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Second entanglement and (re)Born wave functions in Stochastic Electrodynamics

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Abstract. The wave function in Quantum Mechanics has properties that render its physical interpretation unclear. On the one hand, its modulus squared is interpreted as a probability density, but, unlike conventional probabilities, it interacts with the material world. In addition, it is thought to have encapsulated nonlocal correlation and, according to modern thought, parallel information in a manner impossible according to classical physics. In this paper, an extended version of Stochastic Electrodynamics is presented which offers conventional models for these otherwise spooky features.

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PHILOSOPHY

Quantum Mechanics (QM) was discovered by trial-and-error. There is no unifying principle put forth by a single inspired 'founder.' The main phenomena captured by the early forms of QM, were the discrete spectral lines of atomic emissions. This structure was encoded by Heisenberg in a fully algebraic theory employing what turned out to be, essentially, group theory. An interpretation for, or the philosophical meaning of Heisenberg's theory was fully opaque; no model or paradigm was suggested to make it seem intuitive. Parallel to this development, Schrödinger discovered an equation for which the eigenvalues of the solutions corresponded to the terms in Heisenberg's formulation, that match up so well with spectral lines. This parallel development was welcomed by the philosophically inclined because the equation was a wave equation, and all wave equations theretofore had more or less obvious intuitive content. The big problem then was, however, that although the eigenvalues mated well with physical observations, just what was 'waving,' so as to yield these eigenvalues, was completely obscure.

Schrödinger himself struggled to find a physical identity for the solutions, and played with the notion that they (usually nowadays called 'wave functions') might represent charge density. Unfortunately, this conception evaded all consistent interpretation and the identity question remained open until Born suggested interpreting the modulus squared of a wave function as a "probability density of presence." What is truly remarkable about Born's idea is, that it has nearly nothing to contribute to the resolution of practical problems or understanding the application of QM to experiments. It is largely a matter of 'philosophy,' and can, in fact, be ignored.

Although as philosophy, it introduces another fundamentally mysterious feature. It is this: wave functions interfere; probabilities do not. Moreover, wave functions respond to physical barriers and boundaries. Interference and interaction with physical boundaries imply that wave functions somehow represent some kind of *physical* phenomena; they can not be just calculational aids, as are probabilities. Why, where and how does this *physics* as a mechanical input enter into the theory? Or, in short: how is it that a wave function is distinguished from a mathematical expression? This is the basic issue recognized by Schrödinger, and still of intense interest today. Indeed, the consternation that this feature introduced for students of QM is arguably the single most discussed theme, in one form or another, at the Växjö conferences.

There are surprisingly few attempts to identify possible *mechanical* inputs among the hypotheses of QM; most attempts, e.g., hydrodynamical models, etc., were made very early in the last century but eventually disregarded for lack of consistency. Nowadays, the general trend, based on the well appreciated success of QM to deliver empirically verified calculations, is to seek the answer within QM itself. Such attempts include Bohmian Mechanics, Consistent Histories, etc. In the end, none of these paradigms offers an explanation of the origin of the physical aspect of wave functions; they just find terms in the symbolics that imply or require its existence.

Aside from the issue of mechanical input, interpreting the modulus squared of a wave function as a probability density introduces the additional and perplexing feature that quantum 'probabilities' exhibit complex spatial patterns resulting from interference of the underlying wave function. The hallmark of the peculiar nature of this spacial dependence, is both its nonlocal response[1] to measurement and its spacial correlations which are symbolically manifested by the fact that a wave function, $\psi(x)$, and the resulting Born probability density, can not always be factored, i.e., there are circumstances when one can not write $\psi(x,y) = \psi(x)\psi(y)$. In terms of the resulting Born probabilities, this parallels statements concerning 'correlation' in probability theory, from which it is distinguished by a complex of prescriptions known as 'von Neumann's measurement theory.' According to this 'theory,' upon measurement of position, for example of a particle in a beam, the diffuse wave function "collapses" to the spacial point where the particle is actually observed. Since this collapse is imagined to instantaneously reduce a wave function which is finite on a large domain to a point, the process of collapsing transpires superluminally, i.e., non-locally.

This feature is unknown in ordinary physics of material objects and leads to many conundrums, perhaps the most famous of which has been rendered as Feynman's advice to students of QM: "Don't keep saying to yourself, if you can possible avoid it, 'But how can it be like that?' because you will get down the drain into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that." This advice was the best Feynman could offer for explaining the wave-like behavior of particles passing through a double slit as in Young's experiment. The conditional dependence of the wave function beyond the diffracting slits on both slits regardless of which slit the putative particle in fact must have passed through, reveals a kind of correlation (or in the jargon of QM: 'entanglement') of particles with themselves across spacial extentions that must be considered a unique, essential and intrinsic feature of QM.

