

Nonlocality, Bell's Ansatz and Probability*

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Quantum Mechanics lacks an intuitive interpretation, which is the cause of a generally formalistic approach to its use. This in turn has led to a certain insensitivity to the actual meaning of many words used in its description and interpretation. Herein, we analyze carefully the possible mathematical meanings of those terms used in analysis of EPR's contention, that Quantum Mechanics is incomplete, as well as Bell's work descendant therefrom. As a result, many inconsistencies and errors in contemporary discussions of nonlocality, as well as in Bell's Ansatz with respect to the laws of probability, are identified. Evading these errors precludes serious conflicts between Quantum Mechanics and both Special Relativity and Philosophy.

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I. A CONUNDRUM: NONLOCALITY

Obscurity in the meaning, or potential meaning, of the Quantum Mechanical (QM) wave function led EINSTEIN, PADOLSKI and ROSEN to claim that BORN's interpretation of the modulus squared of a wave function as a probability of presence, implies that QM is incomplete.[1] BOHR disagreed; but his arguments are not transparent.[2] The position argued by BOHR is developed more transparently, perhaps, by a line of reasoning beginning with VON NEUMANN and carried forth by BOHM and finally expounded elegantly by JOHN BELL. They sought to either prove, or disprove by example, that completing QM could be accomplished only at the cost of introducing elements into the envisioned extended theory more disagreeable even than the putative incompleteness of QM; in the case of BELL specifically, this feature is *nonlocality*.[3]

In the end, all of BELL's considerations on this issue, are based on the following argument and formula. We paraphrase:

Consider the disintegration of a spin-0 Boson into two daughters, comprising a system described by the singlet state. If now a measurement made of the spin of one daughter, let it be denoted "A", in the \vec{a} direction, i.e., $\vec{\sigma}_A \cdot \vec{a}$, yields $A(a) = +1$, then a measurement of "B", $\vec{\sigma}_B \cdot \vec{b}$, must yield $B(b) = -1$. These outcomes are, in the usual formulation of QM, fundamentally probabilistic, either side for any given pair could be ± 1 , the only constraint here is the subsequent deterministic anti-correlation of its partner.

Completion of QM, BELL takes it, means that there should exist further, as yet undiscerned, variables λ , which, were they knowable and measurable, would enable the deterministic prediction of the outcomes, thereby eliminating the statistical character now intrinsic to QM. Thus, BELL writes altered symbols denoting the outcomes of each measurement for the proposed experiment: $A(\vec{a}, \lambda)$ and $B(\vec{b}, \lambda)$; that is, they are given the form of deterministic functions of their argu-

ments. More importantly, he takes it, that by virtue of *locality*, what happens at station "A" cannot depend on the value of b , the settings of the measurement device at station "B"; and of course, *visa versa*. Then, without further ado, BELL set forth his now famous Ansatz, that the expectation of their product should be, *if it is to be local and realistic*, given by

$$P(\vec{a}, \vec{b}) = \int d\lambda \rho(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda), \quad (1)$$

where $\rho(\lambda)$, is a putative distribution of the still undiscerned¹ variables. Finally, BELL notes, that by the algorithms of QM, this should equal the expectation:

$$\langle \vec{\sigma}_A \cdot \vec{a}, \vec{\sigma}_B \cdot \vec{b} \rangle = -\vec{a} \cdot \vec{b}. \quad (2)$$

On the basis of Eq. (1), BELL (and others) derived several versions of his renowned inequalities which have been shown empirically to be incompatible with Eq. (2). That is, the data taken in actual experiments is accurately predicted by Eq. (2), but does not satisfy those inequalities derived by BELL et al. from Eq. (1). As satisfying these inequalities is considered a test for the assumptions going into their derivation, disagreement means that both QM and experiments do not meet the assumptions. The final conclusion usually drawn from all this is, that QM is incompatible with a *local* extended or deeper theory eliminating the statistical aspects of quantum theory, because locality is thought to have been the only disputable hypothetical input into the derivation of Bell inequalities.

All of this is very well known and generally considered utterly beyond dispute.

There are, however, disturbing features in BELL's Ansatz (even ignoring, momentarily, sharply critical, contrarian analysis[4]), that are captured by the observation that Eq. (1) exactly as BELL wrote it, cannot be found in treatises on probability and statistics! This, we believe, very revealing curiosity, is the motivation for this study.

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¹ The standard term "*hidden*," instead of *undiscerned*, presupposes some external agent other than incapability or inattention of observers; a logically unjustified covert assumption.

II. BACK TO BASICS

The source of this ‘‘curiosity’’ must be in the, partially implicit, relationship between the imputed physics in BELL’s arguments and its encoding in symbolics for mathematical manipulation. It is the purpose here to parse precisely such relationships, in particular to identify any implicit or covert aspects.

