lies in the first instance, modifications should be restricted simply to adding additional parameters to QM.

What is legitimately arguable, however, is that an altered statement, i.e., a change in BELL's definition of the task, is definitely not true. The facilitating alteration is to replace the phrase extracted above with: "parameters are added to *those of* quantum mechanics". This change admits consideration of alternative theories free of vNMT, i.e., superposition, wave collapse and the like. Returning to pre-quantum physics precludes from the start the philosophical and interpretational issues that continue to plague QM interpretations. In addition, this modification seems to be in accord with contemporary understanding, in that virtually all literature on BELL's analysis of the EINSTEIN, PODOLSKY and ROSEN (EPR) conundrum, tacitly interprets BELL's statement to mean that all "local realistic" alternatives, vice just extended QM with additional variables, are excluded. Actually, in the end, all a successful extension of QM must provide is accurate numerical results for measurable physical quantities.

## CRITICAL REVIEW OF INEQUALITY DERIVATIONS

In this spirit, let us reexamine the derivation of a Bell Inequality, now with the intention of assuring that every step is sensible from the point of view of pre-quantum physics. To facilitate this approach, let us jump immediately to the

EPR-B (EPR as modified by BOHM) variant as applied to polarization phenomena, as they have no taint of quantum structure, in contrast to 'spin'. (The physics of such experiments is very well known; it shall not be repeated here again. See, *e.g.*: [2])

First, recall a mathematical technicality concerning the product of two Dirac delta functions, which is essential for what follows. It is that the integral of the product of two delta functions displaced from one another, is zero; i.e.:

$$\int dx\, f(x)\delta(x-l)\delta(x-m) = 0, \tag{1}$$

whenever $l \neq m$.

The derivation of a Bell Inequality starts from BELL's fundamental *Ansatz*:

$$P(a,b) = \int d\lambda \rho(\lambda) A(a,\lambda) B(b,\lambda), \tag{2}$$

where, per *explicit assumption:* $A$ is not a function of $b$; nor $B$ of $a$; and each represents the appearance of a photoelectron in its wing, and $a$ and $b$ are the corresponding polarizer filter settings.[1] This is motivated on the grounds that a measurement at station $A$, if it respects 'locality', so argues BELL, can not depend on remote conditions, such as the settings of a remote polarizer. In addition, each, by definition, satisfies

$$|A| \leq 1, \ \ |B| \leq 1, \tag{3}$$

which in this case effectively restricts the analysis to the case of just one photoelectron per time window per detector. Eq. (2) encodes the desideratum, that when the hidden variables are averaged out, the usual results from QM are to be recovered.

The $\lambda$ above in BELL's analysis stands for a hypothetical set of "hidden variables", which, if they exist, should render QM deterministic. This set may include many different types of variables, such as discrete, continuous, tensor or whatever.

Extraction of inequalities proceeds by considering differences of two such correlations where $(a,b)$, *i.e.*, the polarizer axis of measuring stations, left and right, differ:

$$P(a,b) - P(a,b') = \int d\lambda \rho(\lambda)[A(a,\lambda)B(b,\lambda) - A(a,\lambda)B(b',\lambda)], \tag{4}$$

to which zero in the form:

$$A(a,\lambda)B(b,\lambda)A(a',\lambda)B(b',\lambda) - A(a,\lambda)B(b',\lambda)A(a',\lambda)B(b,\lambda) = 0, \tag{5}$$

is added to get:

$$P(a,b) - P(a,b') = \int d\lambda \rho(\lambda) A(a,\lambda)B(b,\lambda)[1 \pm A(a',\lambda)B(b',\lambda)] - \int d\lambda \rho(\lambda) A(a,\lambda)B(b',\lambda)[1 \pm A(a',\lambda)B(b,\lambda)], \tag{6}$$

which, in turn, upon taking absolute values and in view of Eqs. (3), BELL wrote as:

$$|P(a,b) - P(a,b')| \leq \int d\lambda \rho(\lambda)[1 \pm A(a',\lambda)B(b',\lambda)] + \int d\lambda \rho(\lambda)[1 \pm A(a',\lambda)B(b,\lambda)]. \tag{7}$$

Then, using Eq. (2), and the normalization condition $\int d\lambda \rho(\lambda) = 1$, he got, for example:

$$|P(a,b) - P(a,b')| + |P(a',b') + P(a',b)| \leq 2, \tag{8}$$

a 'Bell inequality'.