Historically some researchers have endeavored to understand this feature by denying the identity of the particles as particles. No such attempt on the basis of *physical principles* has survived continued scrutiny; lexical concoctions such as 'wavicle' notwithstanding. A recent attempt by Khrennikov to capture this feature *mathematically* within the context of probability theory has lead to very interesting formal structures for novel variants of the probability of events given their *contextual* conditions.[1] Fascinating though this is as a logical structure, as physics it has the nature of a 'solution looking for a problem' as it does not provide insight into *how* such contextual information is *physically* insinuated, and then, evidently, on why only at the scale of atomic effects.

STOCHASTIC ELECTRODYNAMICS AND KINEMATIC TUNING

An exception to the trend in modeling QM is Stochastic Electrodynamics (SED).[2] This rubric is applied nowadays to a collection of notions that has a long history because of its rather natural motivation. The basic idea is that there exists a residue of random electromagnetic background radiation, that is stochastically perturbing atomic scale systems such that it should ultimately account for 'quantized' behavior. Although Nernst appears to have been the first to have suggested the notion, the first application of this idea is usually thought to be Welton's calculation of the Lamb shift. Thereafter in the 1960's several researchers took up the concept and applied it to various phenomena. Perhaps the most widely published of these efforts were made by Marshall[3] and Boyer[4]. The central addition to the conceptual foundations that they exploited to achieve progress was the additional hypothetical input that the stochastic background has a Lorentz invariant spectral power distribution, i.e., $E(\omega) = \hbar\omega$, where $\hbar$ is an empirically determined scale factor.

This stipulation was very fruitful. It lead to an SED variant for rationalization for the Planck black body spectrum, and a whole series of effects otherwise described by Quantum Electrodynamics, or in other words: 'second quantization.' The one thing it did not do, however, is offer any insight into the seminal quantum phenomenon, the wave like navigation of *particle* beams. It is this aspect, which derives from Schrödinger's equation, and is captured by "Feynman's conundrum," regarding the diffraction of particle beams by a Young double-slit experiment, which is the crucial effect of QM and in the end responsible for its uniqueness.

It was this lack of insight into what can be called 'first quantization,' in spite of the logical and successful application to 'second quantization,' that caught the eye of this writer in the early 1970's. He, motivated by the term "stochastic," tried to model and simulate, on computers, particle beam diffraction with various diffusion processes, an effort that lead to nothing but more unneeded demonstrations of the central limit theorem. Detailed examination of the foundations of diffusion processes eventually lead to the realization that all such processes are regulated by parabolic differential equations, whereas the Schrödinger Equation, the *non plus ultra* of QM, is a hyperbolic differential equation.[5] The

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[1] In the context of QM, the term "nonlocal" is usually taken to mean: superluminal interaction outside the light cone; its antonym is then "causal." This use must be distinguished from *topological* 'locality' for which the antonym is 'global.' The latter is unrelated to interaction, rather to the analytic properties of the equation for which the expression of interest is a solution.

solution spaces of these equation types are *topologically* different. They can not be deformed one into the other or expanded under any microscope so as to expose underlying structural similarities. The distinction in these spaces demands a distinction in the physics of the equations whose solutions 'live' in these spaces. In short, QM can not be modeled by any diffusion process governed by a parabolic equation; the physics is fundamentally different.

In response, the identification of a new *physical* hypothesis was then made by this writer in his thesis in 1973[6]; and subsequently published elsewhere.[7–10] The key idea is that particles exposed to the Lorentz invariant background effectively tune to a single, particular signal in the SED background, with which it then remains in energetic exchange, even as this signal itself is modified by physical boundary conditions.[10] The preferential tuning is simply a consequence of dynamic equilibrium and is supposed to occur at the *Zitterbewegung* frequency. The signal with privileged contact with the particle is seen, essentially tautologically, as a standing wave in the particle's rest frame. In a relatively moving frame, say that of Young's slits, this standing wave is a traveling wave for which the modulation envelope is what is otherwise known as the particle's de Broglie wave. In other words, this complex of effects gives rise to a pilot wave that accounts for wave-like navigation of particles. The complex of effects leading to the attachment of a pilot wave, conventionally called a 'matter wave,' can be denoted 'kinematic tuning,' as it is a result of the motion of a particle through the Lorentz invariant, electromagnetic, stochastic background, where, however, the crucial feature is *not* the stochastic element, but the coherent modulation resulting from relative motion. The question of the origin of this electromagnetic background is, of course, fundamental. In the historical development of SED, its existence has been posited as an operational hypothesis whose justification rests *a posteriori* on results. Nevertheless, lurking on the fringes from the beginning, has been the idea that this background is the result of self-consistent interaction; i.e., the background arises out of interactions from all other electromagnetic charges in the universe.[11]

For somewhat deeper, more quantitative insight into this paradigm, consider the following: Although any system with charge structure, not necessarily a net charge, will suffice, let us take as a prototype system a dipole with characteristic frequency ω. Equilibrium with the background for such a system in its rest frame can be expressed as

$$m_0 c^2 = \hbar\omega_0. \tag{1}$$

This statement is actually tautological, as it just defines ω_0 for which an exact numerical value will turn out to be practically immaterial.