To begin, it is imperative to understand the structure of BELL’s argument, i.e., its overarching logic, which was to determine the possibility of extending QM in such a way that its weird features would be eliminated. That is to say, the *hypothetical* extended formalism, the goal, ideally should contain no ill-defined, ambiguous or contradictory aspects. While perhaps not the only option, the most obvious first choice for searching afield for this ideal structure would be the domain of classical physics, for which the only known defect was, that it did not fully explain all observed phenomena, not that it, itself, is internally contradictory. In any case, it can be cogently argued, BELL seemed to have assumed implicitly that the target structure would be fully compatible with known mathematics. Let us join him now explicitly in this latter assumption.

What this entails is that all the symbols in Eq. (1) must be given meaning in conformity with the practices as presented in standard texts on probability and statistics. All but the undiscerned variables, λ , are operationally defined and, therefore, already have fully compatible identities devoid of pre-natural properties. Thus, now only for the variables λ , must it be so arranged that they not be ascribed properties unrealizable within classical mathematical physics. This is fulfilled if a device can be found that can measure their values. If this consideration is not imposed, then one of BELL’s primary motivations is frustrated at the start.

Now, it is asserted often that according to QM both $A(\vec{a})$ and $B(\vec{b})$ are random variables, that is, they take on the values ± 1 randomly. $A(\vec{a}, \lambda)$ and $B(\vec{b}, \lambda)$ are, therefore, essentially identical to $A(\vec{a})$ and $B(\vec{b})$, with the difference, that the symbol λ in the set of arguments is a place-holder for that information, which, because it is missing, renders $A(\vec{a})$ and $B(\vec{b})$ random (but appropriately correlated) variables, i.e., were this information available, they would be deterministic functions. This brings one, however, straight up against a source of deep ambiguity in the currently used symbolics.

It is this: in order for Eq. (2) to hold, we must be able also to write

$$\langle \vec{\sigma}_A \cdot \vec{a} \rangle = \int d\lambda \rho(\lambda) A(\vec{a}, \lambda). \quad (3)$$

While this seems innocent enough, it begs an issue, namely what should $A(a, \lambda)$ actually signify? Confusion arises immediately when it is recognized that in experiments no measuring apparatus actually outputs readings of ± 1 (regardless of whatever units are impugned, but overlooked); rather, in optical experiments², the photodetectors simply register the excitation

² As a practical matter, it seems, one need consider only optical experiments,

of a photoelectron in a detector behind some kind of filter. The numerical values ‘ ± 1 ’, then, can be, at most, just labels for the channels in which the photodetectors ‘‘fired,’’ indicating that a signal arrived at the detector that passed through the filter standing in front of it. The mesh size of the filter, as it were, is indicated by the label a or b . In turn, Eq. (3) must be understood as an expression of the ratio:

$$\langle \sigma \cdot a \rangle = \frac{Q_a}{N}, \quad (4)$$

where Q_a then is the *number of times* a photoelectron appeared in the channel in which the polarizer-filter setting was given by a , in N trials, in the limit $N \rightarrow \infty$. This is, of course, virtually the very definition of a probability.

At this point some additional insight can be gained by imagining carrying out an experiment explicitly. Consider an optical experiment in which the source is a parametric down conversion crystal (PDC, type II) producing signals anticorrelated with respect to polarization.[5] In each arm of the setup there is then a polarizer with its axis set in some direction (e.g., a or a' , etc.) perpendicular to the line of flight. Behind each polarizer there is a photodetector. Typically, the source intensity is set so low that the likelihood is that there is only one ‘photon’ in each signal of the pair, that is, operationally this is understood to mean that only one photoelectron appears in the detector circuitry for each arm. Now, in the course of an experiment, what happens exactly? First, for the sake of simplicity, let us assume that we know somehow that the first pair has been generated, i.e. $n = 1$. Then, a check is made at each photodetector to see if a photoelectron was registered. If now the left, A , polarizer’s axis was in the \vec{a}_1 direction, and the B polarizer in the \vec{b}_1 direction, then the data taken for this run would be (where, for example, a hit was registered at A but not B): $n = 1$, $\vec{a} = \vec{a}_1$: *yes*; $\vec{b} = \vec{b}_1$: *no*. To take this data, three things must be determined: 1) that a pair was generated, 2) the settings of the polarizers and 3) the output of the photodetectors.³

Of course, what is not known in this case is the precise polarization of the signals comprising the pair as emitted at the source but before they reach the polarizer-filters. The polarizer settings can be known because they are inputs into measuring devices under the control of the experimenter who selects their orientation before the pair is generated at the source. The effect of a polarizer is described by MALUS’ Law, and the digitized response of detectors is a consequence of the nature of individual photoelectron generation at very low stimulus levels.

as several obstacles have prevented carrying out convincing experiments with particles having ‘entangled spin.’