Now, however, if the $\lambda$ are a complete set[2], thereby rendering everything deterministic so that all probabilities become Dirac or Kronecker delta distributions, then the $A$'s and $B$'s in Eq. (6) are pair-wise; that is to say as individual

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[1] *After-the-fact note*: Bell's notation, e.g., $P(a,\lambda)$ makes no distinction between variables, here $a$, and conditioning parameters, here $\lambda$, customarily separated by a vertical bar rather than a comma. This oversight is the source of much confusion, and possibly even the prime cause of his 'error.' In this paper, Bell's notation is retained whenever referring directly to his formulas.

[2] *After-the-fact note*: Bell used a single symbol: $\lambda$ to denote what could be a complicated set of variables of possibly different types even. Thus, a "particular values for $\lambda$" means that each entity in the whole set must have a value.

events comprising the generation at the source of one pair, are non-zero for distinct values of $\lambda$, which, by virtue of completeness, do not coincide for distinct events, *i.e.*, for different pairs. That is, for each pair of settings $(a, b)$ and iteration of the experiment, there exists a unique set of values, $\lambda_{(a,b)}$ say, for which $A(a|\lambda_{(a,b)})B(b|\lambda_{(a,b)})$ is non-zero ($\pm 1$ in the discrete case, $\pm\infty$ in the continuous case). In other words, each product $A(a|\lambda_{(a,b)})B(b|\lambda_{(a,b)})$ can be written in the form $f(x)\delta(x-\lambda_{(a,b)})$, so that all quadruple products

$$A(a|\lambda_{(a,b)})B(b|\lambda_{(a,b)})A(a|\lambda_{(a',b')})B(b|\lambda_{(a',b')}), \tag{9}$$

are of the form:

$$f(x)\delta(x-\lambda_{(a,b)})g(x)\delta(x-\lambda_{(a',b')}), \tag{10}$$

where $x$ is a dummy variable of integration to run over all admissible values of $\lambda$. Therefore, such terms with pair-wise different values of $\lambda_{(ab)}$ in Eq. (6), *i.e.*, when either $a \neq a'$ or $b \neq b'$, are, in accord with Eq. (1), identically zero under integration over $\lambda$. This annihilates two terms on the left of Eq. (8), so that the final form of this Bell Inequality, *resulting from the above complex of hypotheses,* (i.e., no vNMT) is actually, for example, the trivial identity[3, 4]:

$$|P(a,b)| + |P(a',b')| \leq 2. \tag{11}$$

Transferring this whole argument to a purely classical venue, precludes certain complications that are difficult or impossible to evade in QM. An example here is that the expressions $A(a|\lambda_{(a,b)})$ and $B(b|\lambda_{(a,b)})$ as quantum objects are superpositions of possible outcomes for various values of $\lambda_{(a,b)}$. Indeed, the source signal for a quantum model of an EPR experiment is the singlet state which is considered to be a superposition of mutually exclusive outcomes. On occasion this notion is extended such that information for so-called "counterfactual" measurements is considered contained in wave functions. That is, wave functions are presumed to have within them whatever is needed to give measurement results for both the setting $a$, and also for the setting $a'$, distinct from $a$, if, contrary to fact, it *had been measured.*

This notion relates to the issue of "reality", which is the philosophical position briefly captured by EINSTEIN's declared preference: the moon is in fact 'there', even when nobody is looking. From this position, the possibility that a wave function has no material identity before somebody "looks", *i.e.*, before it is collapsed by measurement, is excluded from the start. In other words, in a classical model of EPR experiments, 'reality' is encoded into the model by the assumption that instead of a superposition of possible outcomes as represented by the singlet state, the states are just a (random) selection of the base states that go into the superposition. While it is imaginable that the source signal could encompass information needed to determine the measurement outcome for different settings on each side separately, it seems clear that the vertical::horizontal ambiguity between the two terms of the singlet state can not be maintained under local realism. Presumably, just which basis state (term) represents any particular pair can be specified by one of the $\lambda$ parameters.