This equilibrium in each degree of freedom is achieved in the particle's rest frame by interaction with counter propagating electromagnetic background signals in both polarization modes separately, which on the average, add to give a standing wave with antinode at the particle's position:

$$2\cos(k_0 x)\sin(\omega_0 t). \tag{2}$$

Again, this is essentially a tautological statement as a particle doesn't 'see' signals with nodes at its location, thereby leaving only the others. Of course, everything is to be understood in an on-the-average, statistical sense.

Now consider Eq. (2) in a translating frame, in particular the rest frame of a slit through which the particle as a member of a beam ensemble passes. In such a frame the component signals under a Lorentz transform are Doppler shifted and then add together to give what appears as modulated waves:

$$2\cos(k_0\gamma(x - c\beta t))\sin(\omega_0\gamma(t - c^{-1}\beta x)), \tag{3}$$

for which the second, the modulation factor, has wave length $\lambda = (\gamma\beta k_0)^{-1}$. From the Lorentz transform of Eq. (1), $P = \hbar\gamma\beta k_0$, the factors $\gamma\beta k_0$ can be identified as the de Broglie wave vector from QM as expressed in the slit frame.

In short, it is seen that a particle's de Broglie wave is modulation on what the orthodox theory designates *Zitterbewegung.* The modulation-wave effectively functions as a pilot wave. Unlike de Broglie's original conception in which the pilot wave emanates from the kernel, here this pilot wave is a kinematic effect of the particle interacting with the SED Background. Because this SED Background is classical electromagnetic radiation, it will diffract according to the usual laws of optics and thereafter, modify the trajectory of the particle with which it is in equilibrium. See references [6–10] for a more detailed development of these concepts.

The detailed model of the mechanism for pilot wave steerage is based on observing that the energy pattern of the carrier signal that pilot waves are modulating, and to which a particle tunes, comprises a fence or rake-like structure with prongs of varying average heights specified by the pilot wave modulation. These prongs, in turn, can be considered as forming the boundaries of energy wells in which particles are trapped; a series of micro-Paul-traps, as it were. Intuitively, it is clear that where such traps are deepest, particles will tend to be captured and dwell the longest. The exact mechanism moving and restraining particles is radiation pressure, but not as given by the modulation, rather by the carrier signal itself. Of course, because these signals are stochastic, well boundaries are bobbing up and down somewhat so that any given particle with whatever energy it has will tend to migrate back and forth into neighboring cells as boundary fluctuations permit. Where the wells are very shallow, on the other hand, particles are laterally (in a diffraction setup, say) unconstrained; they tend to vacate such regions, and therefore have a low probability of being found there.

The observable consequences of the constraints imposed on the motion of particles is a microscopic effect which can be made manifest by observing many similar systems. For illustration, consider an ensemble of similar particles comprising a beam passing through a slit. Let us assume that these particles are very close to equilibrium with the background, that is, that any effects due to the slit can be considered as slight perturbations on the systematic motion of the beam members.

Given this assumption, each member of the ensemble with index, n say, will with a certain probability have a given amount of kinetic energy, E_n, associated with each degree of freedom. Of special interest here is the direction perpendicular to both the beam and the slit in which, by virtue of the assumed state of near equilibrium with the background, we can take the distribution, with respect to energy of the members of the ensemble, to be given in the usual way by the Boltzmann Factor:$e^{-\beta E_n}$ where β is the reciprocal product of the Boltzmann Constant k and the temperature, T, in degrees Kelvin. The temperature in this case is that of the electromagnetic background serving as a thermal bath for the beam particles with which it is in near equilibrium.

Now, the relative probability of finding any given particle; i.e., with energy $E_{\{n,j\}}$ or $E_{\{n,k\}}$ or $\ldots$, trapped in a particular well will be, according to elementary probability, proportional to the sum of the probabilities of finding particles with energy less than the well depth,

$$\sum_{\{l|E_{n,l}\leq d\}} e^{-\beta E_{n,l}} \simeq \int_0^D d(\frac{E_n}{E_0})e^{-\beta E_n} = \frac{1}{\beta E_0}(1-e^{-\beta D}), \tag{4}$$

where approximating the sum with an integral is tantamount to the recognition that the number of energy levels, if not *a priori* continuous, is large with respect to the well depth.

If now d in Eq. (4) is expressed as a function of position, we get the probability density as a function of position. For example, for a diffraction pattern from a single slit of width a at distance D, the intensity (essentially the energy density) as a function of lateral position is: $E_0\sin^2(\theta)/\theta^2$ where $\theta = k_{pilot\,wave}(2a/D)y$, and the probability density of occurrence, $P(\theta(y))$, as a function of position, would be

$$P(y) \propto (1-e^{-\beta E_0\sin^2(\theta)/\theta^2}). \tag{5}$$

Whenever the exponent in Eq. (5) is significantly less than one, the right hand side is very accurately approximated by the exponent itself; so that one obtains the standard and verified result that the probability density of occurrence, $P(y) = \psi^*\psi$ in conventional QM, is proportional to the intensity of a particle's de Broglie (pilot) wave.