³ In the experiment described here a ‘no-hit’ at a detector in one arm is usually understood as a positive detection in a companion channel, e.g., one labeled ‘ -1 ’. By using a polarizing beam splitter, with each output face feeding a separate detector, both outcomes can be registered as positive events, thereby reducing ambiguity. Such refinements do not affect the points being made in the present argument, however.

Seen this way, it is absolutely indisputable that such detector settings have no effect on the source, and, therefore, have no effect on the pair of signals before they enter the polarizers. They do, of course, have an effect on whether particular signals will pass the polarizers and trigger ‘hits’ in the photodetectors, but they do so independently, without collaboration. They do have, therefore, a direct contributing affect on those photo-detection events counted for the experiment.

By continuing the experiment, additional data can be taken for additional runs $n = 2, 3, \dots, N$ leading to ratios for which the numerator is the number of photo-detections when $a = \vec{a}_1$, or \vec{a}_2 , and the denominator is the number of runs, N , i.e., n_{a_1}/N , and likewise on the other side, n_{b_1}/N . Reasonably, after a sufficient number of runs, we expect these ratios to converge to the probabilities of photo-detections under the circumstance of the experiment. Thus, we may write them as, e.g., $P(a_1)$.

This is a standard application of probability theory; BELL’s program was to ask: what information is needed to so extend the quantum theory of this experiment that it becomes deterministic (and fault free)? He supposed that the variables denoted λ above, when known, would specify the conditions of the experiment so that the outcome with each run could be unambiguously determined in advance, at least in principle. Such a circumstance can be imagined as follows: First we shall take it that the source has fixed orientation; for a source exploiting parametric down conversion, this is achieved if the known axis of the crystal is fixed in a given direction so that vertically polarized signals are vertical with respect to both the crystal and laboratory. This implies that horizontal signals will be likewise, also with respect to the crystal and laboratory. Then all the information that is needed to render each experimental run fully deterministic is the polarization orientation of one of the signals as it departs the source, either vertical or horizontal, as the other must be complementary.

In the usual setup, exactly this information is not available to the experimenter; it is for him, “hidden.” But, presumably ZEUS knows it, and, were he the experimenter, he could discern the value of the ‘hidden’ variable, λ , such that for him the outcomes would be deterministic and predictable in advance. For him, the $P(a_1, \lambda)$ becomes then a Kronecker delta function equal to “yes” (which means that a photoelectron is elevated into the conduction band in a detector) for that value of λ that corresponds to a specific event comprising a particular pair of anticorrelated signals. Further, ZEUS could take complete data, that is, for each pair he could write down, in addition to the run, n , and the settings of the polarizer filters, \vec{a} and \vec{b} , also the values of λ , for example, left signal: 0° ; right signal: 90° . Moreover, he would know the intensity of background fluctuations, and whatever other signals contribute to the generation of a photoelectron in each channel, right and left, so that he could predict with certainty whether a photoelectron is in fact to be generated in each arm after passing a polarizer with a given orientation. By knowing all this information, ZEUS would then also be in position to sort the final data stream in terms of the values of λ into groups of similar values; and, in that case (and only in that case) by the precepts of standard probability theory, the factorization evident

in Eq. (1) with respect to $A(\vec{a}, \lambda)$ and $B(\vec{b}, \lambda)$, could be carried out. In the language of a probability theorist, ZEUS can “screen off” the variable λ .

The mortal experimenter, however, with no means of knowing the values of λ , cannot sort the data into groups within which the value of λ is constant. Thus, even though there are underlying specifiable causes, insofar as for the mortal analyst they are in fact unknowable, sorting on λ is *for him* impossible, therefore the factors $A(a|\lambda)$ and $B(b|\lambda)$ in the form of data are *for him* sortable only with respect to the values of \vec{a} and \vec{b} . At this stage the purpose of an experiment is to study the correlations seen in the pattern of joint hits or detections given the settings, and because the source was selected on the basis of providing correlated pairs of output signals, it is absolutely necessary to use Bayes’ formula, namely:

$$P(a, b|\lambda) = A(a|b, \lambda)B(b|\lambda), \quad (5)$$

for the product in the integrand in Eq. (1). [6] The outcomes of the measurements are correlated (indeed, observing this correlation is the purpose of the experiment), because the inputs were correlated, and this fact necessitates using Eq. (5) whatever the significance of λ . In this application, the factor $A(a|b, \lambda)$ is no longer an independent probability, but a *conditional* probability, which answers the question: what is the probability of a hit at station A when set to \vec{a} , given that a hit *was* registered at station B when its polarizer is set to \vec{b} . In this case, carrying along the symbol λ is just a reminder that additional but unknowable information could obviate the need for statistical analysis—as is always true. Again, there is, contrary to BELL’s argument, no implication whatsoever that the settings \vec{a} and \vec{b} affect the signals as generated at the source before they encounter polarizers, or that they affect each other during detection just because the term $A(a|b, \lambda)$ has the parameter \vec{b} within its complement of arguments. All this means is that the total polarizer filter setup, before the signal pair was even generated, was so chosen that it gives the *conditional* probability of correlated signals for this particular polarizer setting combination given by a and b . If the source pair is not compatible with the preselected polarizer settings, then as filters, the polarizers do not pass the signals to the detectors to generate detection events that can be counted; an inappropriate pulse pair simply does not contribute to the data stream; it is rejected by the logic of the coincidence circuitry as spurious background. Mathematically, the distinct form of Eq. (5), reflects the restriction for the association of factors $A(a|\lambda)$ with $B(b|\lambda)$ so as to take into account that not just any outcome at station A can be a cofactor with a particular value of $B(b|\lambda)$ because of the *correlation* invested in the pair by a *common cause* within the past light cones of both measuring stations. In EPR-B experiments this correlation results from limits imposed on the pairs, as generated at the source, to being *anti-aligned* in terms of polarization, i.e., to being anticorrelated. In any case, the role that a and b have in the symbolics is totally passive; and, association of these variables with the determination of any property of the signals generated at the source is, as simply a matter of probability theory, misplaced. The appearance of a parameter for distinct events

or remote objects in a *conditional* probability does not imply the existence of a continuing connection (vice structural compatability) of any kind, much less specifically non-local interaction.[6]

This restriction in the pattern of cofactors is expressed in the symbolics by employing the formula Eq. (5). By correctly employing this formula, one finds as an immediate consequence, that derivations of Bell inequalities do not proceed. On the other hand, it is just as clear that they do proceed when, as a restricted case, Eq. (1) does pertain; that is, when there is *no correlation* between the factors $A(a|\lambda)$ and $B(b|\lambda)$. Therefore, on the basis of these considerations, Bell inequalities are applicable only to ensembles of *uncorrelated* pairs. Clearly then, testing them with *correlated* signal pairs must lead to invalid conclusions.[7–9]

III. DOUBLE CONUNDRUM: IRREALITY

This story, so far, is too simple for direct application to QM! There is an additional and serious complication. It is brought into the matter, although BELL did not make explicit mention of it in most of his papers, by VON NEUMANN’s measurement theory with its ‘projection hypothesis.’[10]

For reasons (that we shall try to analyze below), it is taken often that the state of a single signal pair in QM is given by:

$$|l, r \rangle = \frac{1}{\sqrt{2}}(|v, h \rangle - |h, v \rangle), \quad (6)$$

where l and r indicate the left and right arm of an EPR experiment, and v and h indicate which polarization the signal sent into the respective arm is to have. This state, for reasons clear from the QM analysis of spectroscopy data, is called the ‘singlet state’ and is, as indicated, to be comprised of two mutually exclusive possibilities, i.e., it is “irreal.” According to the orthodox, ‘Copenhagen’ interpretation of QM, which is based on the presumption of the completeness of QM, this is the state *actually* (in the full sense of ontology) describing the signal pair as they depart the source, but before they trigger detections. At the detectors, however, what is known as “VON NEUMANN’s measurement theory” which includes the “projection hypothesis” is invoked to account for the fact that a detection on either side always finds only one of the possibilities; it is asserted, by authority of this theory, that measurement itself projects the state onto the base states such that only one outcome (vertical, say) is realized in one arm. Further then, by symmetry, the state of the signal in the other arm is also determined by collapse (e.g., horizontal). This ‘projection,’ or ‘collapse’ of this wave packet, is considered to transpire instantaneously regardless of the separation of the measuring stations on the two sides of the experiment; which is, again, as is very well known, a violation of the principle of Relativity according to which *no interaction* can transpire faster than the speed of light. ⁴

For the moment, let us not question any of this. Instead let us see what consequences imposing these considerations on those in the previous section might have.

Recall to start, that above we observed that Bayes’ formula:

$$P(a, b) = P(a|b)P(b), \quad (7)$$

does not imply any particular type of *physical* interaction between the stations A and B . In standard probability theory, it is never a point of contest, because it is simply taken that the correlation was invested by a “common cause” in the past, so that no violation of Special Relativity is involved. The fact is, however, this need not be true to comply with the logic of conditional probabilities. In other words, from strictly the mathematical point of view, it would be acceptable to call on a ‘projection hypothesis’ to resolve any essential ambiguity of singlet-type states; so, mathematically this issue is no obstacle.