## CRITICAL REVIEW OF ABSTRACT INEQUALITIES

Besides the above line of analysis, there is a more abstract approach that, seemingly, is even independent of the specific physics of any EPR experiment. It is based on the character of dichotomic sequences of the sort produced by such experiments. It proceeds as follows:

Consider four dichotomic sequences comprised of $\pm 1$'s and length $N$: $a$, $a'$, $b$ and $b'$ ($a$, $b$ here are unrelated to those above) . Now compose the following two quantities $a_ib_i + a_ib'_i = a_i(b_i + b'_i)$ and $a'_ib_i - a'_ib'_i = a'_i(b_i - b'_i)$, sum them over $i$, divide by $N$, and take absolute values before adding together to get:

$$|\tfrac{1}{N}\textstyle\sum_i^N a_ib_i + \tfrac{1}{N}\sum_i^N a_ib'_i| + |\tfrac{1}{N}\sum_i^N a'_ib_i - \tfrac{1}{N}\sum_i^N a'_ib'_i| \leq \tfrac{1}{N}\sum_i^N |a_i||b_i + b'_i| + \tfrac{1}{N}\sum_i^N |a'_i||b_i - b'_i|; \tag{12}$$

or, in compact notation:

$$| < ab > + < ab' > | + | < a'b > - < a'b' > | \leq < |a||b+b'| > + < |a'||b-b'| > . \tag{13}$$

The right side equals 2, so this is in fact a Bell inequality. [4]

Here the question is: what do the physical circumstances of an EPR experiment dictate with regard to these terms? First, it is obvious that each term on the left individually is a correlation. Thus, the first term is the correlation of the appearance of photoelectrons right and left for the settings regime $a$ and $b$. The second term then, is the correlation that *would* obtain if instead of $b$ the one setting *had* been $b'$, *i.e.*, it is a *counterfactual* correlation. Likewise between the other two settings.

In reality, however, in a do-able experiment, only two of the product sequences can be taken as data. The other two would require a perfectly faithful replay of a factor sequence. In principle, if hidden variables can be manipulated, not just passively known, then it might be possible to carry out exactly such a replay to obtain, for example, that factor sequence $b'$ that is also correlated with $a$. As a practical task, however, this seems to be impossible, so that the best that can be obtained from do-able experiments would be only one or the other, either $b$ or $b'$ would be correlated with $a$ so that then either $b'$ or $b$ respectively, would be random or uncorrelated with respect to $a$. Of course, for the uncorrelated factor sequence, $b'$ say, it is true that $< ab' >= 0$. (This is most easily seen, perhaps, by recalling that the percentage of $+1$'s, say, in the sequence $b'$ will be proportional to $\cos^2(\beta)$ where $\beta$ is the angle that the signal polarization makes with the polarizer setting yielding sequence $b'$. Now if the two sides are uncorrelated, this proportionality factor will be $\sin^2(\beta)$ in half the iterations so that the sum of observed $+1$'s will be constant or equal to a purely random, uncorrelated distribution.) Applying this reasoning to all terms in Eq. (12), then, nullifies two of them, so that the inequality *for experimental output* is revealed to be of the same form as Eq. (11); *e.g.*,

$$| < ab > | + | < a'b' > | \leq 2, \tag{14}$$

a trivial tautology.

To further illustrate the logical impossibility of relating data taken from a do-able experiment with sequences required for Eq. (12), consider the following. Suppose there is a string of data available from an experiment. It will be comprised of four virtually equal length subsets, one for each setting combination; let the first subset be denoted $a_1b_1$, the second $a_2b'_2$, etc. (and where another serial or iteration counter subscript is implied). Now, it is obvious that for a particular setting, the percentage of $+1$'s in the total of long enough samples will be equal; *i.e.*, the number for $a_1$ equals the number for $a_2$ etc.; so that one can imagine re-sorting $a_2$ so that it has nearly the identical pattern as $a_1$. Denote the re-sorted version as $\tilde{a}_2$. Thus, the re-sorted second term in Eq. (12), for example, becomes $a_2b'_2 \Rightarrow \tilde{a}_2\tilde{b}'_2 \cong a_1\tilde{b}'_2$. Likewise the third and fourth terms become: $a'_3b'_3 \Rightarrow \tilde{a}'_3\tilde{b}'_3 \cong \tilde{a}'_3\tilde{b}'_2$ and $a'_4b_4 \Rightarrow \tilde{a}'_4\tilde{b}_4 \cong \tilde{a}'_3\tilde{b}_4$, respectively, so that the right side of Eq. (13) becomes:

$$< |a_1||(b_1 + \tilde{b}'_2)| > + < |\tilde{a}'_3||(\tilde{b}'_2 - \tilde{b}_4)| > . \tag{15}$$