On the basis of this kinematic tuning, the question can be posed: how is an ensemble of similar systems to be analyzed in the spirit of statistical physics. The answer lies in the fact that kinematical tuning associates a wave with the velocity of a particle so as to suggest the existence of a *physically* motivated Fourier kernel, which, when applied to the Liouville equation for the ensemble, leads to a transformation to Schrödinger's Equation.[8] This part of the story turns out to be essentially the reverse of Wigner's derivation of what has become known as "Wigner densities." As such, it makes contact with recognized and verified useful analysis.[2] In effect, this particular association of Schrödinger's Equation with the Liouville equation is a rationalization for Born's interpretation of wave functions. From this view point, the historical debate between Einstein and Bohr is adjudicated completely in favor of Einstein, as a Born probability density is seen to be a phase space density, i.e., an average over underlying *Zitter* motion of extreme complexity, but in principle describable with additional variables; in other words: QM is incomplete as surmised by Einstein, Podolsky and Rosen.[12]

A consequence of the attachment of a de Broglie pilot wave to each particle is that there exists a Fourier kernel of the following form:

$$e^{\frac{i2\mathbf{p}\cdot\mathbf{x}'}{\hbar}}, \tag{6}$$

[2] To be sure, other "derivations" of Schrödinger's Equation can be found in the literature. None to this writer's best information, however, have a classical *physical* rationalization. They depend on formalistic manipulations employing fortuitous definitions of one or another quantity, typically velocity.

which can be used to decompose the density function of an ensemble of similar particles. Consider an ensemble governed by the Liouville Equation:

$$\frac{\partial \rho}{\partial t} = -\nabla \rho \cdot \frac{\mathbf{p}}{m} + (\nabla_p \rho) \cdot \mathbf{F}, \quad \left[\nabla_p \equiv \sum_{i=x,y,z} \frac{\partial}{\partial p_i} \right]. \tag{7}$$

Now, decompose $\rho(x, p)$with respect to p using the De Broglie-Fourier Kernel:

$$\widehat{\rho}(\mathbf{x}, \mathbf{x}', t) = \int e^{\frac{i2\mathbf{p}\cdot\mathbf{x}'}{\hbar}} \rho(\mathbf{x}, \mathbf{p}, t) d\mathbf{p}, \tag{8}$$

to transform the Liouville Equation into:

$$\frac{\partial \widehat{\rho}}{\partial t} = \left(\frac{\hbar}{i2m} \right) \nabla' \nabla \widehat{\rho} - \left(\frac{i2}{\hbar} \right) (\mathbf{x}' \cdot \mathbf{F}) \widehat{\rho}. \tag{9}$$

To solve, separate variables using:

$$\mathbf{r} = \mathbf{x} + \mathbf{x}', \quad \mathbf{r}' = \mathbf{x} - \mathbf{x}', \tag{10}$$

to get

$$\frac{\partial \widehat{\rho}}{\partial t} = \left(\frac{\hbar}{i2m} \right) (\nabla^2 - (\nabla')^2) \widehat{\rho} - \left(\frac{i}{\hbar} \right) (\mathbf{r} - \mathbf{r}') \cdot \mathbf{F} \left(\frac{\mathbf{r} + \mathbf{r}'}{2} \right) \widehat{\rho}, \tag{11}$$

which can (sometimes—See Ref. [8] for details.) be separated by writing:

$$\widehat{\rho}(\mathbf{r}, \mathbf{r}') = \psi^*(\mathbf{r}')\psi(\mathbf{r}), \tag{12}$$

to get Schrödinger's Equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi + V\psi. \tag{13}$$

Wave-like quantum phenomena imply *physical* perturbation or guidance of particles, so there must be a source of energy to do the necessary work. Since the SED background is taken to be an electromagnetic field, it can provide this energy, which represents an advantage over competing interpretations for QM in which the peculiar quantum effects are often thought to arise just from internal, usually mathematical, features; e.g., the 'quantum potential.' SED clearly addresses one of the fundamental questions regarding the nature of quantum wave functions, namely, how does *physics* as a mechanical agent enter into the theory?

KINEMATIC TUNING VERSUS DIFFUSION OR STOCHASTIC AGITATION

As implied above, the original motivation for SED takes much inspiration from the the theory of Brownian Motion. Some proponents of SED seemingly continue to hope that one or another version of diffusion or stochastic agitation alone will suffice to formulate a foundation for QM on the basis of SED. But, the fact that the solution spaces of the relevant equations are topologically distinct, arguably torpedos all hope in this direction. For this reason this writer takes a very pessimistic view of the prospects for success for simulations of quantum effects on the basis of SED, without explicit use of kinematic tuning.