The logic of BELL’s analysis is absolutely opposed, however. BELL’s intention when conceiving of his “proof,” excluded insinuating, at the meta-level where the inequalities are being derived, any hypothesis not found in classical, local and realistic physics as it was understood before the discovery of QM, where the interpretation issues of QM do not exist. His explicit purpose was to examine the question of the existence of a covering theory that has just that structure exploited by classical, pre-quantum theories. His tactic taken in the proof of what has become known as a “theorem,” although he himself never so designated his argument as such, was to assume that a *problem free* super theory exists in principle, and then derive constraints from it that then should percolate down to the lower quantum theory as it is understood nowadays to see if they are compatible with what exists, at that coarser level. The point here is, that this motivation precludes altogether bringing the VON NEUMANN measurement theory with its ‘projection hypothesis’ (and all else unique to QM) into the higher level, insofar as this structure is both unknown in classical physics and in violation of Special Relativity on its face. Of course, one might dispute the necessity of wave function collapse even at the level of QM, but that is a separate issue.

This admonition deserves strong emphasis. Many authors discuss the EPR conundrum and BELL’s analysis without scrupulous attention to and explicit mention of BELL’s overarching logic and thereby fall into implicit ambiguity. They bring these special and non-classical features, occasionally even directly at the hypothetical meta-level, implicitly into the story as if they were somehow germane and fully legitimate, contrary to BELL’s primary objective.

cate, and therefore that it does not violate Relativity, can be challenged on the grounds that Relativity constrains all *interaction*, not just communication. Communication, after all, is just modulation on interaction of some sort.

⁴ Arguments to the effect that this transmission cannot be used to communi-

IV. UNDERLYING ALGEBRA

Given that the two signals in an EPR-B experiment can be in two states each, there will then be four possible combinations for the pair. The fact that for these experiments the paired signals must be anticorrelated eliminates the two even combinations, leaving the two pair-states:

$$|v, h \rangle \text{ and } |h, v \rangle. \quad (8)$$

Insofar as these are mutually exclusive states, they may be regarded as orthogonal vectors spanning a two-dimensional space. This is, of course, a formal association, the utility of which depends on full compatibility of the physical and mathematical structures, a proposition to be examined. Nevertheless, accepting this formality allows considering a rotation of the axis of the two-dimensional space to get the superposition states:

$$\frac{1}{\sqrt{2}}(|v, h \rangle + |h, v \rangle), \text{ and } \frac{1}{\sqrt{2}}(|v, h \rangle - |h, v \rangle). \quad (9)$$

Although a perfectly legitimate vector space operation, as an ontological statement about the physical objects associated with the vectors, it is “otherworldly.” Consider, for example, a room in a building in which a coordinate system is defined to be such that one wall is the abscissa, x , and a perpendicular wall the ordinate y . Here each dimension is associated with a material object, a wall; but, in the rotated system obtained by employing the transformations $x' = x + y$; $y' = x - y$, there is no material association with x' , indeed; it runs diagonally through the middle of the room where there is no wall. The same principles apply to the Hilbert space comprising the solution space to a Sturm-Liouville differential equation. Moreover, even before this consideration can be brought to bear, for a Hilbert space there is no natural, intrinsic association of the basis vectors for the solution space with ontological entities. Therefore, *a fortiori*, there is no implicit association with ontological objects in transformed bases. Such associations must be individually established and physically justified for each application, presumably by deliberate matching superpositions of eigen functions with physically valid boundary or initial conditions; it cannot be expected that ontologically valid states fall out of a formalism mindlessly.

Superpositions (sums) of mutually exclusive objects, such as Eqs. (9), do not enjoy ontological existence with respect to ordinary logic. How is it, then, that such “irreal” objects are taken as ontologically acceptable within the QM formalism?

Eqs. (9) are deduced usually, i.e., in textbooks, by considering symmetry requirements implied by the indistinguishability of perfectly identical particles. The explication virtually always begins with the observation that, because such particles cannot be individually distinguished, the Hamiltonian for a system comprising two such identical particles, for example, must be perfectly symmetric with respect to the exchange of the coordinates of the particles. (See, e.g., [11].) Thereafter, however, in spite of general similarities, the hypothetical inputs are delineated and laid out with enormous variety and often imprecision.

The first problem arising here is due to the fact that the application of this logic to electromagnetic pulses or signals with different states of polarization ignores the fact that polarization states are derived from classical electrodynamics; no QM is involved, so that the Hamiltonian for this structure is classical, and the indistinguishability of identical entities is not an issue. Classical entities, described by classical Hamiltonians, always can be identified and distinguished in principle, so that this consideration should not be brought up for these states in the first instance.[12]

Then, the demonstration that state vectors or wave functions for the system as a whole need be symmetric or anti-symmetric, typically tacitly assumes that physical (i.e., ontologically valid) states are eigen vectors of the Hamiltonian. Although this assumption is very widely made by authors on QM, it is still dubious. Eigen states of the quantized harmonic oscillator, for example, do not oscillate; an alarmingly embarrassing feature! The argument that this is a “mystery” endemic to the microword of QM is seriously undermined by MOYAL’s observation that the eigen states of the dynamical equation of a Markoff process, even in classical physics, do not yield positive definite Wigner densities.[13] Thus, the failure of the eigen states of the Schrödinger equation for the harmonic oscillator to yield everywhere positive definite Wigner densities cannot be taken as evidence of ineluctable *quantum* incomprehensibility at a microscopic level. Insofar as coherent states both oscillate and yield positive definite Wigner densities, perhaps it should be taken that only they are legitimately identified as ontologically meaningful states.