Obviously, as $b_1 \cong \tilde{b}_4$ is not necessarily true identically, that is by cause of physical requirements from the experiment's setup, the loop can not be closed and the whole expression can not be limited always (nor even mostly) to being $\leq 2$.

What this means in toto is, that even if Eqs. (8) and (12) are regarded as valid sufficient constraints on correlations imposed by "local realism", they can not be applied in experiments that can be done. That is in short: Bell inequalities, *vís-a-vís the modified statement above* (no vNMT), and EPR experiments, are disjoint. The violation of Bell inequalities by both calculations done with QM and by data taken in past experiments can not be given significance, because the data streams do not meet the conditions of derivation of Bell Inequalities.

## ALTERNATE CRITICISMS

Quite independent of this line of reasoning, which, to this writer's best information, originated with DE LA PEÑA, CETTO and BRODY in 1972[5] and continues to the present[6–8], the extraction of Bell Inequalities has been criticized on completely different grounds by JAYNES[4, 9]. His point is that Eq. (2) results from a misconstrual of Bayes' formula or the 'chain rule' for conditional probabilities, namely:

$$\rho(a, b, \lambda) = \rho(a|b, \lambda)\rho(b|\lambda)\rho(\lambda), \tag{16}$$

where $\rho(a, b, \lambda)$ is a joint probability distribution and $\rho(b|\lambda)$ is a *conditional* probability distribution. JAYNES points out that BELL takes it that the presence of the variable $b$ in the factor $\rho(a|b, \lambda)$ implies instantaneous action-at-a-distance. This is true, however, only for the quantum case for which it is understood according to VON NEUMANN's measurement theory that wave functions are superpositions of the possible outcomes (even when mutually exclusive) whose ambiguity is resolved by collapse precipitated by the act of measurement. Eq. (16), however, for application in non-quantum circumstances implies no more than that there was a *common cause* for a *coincidence* in the past light cones of both measuring stations, a precondition which in QM is preempted by superposition.

The upshot is again that Eq. (2) does not pertain to non-quantum models; it is not a faithful encoding of locality as needed for theories not employing the irreality of wave functions interpreted according to VON NEUMANN's theory.

Either of these arguments lifts the ban imposed by BELL's analysis on local realistic alternatives to QM. It turns out, however, this is not the end of toxic covert hypotheses vexing this issue. Another one was concealed in BOHM's modification.[10] Originally EPR considered a *Gedanken* experiment in phase space, which in QM is spanned by the operators $\hat{x}$ and $\hat{p}$. Because realizing the experiment as envisioned was virtually impossible, BOHM proposed changing venue to polarization space spanned by $\hat{v}$, vertical, and $\hat{h}$ horizontal, (with an intermediate stop at spin-mechanics—which shall be ignored for the moment). This was done without noticing that while phase space operators do not commute (ultimately because of Heisenberg uncertainty), for a fixed direction of propagation, *i.e.*, fixed **k** vector, the latter do commute.[3] Thus, polarization space is a venue without quantum structure (*i.e.*,Heisenberg uncertainty) and can not, therefore, be used for the purpose of plumbing the mysteries of QM.[4, 13]

## A COUNTEREXAMPLE

Building on this understanding, it is possible to construct a model of EPR-B experiments that is fully local and realistic, but that nevertheless, delivers the correlations as calculated by QM and measured in experiments, contrary to the (modified) statement of BELL's conclusion.