One such study was reported by Cole and Zou.[13] In that study of the ground state of hydrogen, the SED contribution from the background was limited to frequencies near the orbital frequency of the electron in the ground state. It was taken that only these frequencies are likely to have a large cross section with the electron. This assumption does not accord with the fact that an electron has a very high *Zitter* resonance for which the signals in the background are correspondingly of much higher energy. On this basis, it can be expected reasonably that effects derived from energetic exchange at the *Zitter* frequency should overwhelm those at the lower orbital frequency. Moreover, it could well be that certain energy balance considerations insensitive to the details of the exact interaction, in particular the frequency, yield the coarse behavior of the ground state; and, indeed this may well account for the rather good approximation achieved with the reported simulations for the radial distribution of the ground state. Yet, what it seems 'parabolic' effects will never do, is exhibit the interference needed to mimic the structure of the excited states. Without the ‘hyperbolic’ structure of (pilot) waves, the essentially ‘parabolic’ nature of stochastic agitation alone is intrinsically insufficient.

These remarks pertain also to the study reported by Nieuwehuizen in which relativity was taken into account.[14]

The full relationship of SED with kinematic tuning to the so-called Linear Stochastic Electrodynamics (LSED) as propounded by Cetto and de la Pena, is less clear.[2, 15] These authors describe these two approaches as “complimentary.” The development they present, however, is based on just low frequency interaction with the background. If their concepts are correct, it is still not clear how the needed hyperbolic structure enters into LSED as they explicate it. Their arguments feature formulas seen for ordinary resonance of waves with charged particles, but with the difference of being based on a “dipole approximation,” which they describe as neglecting the the spacial dependence of the background signals agitating an atom, say. From a point of view exploiting kinematic tuning, however, it is just the spacial dependence of the background signals that brings in the ‘hyperbolic’ aspects required for quantum behavior. The added structure they draw upon to deal with this aspect, is the claim that several such waves interact, i.e., “nonlinear terms entangle the frequencies,” so as to result in more than just ordinary resonance phenomena. This assertion is not easily evaluated; LSED’s development proceeds through a sequence of assumptions and approximations that could easily conceal surprises, the least unexpected of which would be the physical analog of the central limit theorem: Gaussian agitation. It is hard to see just how among the plethora of feasible nonlinearities, just that one and only that one that results in ‘hyperbolic’ or wave-like time evolution would be privileged. Thus, should any one of such difficulties obtain, LSED could not evade the ‘parabolic’ dead-end; and, in any case, as currently developed, in the LSED there is no explicit reference to kinematic tuning or identification of a unique and inevitable functional equivalent.

Kinematic tuning has the appealing feature of a good pedigree by virtue of its close relationship to Wigner’s work on quantum phase space densities. This relationship has proved its value in numerous studies[17], so the link between Liouville’s Equation and Schrödinger’s Equation, entirely aside from its use for the kinematic tuning model, reflects substantial significance. For this reason, this writer is suspicious of all derivations of Schrödinger’s Equation that make no mention of this connection. It is highly likely that “derivations” that do not explicitely involve some *physical* rationalization for ‘Wigner’s’ Fourier kernel, especially if they are based on motivation making reference to statistics, probability or diffusion, are in fact vacuous tautologically by fault of covert hypotheses.

ENTANGLEMENT

On the basis of the mapping from phase space to the Schrödinger Equation by means of kinematic tuning, it is seen that the modulus squared of a wave function started life as a density on phase space, which is precisely the same thing as a probability density of presence as foreseen by Born. Since a density on phase space pertains to either non-interacting extant or conceptual (Gibbsian) replications of a system, the meaning of a wave function is specified; it does not pertain to a single system. If the opposite viewpoint is taken, that the wave function is ‘complete’ and refers to a single system, then the phenomena of internal entanglement arrises. Historically this problem was discussed in terms of wave collapse where the difficulty arises most clearly in connection with particle beams. If the wave function represents the ontological essence of a single particle, then upon measurement, which for example might mean registration at a point on some detector matrix in the form of a flash say, the wave pulse must “collapse” instantaneously to the point at which a particle was observed to impact the detector. As the wave could have been finite over a broad region, such a collapse would be nonlocal. From the perspective of SED (with kinematic tuning), however, none of these features is problematic. The wave function’s modulus squared is simply a usual density on phase space. Collapse represents just a change in knowledge, here no longer problematical, because the background waves have, so to speak, taken over the task of doing the work on the particles, and the Born probability density is allowed to be an unphysical calculational abstraction, albeit a very special one pertaining to systems in equilibrium with waves via the mechanisms described above.

With respect to this point there has arisen long ago an objection to the association of the modulus squared of wave functions to a phase space density; it is this: eigenfunctions converted to Wigner densities can exhibit regions where they are negative on phase space, and therefore can not be densities. The response to this observation is to note that there are states that do give positive Wigner densities; for example for the harmonic oscillator, these include coherent states and thermal states.[16] Given these facts, it is not unreasonable to surmise that only such states can actually constitute physically realizable states; others, in particular those that give negative densities, are taken then to be states which do not satisfy all the boundary conditions pertinent to the problem; they are simply mathematical intermediary steps.