Likewise, the eigen functions of the wave equation, the trigonometric functions, being finite on an infinite domain, are also clearly non physical; indeed these states are considered unphysical simply because they cannot be normalized. It is arguably reasonable, therefore, to presume that physically relevant solutions for the equations of mathematical physics are restricted to only those combinations of eigen functions that satisfy some physically realizable initial or boundary conditions.

This reasoning brings analysis of the physical significance of Eqs. (9) to the question: what do in fact the hypothetical inputs into the arguments introducing such superpositions actually imply about their ontological or physical meaning? To begin, let us ignore the technical problems just mentioned, and reason simply from “ground up.” It is a tautology that either a wave function pertains to a single system (perhaps comprised of several parts), or it does not. If it does not, then the superposition can be understood easily. In this case, a wave function can be taken to specify not the named object, but rather the preparation procedure that yields this object for observation.[14] The presence, then, of mutually exclusive sub-items or terms, is not problematic, it means just that the procedure can produce either of the options at separate times such that each is a separate member of an ensemble. This option appears to be entirely problem free,⁵ but still is the minor

⁵ Single object (particle) wave functions can be interpreted as ensemble den-

ity view. Because it implies that QM is incomplete, it supports the thrust of the EINSTEIN, PODOLSKY and ROSEN incompleteness argument, which, supposedly, was rebutted successfully by BOHR, and so nowadays is rejected routinely; but, we argue, falsely.

On the other hand, taking a wave function to pertain to single systems necessitates introducing the projection hypothesis to account for the fact that no superposition of mutually exclusive observable states is ever seen in the laboratory (ignoring the logical absurdity of the expectation that it could). In addition, it seems that the singlet-triplet state structure is not universally applicable. In particular for EPR setups, where it is taken that the outcomes pairwise are deterministically anti-correlated, two of the triplet states are not available at the start for contribution to the relative frequencies of the observations, thereby calling into question the use of the term ‘singlet state’ for these applications, i.e., the ‘singlet’ distinction is germane only *vis-a-vis* the presence of ‘triplet’ states which engender three contrasting signals. (Below, we shall give an even better reason for this contention.) Moreover, the logic used to motivate the conception of such states, i.e., the indistinguishability of identical particles, does not pertain to polarization states; they are always distinguishable.

Perversely, the essentially probabilistic nature ascribed to wave functions precludes empirically determining the validity of their completeness and the incumbent projection hypothesis. A single data point (for a single pair) in an EPR experiment resolves nothing. Likewise, Heisenberg uncertainty cannot be tested with a single data point. A numerically significant sample of data points is needed to permit doing the statistical analysis implicit in these very concepts. In plain talk, there is no measurement to be made on a single system that can verify the presumption that it is *completely* described by its quantum wave function. By all logic, all qualities of wave functions can be fixed only using statistical analysis of an ensemble. Here it would seem, therefore, that OCCAM’S razor should leap into action to preempt POPPER’S objection to introducing untestable hypothesis into scientific theories, in this case, being the completeness of wave functions with the then incumbent ‘projection hypothesis.’

V. UNDERLYING PHYSICS

Eqs. (9), whilst being superpositions of mutually exclusive outcomes, were not taken into the tool kit of QM just because they enjoy algebraic legitimacy. They have very useful applications, apparently the first of which was considered by HEISENBERG in 1926, where he applied the then new techniques introduced by QM to explain the spectrum for multi-electron atoms.[16] Helium, with two electrons, is the simplest case; and, its first excited state—with either one or the

other of its two electrons in an excited $2P$ -state, while its partner is in the $1S$ -ground state—is degenerate, because there are two possible combinations leading to the same circumstance.

Using degenerate perturbation theory one finds directly but tediously, that the eigen vectors for the coupled case have the form of Eqs. (9), and that the correction to the energy i.e., $E_{\text{correction}} = E_{\text{Coulomb}} \pm E_{\text{exchange}}$, where the first term is given by:

$$E_{\text{Coulomb}} = \int u_1^*(r_1)u_2^*(r_2) \frac{e^2}{r_1 - r_2} u_1(r_1)u_2(r_2) d^3r_1 d^3r_2, \quad (10)$$

and can be interpreted classically as the expectation of the Coulomb interaction of the charge distributions for both electrons. The second term is the result of the so-called “exchange force:”

$$E_{\text{exchange}} = \int u_1^*(r_1)u_2^*(r_2) \frac{e^2}{r_1 - r_2} u_1(r_2)u_2(r_1) d^3r_1 d^3r_2, \quad (11)$$

where in this case the interaction is for electrons in an atom and is given by $V = e^2/(r_1 - r_2)$. Clearly, as an indisputable historical matter, this is the logic and structure that led to the introduction of such states into QM in the first instance.