The basic premise of the model is that EPR-B correlations arise from purely non quantum structure and can, therefore, be accurately calculated using the classical formula for high order correlations:

$$\Gamma^{(N)}(r_1,t_1,\ldots,r_N,t_N) = \frac{< \prod_{j=N}^{1} E_j^*(r_j,t_j) \prod_{j=1}^{N} E_j(r_j,t_j) >}{(\prod_{j=1}^{N} < |E_j(r_j,t_j)|^2 >)^{(1/2)}}. \tag{17}$$

The application of this definition to any particular experimental setup is not always transparent. Essentially it calls for an average of the tensor products squared of the output signals normalized by the intensity of the input signals. For an EPR-B setup, the following MuPAD[11] routine calculates the full correlation using Eq. (17):

```
//**********begin: eprb.mb**********

Matrix:=Dom::SquareMatrix(2):  vector:=Dom::Matrix(): //Function definitions.

Proj:=z->Matrix([[cos(z),sin(z)],[-sin(z),cos(z)]]): //Projection operator defined.

Sl:=n->vector(2,1,[[n],[1-n]]): Sr:=n->vector(2,1,[[n-1],[n]]): //Source signals defined.

El:=(n,z1)->Proj(z1)*Sl(n): Er:=(n,z2)->Proj(z2)*Sr(n): //Detector inputs calculated.

//Numerator for correlation  function defined:
 Num:=(n,m,o,p,q,z1,z2)->(-1)^c*El(n,z1)[m]*Er(n,z2+c*PI/2)[o]*
 Er(n,z2+c*PI/2)[p]*El(n,z1)[q]:
//Denominator left implicit, as for the above sources it equals 1.

//Calculate correlation; inner 4 sums over the physically realizable products
//of output signals; sum on "n" over input signals; sum on "c" to effect the
//weighted sum over "like" and "unlike" combinations to get setup correlation.
 _plus(sum(_plus(_plus(_plus(_plus(Num(n,m,o,m,o,z1,z2)
 $ m=1..2) $ o=1..2) $ p=1..2) $ q=1..2),n=0..1) $ c=0..1)/2^3:  simplify(%);

//**********end**********
```

The essential difference between classical and quantum correlations in Eq. (17) is that in the quantum version the factors $E(r,t)$ are creation or annulation operators and do not commute, whereas in the classical version they are simply

[3] Here many are misled often by the fact that when **k** is not fixed, it induces the non-commutivity of rotation on the sphere onto the $\hat{v}$ and $\hat{h}$ vectors orthogonal to it; this non-commutivity is a geometric effect, totally unrelated to QM.

scalar functions and do commute. It is easy to verify with the above routine that the factors $E(r,t)$ can be rearranged arbitrarily without affecting the outcome, namely: $-\cos(2(\theta_l - \theta_r))$, thereby confirming that EPR-B correlations are non quantum in nature.

The structure of an EPR-B experiment can be simulated to give a data-point-by-data-point (or photoelectron-by-photoelectron) exposition of the structure.[12, 13] It is best described in reverse order, so to speak.

The EPR-B experiment consists of a source emitting pairs of photons in the singlet state. As here a classical model is sought, instead of the singlet state comprised of a superposition, it is taken that one of two classical states, namely a horizontal pulse to the left and a vertical pulse to right, or vice versa, is randomly emitted. Then, after the source signal is determined, a random selection of two angular values of the axis of the polarizer filters left and right is made. Finally, these signals as modified by Malus' Law are sent to detectors.

In the usual way, the final step in the analysis of data taken in an EPR-B experiment, is to calculate the CHSH contrast:

$$S = k_{12} + k_{11} + k_{21} - k_{22}, \tag{18}$$

where the individual terms, $k_{ij}$ are the correlation functions when the polarizer on the left has angular setting $\theta_i$ and that on the right, $\theta_j$. Recall that "Bell's theorem" is taken to state that $|S| \leq 2$ for all local realistic theories.