The intricacies of wave function characterization get particularly interesting when two interacting particles are involved, because then the rules and algorithms yield an additional and also mysterious feature. When the two particles interacted while in contact and thereafter separate, the wave function for the system, i.e., the wave functions for the separate daughters after the interaction, can maintain a correlation. The prototypical example is that of the disintegration of a Boson into two 1/2 spin Fermion daughters. Conservation principles imply that the sum of the spins after a disintegration should remain null.

Historically we know that analysis of spectral phenomena lead to a complex of algorithms that yielded a very good match up with eigenvalues of the Schrödinger's Equation. These prescriptions had nothing to do with spin, for which the theoretical foundation seems to derive from Dirac's relativistic equation. Again, without a guiding principle, it was found by trial-and-error that wave functions needed to be doubled and that this structure was governed by the group SU(2), which also falls out formalistically of relativistic considerations. The point here is, that the addition of this SU(2) structure, although required to meet empirical facts and supported by Relativity, was not motivated *a priori* by a physical model. Subsequently, it has been vested with various folk notions resembling models, but it remains essentially mysterious.

In contrast, SED provides natural imagery for spin: it is the result of the polarization of the background signals to which a particle is tuned.[6, 8] That is, in a magnetic field, a particle can gyrate around the field line in equilibrium with either polarization mode of its background signal, where this effect is to be understood, not in a rigid sense, but statistically, and energetically in accord with the Boltzmann distribution. This imagery provides a happy rationalization for geometric and algebraic conformity of these two separate appearing phenomena, both of which are described by the same $SU(2)$ group structure.

Now, for two particles, the two state structure for each separately leads directly to a tensor product, or four state structure for the combined system. Set within quantum mechanics, this implies that for the two particles after the interaction there are four distinct combinations of densities, of which for any particular realization or event, intuitively it might be expected that only one is physically realized. It is here, that again another trial-and-error element imposing useful but uninterpreted structure has been insinuated into the complex of algorithmic rules-of-thumb constituting QM. It is this: while the natural basis for the tensor space of two two-state systems is simply the tensor products of the base states for the individual systems, this is augmented by the additional requirement that the proper basis be in fact that which is simultaneously comprised of eigenstates for the square of the sum of the individual states. Thus, where, say, spin is under consideration, instead of the states being simply eigenvectors of just $S = S_1 + S_2$, they are to be eigenstates of S^2 also.[3] This is 'explained' with, for example, the simple remark that: "it is sometimes convenient to consider the operator S^2." Fine! But, why? Especially, why in view of an interpretation for QM? And, what does this say about the identity of wave functions?

The first thing to note is, that while the basis for S is

$$|+,+>, \; |+,->, \; |-,+>, \; |-,->, \tag{14}$$

the basis for S^2 is

$$|+,+>, \; |-,->, \; \frac{1}{\sqrt{2}}(|+,-> + |-,+>), \; \frac{1}{\sqrt{2}}(|+,-> - |-,+>). \tag{15}$$

Among these latter states, the two on the right introduce, again, a kind of correlation or entanglement (non factorability) among system states that is over and above that present within each of the subsystem densities made from individual states of the first kind, e.g., Eq. (14). [4]

If follows from the well known fact that a measurement on either of the two-state subsystems will yield a unique output, either $|+,->$ or $|-,+>$, so that measuring just one subsystem alone dictates what the other must be, instantly, even across a space-like interval. As the two superposition states can not be factored, they are said to be "entangled" in a way that is not seen in classical physics, and thus held to be a "quantum phenomenon," which it can not be in reality as there is no *a priori* quantum hypotheses underlying it, although this matter has been ensconced retrospectively in QM as the "principle" of superposition in von Neumann's measurement theory. The latter comprises 'principles' distilled from the complex of calculational algorithms used in QM, but now without explicit restriction to Hamiltonian conjugate variables, and therfore without restriction to truly 'quantized' phase or quadrature space.

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[3] Upon reflection, this last stipulation can be seen to imply much of 'von Neumann measurement theory.' Both of these suppositions lack natural physical motivation.

[4] Sometimes the first two states in the set Eq. (15) are, for formalistic reasons again without *a priori* paradigmatic justification, from QM or elsewhere, replaced by $(|+,+> \pm |-,->)/\sqrt{2}$, in which case the set is known as the "Bell basis."

From these remarks, it can be seen that in fact this additional entanglement arises, not from any strictly QM hypothesis, i.e., *optional* non commutivity in phase (or quadrature) space, but from a group theory feature, which does not require taking account of the source and character of the basis states in the two-state subsystems. It is, therefore, an extra hypothetical input into the complex of algorithmic rules-of-thumb that sociologically constitute what is taught under the rubric of QM. Like the other inputs into this complex, it has no *a priori* paradigmatic justification. The natural question at this point is: can such a justification be imagined? This shall be considered below.