An absolutely crucial condition for application of degenerate perturbation theory is that there exists a physical interaction between the electrons. When interaction is absent, then both the term for Coulomb interaction and that for the exchange force vanish, as they do for noninteracting light pulses (doubly true with different polarizations). Then the eigen values need no corrections and revert to simple products of the unperturbed eigen values, and the eigen functions are then simple (factorable) products of the unperturbed eigen functions. It is the interaction between the electrons that induces correlations spoiling the factorability. Seen in these terms, it is the interaction that spoils the statistical independence of the wave functions for the electrons. In other words, the interaction induces the correlations between the electrons that mandate introduction of conditional probabilities and therefore BAYES’ formula, Eq. (7).

The form of Eqs. (9) can be taken as the structurally simplest form that captures the nonfactorability that is inevitable by cause of interaction. Once again, however, there is nothing in degenerate perturbation analysis that requires that eigen functions in this form be valid ontological states. They can be just the most elementary, nonfactorable form; ontological states must be those superpositions of such states that satisfy initial or boundary conditions.

When there is no interaction, then degenerate perturbation theory does not lead to states of this form. Thus, it would seem that EPR/Bell analysis cannot call on the structure of two polarization states because electromagnetic signals of differing polarization, according to standard electrodynamics, do not interact. Because they do not interact, both the ‘Coulomb’ and exchange term are zero, and the logic leading to the singlet-like states, as a superposition of mutually exclusive options, is not applicable. Perhaps one can go a step further and question even whether the so far hypothetical experiments on particles with spin would be appropriate, as in EPR experiments,

sities only if individual particles are vested with pilot waves to replace the wave character otherwise seen in ‘wave’ functions. See [15] for a paradigm suggesting a physical rationalization for such a hypothetical addenda to QM.

the particles comprising a pair are separated by relatively immense macroscopic distances, and the technicalities of the generation of such states at the source, where interaction does take place, is immaterial for purposes of testing EPR's hypothesis. Of course, the logic for the anticorrelated polarizations remains intact; it is just the logic for the singlet state *as an ontological entity* that is set aside.

Moreover, it is often pointed out that the superposition states correspond to the mechanical analogue involving two coupled oscillators for which there are two 'eigen modes,' i.e., oscillation both in and out of phase. When coupled, the oscillators can exchange energy and slosh back and forth between these modes. This 'classical' model, by example, indirectly supports the understanding of wave functions as being incomplete.

Once again, the identification of eigenstates with ontological states is a matter for specific examination. Obviously, when the two states (for electrons or whatever) are coupled, they influence each other so that the individual states are no longer statistically independent. The state functions for the system then cannot be the simple product of the state functions for the parts, and are so rendered nonfactorable. Eqs. (9) can be taken as the most fundamental form satisfying this requirement; but ontological states, then, would be superpositions of such states that satisfy the relevant physically specified boundary or initial conditions.

Finally, we note that spectral analysis of absorptions and emissions from atoms cannot be taken as evidence that the ontological states of the electrons between such absorptions or emissions are given by the eigen functions. That these spectra comprise multiple lines, leaves open the interpretation that the absorption or emission itself was comprised of a superposition, and only appears to be distinct lines because the total signal is being 'spectrally decomposed' by the optical instrumentation used for observation.

VI. ROTATIONAL INVARIANCE OF THE SINGLET STATE

The state represented by Eq. (6), i.e., the singlet state:

$$|l, r \rangle = \frac{1}{\sqrt{2}}(|v, h \rangle - |h, v \rangle), \quad (12)$$

is said to be rotationally invariant, which is meant to say that if the individual states $|v \rangle_{l,r}$ and $|h \rangle_{l,r}$ are expressed in terms of axes rotated about the fixed wave vector, \vec{k} , e.g., $|v \rangle_l = x_1 \cos \theta + y_1 \sin \theta$, etc., then the system state or wave function preserves its form, namely:

$$|1, 2 \rangle = \frac{1}{\sqrt{2}}(|x_1, y_2 \rangle - |y_1, x_2 \rangle). \quad (13)$$

This equivalence is, nevertheless, still ambiguous. It could mean that statistically all averages and moments calculated with both expressions are equal, in other words only as a statistic is it invariant. Beyond this, however, a physical interpretation can be imposed on the equivalence to the effect that not only are the averages equal, but actually the individual

separate states in both resolutions are the same, at least insofar as the individual anticorrelation is deterministic in both bases. This extra physical assertion is an additional hypothetical input that is independent of the mathematics as necessitated by the statistics; and, it must be independently verified by observation. Symbolic manipulations transforming from Eq. (12) to Eq. (13) do not depend on or address any question of physical interpretation.