From classical optics, the individual terms, $k_{ij}$ , for the intensity correlations for the signals passing through the polarizers, according to Malus' Law are given by:

$$\kappa = \frac{2\cos^2(\theta_r - \theta_l) - 2\sin^2(\theta_r - \theta_l)}{2\cos^2(\theta_r - \theta_l) + 2\sin^2(\theta_r - \theta_l)}. \tag{19}$$

From the fact that the angles from both sides appear in each term, one might be tempted to conclude that a certain non-local effect is involved in determining the values of these correlations. However, this is actually not the case. Each term in Eq. (19) can be expanded using the following trigonometric identities:

$$\begin{aligned} \cos(\theta_r - \theta_l) &= \cos(\theta_r)\cos(\theta_l) + \sin(\theta_r)\sin(\theta_l), \\ \sin(\theta_r - \theta_l) &= \sin(\theta_r)\cos(\theta_l) - \cos(\theta_r)\sin(\theta_l). \end{aligned} \tag{20}$$

Each of the factors in Eq. (20) in turn, is related, again by Malus' Law, to the data stream according to, for example:

$$\cos(\theta_l) \quad = \lim_{N\to\infty} \quad \sqrt{N_{sl}/N}, \tag{21}$$

for positive counts, where $N_{ls}$ is the total number of "hits" or photoelectrons registered in the left, $l$, detector when a signal (pulse) from the source polarized in $s$ mode, and $N$ is the total number of pulses in this regime intercepted by that detector. In typical experiments for which there are two possible angles for the polarizers on each side and two polarization modes (vertical or horizontal) given to the pulses by the source, it follows that $N = T/8$ where $T$ is the total number of pairs considered. Clearly, the compliment of Eq. (21); *i.e.*, $1 - \cos^2(\theta_l)$, gives the probability of a non-hit.

This simulation was realized with the following SCILAB[18] routine:

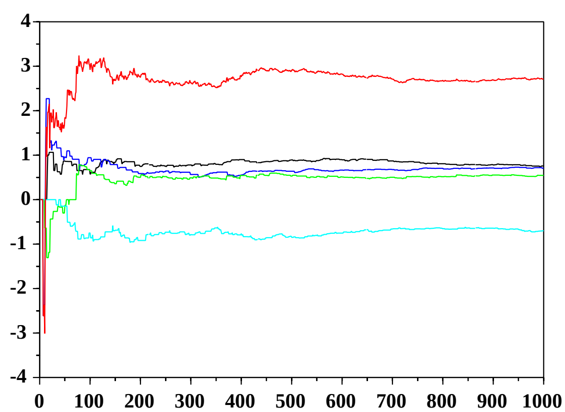
```
//*********begin: eprb_sim.sl*********

xbasc();             //clear graphic window.
getf('cor_fun.sl');  //Load subroutine.
N=1000;              //Set number of iterations.

//Initialize counters:
N11lc=0;N11ls=0;N11rc=0;N11rs=0;N10lc=0;N10ls=0;N10rc=0;N10rs=0;fcor00=0;fcor10=0;
N00lc=0;N00ls=0;N00rc=0;N00rs=0;N01lc=0;N01ls=0;N01rc=0;N01rs=0;fcor01=0;fcor11=0;

a1=0;a2=%pi/4;b1=%pi/8;b2=-%pi/8;   //Set polarizer angles:

for n=1:N;
t1=0; t2=0; p1=0; p2=0;                  //Initialize keys.
if rand()<.5 then k=1; else, k=0; end;  //Select source signal and set its key.
```

```
if rand()<.5 then a=a1; else a=a2; end; //Select left polarizer angle.
if a==a1 then t1=1; else, t2=1; end;    //Set key.
if rand()<.5 then b=b1; else b=b2; end; //Select right polarizer angle.
if b==b1 then p1=1; else p2=1; end;     //set key

//Record signs of angles:
sa=sign(cos(a)); sb=sign(cos(b)); sc=sign(sin(a)); sd=sign(sin(b));

//Given angles, apply Malus' Law to register a "hit" and set counts:
if rand()<k*cos(a)^2 then X=1; else, X=0; end;
if rand()<k*sin(b)^2 then Y=1; else, Y=0; end;
if rand()<(1-k)*sin(a)^2 then W=1; else, W=0; end;
if rand()<(1-k)*cos(b)^2 then Z=1; else, Z=0; end;

//Following 4 blocks update hit counters & correlation calculation
//for the 4 combinations of key (angle) settings, only 1 block responds per trial.

//Block 1:
if k*t1*p1*(X)==1 then N11lc=N11lc+1; end;
if k*t1*p1*(Y)==1 then N11rs=N11rs+1; end;
if (1-k)*t1*p1*(W)==1 then N11ls=N11ls+1; end;
if (1-k)*t1*p1*(Z)==1 then N11rc=N11rc+1; end;

if t1*p1==1 then cor11(n)=Kor(N11lc,N11rc,N11ls,N11rs,sa,sb,sc,sd,n); ...
else, cor11(n)=fcor11; end;  fcor11=cor11(n);

//Block 2:
if k*t1*p2*(X)==1 then N10lc=N10lc+1; end;
if k*t1*p2*(Y)==1 then N10rs=N10rs+1; end;
if (1-k)*t1*p2*(W)==1 then N10ls=N10ls+1; end;
if (1-k)*t1*p2*(Z)==1 then N10rc=N10rc+1; end;

if t1*p2==1 then cor10(n)=Kor(N10lc,N10rc,N10ls,N10rs,sa,sb,sc,sd,n); ...
else, cor10(n)=fcor10; end;  fcor10=cor10(n);

//Block 3:
if k*t2*p1*(X)==1 then N01lc=N01lc+1; end;
if k*t2*p1*(Y)==1 then N01rs=N01rs+1; end;
if (1-k)*t2*p1*(W)==1 then N01ls=N01ls+1; end;
if (1-k)*t2*p1*(Z)==1 then N01rc=N01rc+1; end;

if t2*p1==1 then cor01(n)=Kor(N01lc,N01rc,N01ls,N01rs,sa,sb,sc,sd,n); ...
else, cor01(n)=fcor01; end;  fcor01=cor01(n);

//Block 4:
if k*t2*p2*(X)==1 then N00lc=N00lc+1; end;
if k*t2*p2*(Y)==1 then N00rs=N00rs+1; end;
if (1-k)*t2*p2*(W)==1 then N00ls=N00ls+1; end;
if (1-k)*t2*p2*(Z)==1 then N00rc=N00rc+1; end;

if t2*p2==1 then, cor00(n)=Kor(N00lc,N00rc,N00ls,N00rs,sa,sb,sc,sd,n); ...
else, cor00(n)=fcor00; end;  fcor00=cor00(n);

//Compute CHSH contrast up to "n", and iterate:
D(n)=+cor01(n)+cor11(n)+cor10(n)-cor00(n); n=n+1; end;
```

```
//plot CHSH contrast (red) & correlations:
plot2d([cor11,cor10,cor01,cor00,D])

//*********begin: cor_fun.sl*********

function [cor]=Kor(a,b,c,d,sa,sb,sc,sd,n);
  //Calculate the cos (or sin) of the angle using Malus backwards:
f1=sa*a^.5; f2=sb*b^.5; f3=sc*c^.5; f4=sd*d^.5,

//Compute the system correlation using Malus forwards:
//where, e.g., cos(th_l-th_r)^2 =[cos(th_1)*cos(th_r)+sin(th_r)*sin(th_l)]^2
cor=((f1*f2 + f3*f4)^2 -(f1*f4 - f3*f2)^2)/(n/8)^2;

//*********end*********
```

The critical point here is that all the information needed to evaluate Eq. (21) is provided by the *delayed* signal from the source, the local polarizer setting, and noise at the detector. The last contribution is the one from previously 'hidden variables', here identified and displayable (in a simulation). Nothing is needed from remote locations. It is only in the calculation of the correlations after-the-fact that information from both sides is mixed; there is no role for non-local interaction at the detection events. Further, all of the implied processes are described by conventional, non-quantum physics.[4]

The simulation gives precisely the result for the right side of Eq. (8) calculated for settings on one side of $a = 0, \pi/2$, and on the other of $b = \pm\pi/4$, namely: $2\sqrt{2}$.[13] See: Figure. Its simple existence disproves BELL's assertion by counterexample.




1: **Figure:** Results from a typical run of the EPR-B simulation. The upper curve is the CHSH contrast, the lower four curves are the individual correlations.

This is not the first attempt to construct local-realistic models of EPR type experiments; see for example:[14–16]. These other simulations, however, all use tailored physical models of the detection process in which both channels are not identical; in this sense they are artificial. Alternately, instead of detector symmetry, other features, for example: fair sampling, can be sacrificed.[17] In any case, these modifications were introduced to obtain a model that is *both* local realistic, *and* does not violate Bell Inequalities. The latter stipulation, however, on account of 'disjointness', *inter alia*, is unnecessary. While these models are valid technical counterexamples, evidently because of implausible detection physics, they have not attracted all the attention they deserve.