As mentioned already, the physical status of the two-state subsystems is not determinative for the considerations leading to the 'entangled' states give in Eq. (15). In fact, the actual two-state subsystems are often manifestly non-quantum in character. For experimental purposes the most common choice involves the two polarization states of electromagnetic signals, for which the mathematical structure was worked out by Stokes before the founders of QM even were born.[18] It's all there, the $SU(2)$ structure and everything. Moreover, because this structure is isomorphic to the formalism used to describe spin, the conclusion that the geometric behavior of spin (or angular momentum) is also non-quantum in nature, can not be evaded. Either both are quantum, or both are not, but, because of the isomorphism, they can not be disparate.

This conclusion frequently evokes strong consternation and puzzlement, the basis of which is the observation that the states of angular momentum, including spin, conform to uncertainty relations that are strikingly similar to those pertaining to the incontestably quantized Hamiltonian conjugate variables, $\hat{x}$ and $\hat{p}$ spanning quantized phase space, and that this fact in turn should imply, it is argued, that q-bit space, i.e., polarization or spin space, is just as legitimately a quantum entity as quantized phase space.

However, parallel uncertainty relations are just a coincidence and can be explained as follows. The group structure governing q-bit space, $SU(2)$ is homeomorphic to $SO(3)$, the group structure of rotations on a sphere, which pertains to each and every vector in three dimensional Euclidian space. The non-commutivity evident in $SU(2)$, therefore, is identical to that of $SO(3)$. It can even be thought of as the inherited non commutivity for the polarization vectors orthogonal to a particlar wave vector which itself is rotating on the sphere.

Consider the generators of rotations on quantized phase space, L, which are generalizations of the classical definition

$$L = r \times p, \tag{16}$$

of the form

$$L = \hat{r} \times \hat{p} = r \times \frac{\hbar}{i}\nabla, \tag{17}$$

which leads directly to

$$L \times L = i\hbar L. \tag{18}$$

The only difference with non-quantized phase space, where $L \times L = L$, is the scalar factor $i\hbar$, not the presence of non commutative structure. The factor of $i\hbar$ finds its way into Eq. (18) by way of Eq. (17). The non commutative structure here, unlike with respect to Hamiltonian conjugate variables, is *not* an optional feature, present for quantum theory but otherwise absent. It is an ineluctable geometrical feature of angular momentum and always present.

SECOND ENTANGLEMENT

Regardless of the true status of spin and angular momentum, a fact of modern physics is that fundamental issues are being debated, analyzed and experiments realized in terms of "entangled" *polarization* states. Beyond any contest, polarization is a non quantum phenomena; the non commutivity involved is purely geometric in character.[19] Thus, in the formation of combined systems, the state space is the tensor product of the individual two-state systems. Imposing, then, the additional requirement that the combined system state space be spanned by eigenvectors of the square of the total polarization, gives structure resulting from no uniquely quantum hypothesis. The non-factorable nature of the two base states results from imposing the formalistic requirement that the basic states be eigenstates of the square of the total or system polarization. This has nothing to do with Heisenberg Uncertainty or any other feature unique to QM. Consequently, it can not be denoted legitimately: "Quantum Entanglement." However, since this is an extra degree of non-factorability, over and above that intrinsic to the underlying single-system wave function, it might be denoted "second entanglement."[5]

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[5] It seems that 'second entanglement' is actually what is most often meant, but not always in discussions of fundamentals, by the word 'entanglement.' First entanglement is most often, seemingly, just implicit in the use of the term: 'matter wave.'

This line of reasoning leads naturally to the question: what possible physical significance can be attributed to the requirement that the total system state-space be spanned by states that are eigenstates of the square of total polarization (or angular momentum)? There may be many possible answers to this question, but the only one that this writer has found, is based on observing that these tensor product base states are intended for an multi-particle overlay structure on single densities that individually represent single particle detection probabilities. Keeping this in mind, 'entangled' base states seem to be those for which the statistical measure of the combined states is just what would be obtained from an ensemble of the direct tensor product base states. That is, the entangled base states are those for which the statistics, e.g., averages, is the same as that possible only for an ensemble of direct base states. Indeed, some authors introduce these "maximally entangled" states just *because* of their statistical properties.