The advantage of simulation is that it fosters abandonment of simple trust in formalism in favor of "physical insight". That is, the very act of creating a simulation routine forces one to consider in detail physical models for the processes involved; it forces one to examine various alternatives, until one that yields output mimicking data as seen in laboratory experiments is found. While success is not proof that a physical model is "really" faithful to a phenomenon, unsuccessful models as they stand can be eliminated.

As an illustration, consider a renowned conundrum billed as illustrating, in a particularly "simple but rigorous way, precisely what was extraordinary about quantum correlations".[19] One of the fundamental inputs into this conundrum is use of the 'rotational invariance' of the singlet state, which, by all currently accepted formalistic reasoning, means that regardless of the orientation perpendicular to the **k**-vector, EPR-pairs measured in orthogonal directions, are perfectly anticorrelated.

The impossibility of arranging for this to happen is glaringly obvious to one doing simulations, even while getting

[4] Because spin is homeomorphic to polarization, at root its spacial deportment must also be geometric in character, a point reinforced by the fact that so-called Pauli spin matrices were in use as Stokes' operators decades before spin was discovered.

output consistent with laboratory results is easily possible—thereby undermining claims of the inevitability of quantum mysticism. In other words, while it is possible to set up an arrangement such that, for an arbitrarily chosen direction, the results are perfectly anticorrelated, it is not possible, having made this choice, to then get perfect anticorrelation results in other orientations, because such measurements will exhibit scatter or statistical results governed by Malus' Law. (Calculation shows, however, that a random bias angle, fixed for each, but different between pairs, has no effect on the statistics of large ensembles of pairs.) The rub here consists in assuming that the quantum formalism actually requires turning a specific possibility into an universal ineluctability. However, there is no foundation for this assumption to be found in any precept of QM; it is simply an unexamined legend. When it is abandoned, as simulation forces one to do, the conundrum collapses.

This particular legend engenders much confusion throughout expositions of QM in relation to spin. One reads in virtually every textbook that were spin measurable in more that one direction, the result would be dichotomic in each direction. In fact, there is no support for this notion. One may take it, without fear of contradiction, that spin is actually the resolution of *Zitterbewegung* in terms of clock- and counterclockwise rotation about magnetic field lines, where the proportion of these two components is regulated by Boltzmann's thermodynamic factor. This "spin" motion then manifests itself via its coupling with larger scale motion about the same field line. Thus, while it is possible to resolve the *Zitter* motion about any axis; the observable coupling is restricted to B-field directions. If this model of spin is accepted, it is obviously ontological nonsense to consider spin in more than one direction at once for the same reason that there can never be more than a single B-field direction at once. Thus, the notion of measurements at angles to the axis of rotation, is oxymoronic because spin coupling itself is about the very magnetic field line being used to measure it; measuring at alternate angles rotates the system coordinates, not just an exterior measurement device.

## CONCLUSIONS

Fostered by the demands of simulation,'physical insight', instead of blind hope and faith in abstract formalism, forces identification of implicit hypothetic suppositions that have been insinuated into modern physics theories. Among such covert implicit (and misguided) suppositions pertaining to EPR-B/BELL analysis are the following:

**Hidden null terms:** Failure to identify null terms in the derivation of a Bell-Inequality;

**Statistical independence:** Locality misencoded as statistical independence;

**Data incompatibility:** Failure to observe that irrepeatability precludes taking relevant data;

**Rotational invariance:** Misinterpretation of the rotational invariance of a singlet state, and;

**Geometric non-commutivity:** Failure to see that non-commutivity of Pauli spin operators is actually due to the geometric non-commutivity of generators of rotations on a sphere.

Recognition of any one of these covert assumptions undermines BELL's conclusion that an extention of QM necessarily involves non-locality. The model and simulation displayed above exhibit in detail just how EPR-B experiments can be described using only local realistic concepts from classical physics.

The ultimate conclusion is, that the challenge posed by EPR to find a local realistic completion for QM, is not hopeless, quixotic reverie.

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