This point of view is supported by some technical details concerning the generation of 'singlet states' for, *inter alia*, testing Bell's theorem. From largely formalistic considerations, it is taken that this state is "rotationally invariant." This is meant to convey the fact that, the pairs should be absolutely (anti)correlated regardless of a bias angle in the measurement device, i.e., regardless of the direction one side is measured in, the orthogonal companion signal will be seen to be deterministically (anti)correlated. The singlet state appears to satisfies this criterion, although the actual situation in experiments is not so clear. All of the sources used to test Bell's ideas appear to be oriented, i.e., have a preferred axis; e.g., cascade sources are excited, for example, by electron beams of *fixed* orientation, and parametric down conversion crystals have a *fixed* orientation of ordinary and extraordinary emissions. Thus, only those signals measured so that the axis of the measurement device is parallel to that of the source, will the (anti)correlation be deterministic, for all other angles, just statistical and variable. If, on the other hand, the source has a random bias angle different with each pair, then there will be no detectable variation in the statistics of the as a function of bias angle, i.e., the source will appear 'rotationally invariant.' Checking these details carefully (If at all possible; various 'detection loopholes' render event-wise measurements ambiguous, perhaps almost meaningless.), might reveal something about the nature of such 'entangled' states.

In any case, as these entangled states are obtained from formalistic operations that can not be carried out on material objects, the ontological status of 'entangled' states so constructed is certainly dubious.

CONCLUSIONS

As noted at the beginning, questions regarding the ontological nature of QM wave functions historically have been given provisional answers that are simply mysterious. The most successful attempt at investing some kind of physical ontology onto wave functions is Born's prescription of interpreting the modulus squared of a wave function as the probability density of presence. This, nevertheless, still leaves two features very problematic, as they are foreign to probability theory. One of the these features is that the wave underlying a Born probability density actually interacts with the material world, whereas probabilities are just abstract, calculational aids. On the other hand, in spite of the mutual material interaction of wave function and matter, wave functions also seem to collapse at measurement superluminally, as if they really were still just non-material, mathematical abstractions.

Herein the writer makes a plaidoyer for Stochastic Electrodynamics with Kinematic Tuning as the paradigm most successful at rationalizing these as well as other features of QM wave functions. SED provides a rational and understandable source for the mechanical changes made to the trajectories of particulate atomic scale systems so that they exhibit a wave-like response to environmental obstacles, i.e., initial and boundary conditions. The fundamental hypothesis for SED, namely that there exists a Lorentz invariant residue of background radiation, all by itself accounts for many quantum features, including the Lorentz black body spectrum, Lamb shift, stability of the atom and many many others customarily nowadays taken into account with second quantization. SED with the additional hypothesis of kinematic tuning completes the model by providing a rationalization for first quantization including the Schrödinger Equation.

Although because of its intuitively comprehensible physical motivation, SED with kinematic tuning is arguably the best model for QM proposed to date, it still can not be considered, in this writer's view, a completely successful foundation for QM. This is so because SED is based on an essentially untenable physical premise, namely the existence of a Lorentz invariant power spectrum, $E(\omega) = \hbar\omega/2$, which is divergent when integrated over all ω. It calls for an infinite amount of energy at each point in space. In this respect it is as defective as QED, which predicts the same thing, but as the 'quantized ground state of the free electromagnetic field.' [6] The best one could say for SED then, is that it concentrates all the many other mysterious and puzzling features and effects characteristic of QM interpretations into this one feature. It tidies up the philosophical landscape around QM, but is no final solution to the interpretational problems, and, therefore, fails its principle stated goal, albeit in a very suggestive and interesting way; and, arguably, with the least otherworldly paradigmatic inputs compared to alternatives proposed to date.

Whatever else, however, the formulaic transformation from the Liouville to Schrödinger equation, can be considered a physically motivated 'quantization procedure.' This writer has used this procedure to good advantage in extracting a relativistic wave equation with interaction, and as a procedure to identify unique operator equivalents to classical expressions. [6]

Quite aside from the merits of SED for resolving issues concerning the nature of QM wave functions, mandating that eigenvectors spanning the tensor product space of two multiple-state systems, as being eigenvectors of the total polarization or angular momentum squared, induces an extra layer of 'entanglement' without paradigmatic justification. This extra entanglement, which is widely misattributed to QM somehow, has lead to much exaggerated terminology such as 'quantum computing,' 'teleportation' and the like, that, lacking sobriety, is rather unscientific in character.[20] This is not to say that these fanciful terms do not apply to actual physical effects, but rather that current nomenclature is lexicographically sensationalist and misleading. The intrinsic parallelism that is credited with giving these effects their promise for applications, is already present in Fourier analysis of signals, where only the total signal, not the Fourier components, can be granted ontological meaning. Thus, is a certain sense, internal multiplicity is actually present without QM. In fact, this feature in QM arrises, because Schrödinger's Equation is a hyperbolic differential equation, which it has in common with other Sturm-Liouville equations, in particular the simple wave equation underlying the simplest form of Fourier analysis. In short, these effects, if viable at all, do not exploit quantum structure.

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[6] As has been noted often, QED puts a minimal cut-off on background energy that is a 120 orders of magnitude above the maximal cut-off set by 'cosmological constant' considerations from General Reactivity. As such, this is certainly the biggest problem in theoretical physics!

[7] Preprints of all the author's papers are available for download at: `http://www.nonloco-physics.000freehosting.com`