Rotational invariance of singlet-type states as a *physical* assumption appears to be virtually universally accepted, although sometimes only implicitly, in physics literature. Indeed, it is an essential ingredient in at least one illustration of the mysteries of QM, [17], and if made, leads to mathematical inconsistencies rendering QM truly mysterious. It appears to be functionally equivalent, at least in spirit, to the assumption that QM is complete; i.e., that a wave function represents individual systems, in EPR experiments: to individual signal pairs. For the signals used in EPR-B experiments employing polarization "entanglement," empirical evidence collected by this writer, albeit not at the single 'photon' level, contradicts individual, as opposed to statistical, deterministic anticorrelation, thereby supporting only statistical rotational invariance.⁶ The separate states of polarization are deterministically anticorrelated only in the basis for which the axis of the PDC crystal is parallel or perpendicular to the axis of polarizer. On the other hand, this writer has verified by simulation that statistical rotational invariance is fully valid in the sense that random rotations introduced individually into signals for EPR-B type experiments do not affect the statistical analysis or final determination of correlation functions. [4]

On occasion, rotational invariance is taken even as "spherical invariance," which means that transformations to an arbitrary direction in space, not just rotations about the wave vector, leave the form of Eq. (12) invariant. This mathematical fact is taken to imply yet another physical hypothetical input, namely, that spin is quantized in all directions at once, not just in the direction of the magnetic field (which is required to reveal the existence of spin at all). Obviously, with respect to spin, this is an absolutely untestable proposition, as it is impossible to have magnetic fields in more than one direction at a point simultaneously, making this a pure metaphysical proposition, even oxymoronic. With respect to polarization, the implied physics suffers the same difficulty. Indeed a wave vector has physical significance in only one direction; coordinate transformations from the 'alias' point of view are not the issue. [12]

VII. ENTANGLEMENT VERSES CORRELATION

According to the Born interpretation, QM state functions give the probability of presence as their modulus squared. Since the operation of 'squaring' does not affect factorability,

⁶ Of course, such 'multiple photon' observations are necessary but not sufficient support for the contention that wave functions are not complete.

it seems there should be a direct relationship between nonfactorability of wave functions (a. k. a.: entanglement) and that of the probability densities derived therefrom (native correlation). Nevertheless, they are widely held to be fundamentally distinct. The natural question is: how?

One answer appears to be: entangled states are those that violate Bell inequalities. It is said that, in this case, there are ‘quantum’ correlations ‘stronger’ than admitted by any classical definition. This definition is a derived property rather than a primitive characteristic. Moreover, it presupposes the validity of the derivation of Bell Inequalities, a proposition that can be accepted only by overlooking issues delineated above.

In fact a better grounded reason is that a distinction between these concepts exists because of those hypothetical elements ultimately necessitating the ‘projection hypothesis.’ These elements, in turn, as discussed above, are necessitated by the presumption that QM is complete, that the wave function for an object is its deepest ontological manifestation, and that no finer information than what is given by a wave function is possible, even though, as argued above, this leads to consideration of ‘irreal’ states (a.k.a “cat states”).

The internal consistency of the ‘projection hypothesis’ may be testable. This may have been achieved already using biprisms with electron beams. In these devices an electron beam is sliced by a negatively charged wire arranged perpendicular to its propagation direction and passing through the middle. Upon passing the wire, each half of the beam is repelled somewhat, so that the two half beams diverge slightly. Then, downstream, a second wire, parallel to the first but charged positively, draws the two diverging partial beams together so that they meet on a registration screen, where interference of their de Broglie waves is observed.[18] Now, each of these wires plays the role of an optical instrument, but still they are contrivances created by the experimenter which both act on, and react to, the passing beam particles. In principle, these wires are ‘measuring’ devices in that some beam properties could be read out by observing fluctuations induced in the current in the wire as caused by the passage of the electron beam. As such, these observations should then, according to the precepts of QM (that is, with VON NEUMANN’S contribution), precipitate wave collapse. This in turn should prevent subsequent wave-like behavior of the beam. Interference seen on the screen shows that collapse did not occur, the beam exhibited wave-like behavior downstream from these ‘measurements.’

A separate recent experiment, originally proposed by KARL POPPER, proves the same point for the wave function for systems comprised of two correlated particles that eventually become widely separated, as in EPR experiments.[19] In this experiment, one beam of particles (usually photons, but it could be electrons) is sent through a slit, and the diffraction observed. The correlated beam on the other side is, in contrast, not sent through a slit, but observed for diffraction with an identical set of detectors. If wave function collapse happens in EPR experiments, as envisioned generally, then, although there is no slit on the latter side, it still should exhibit the same spatial pattern of detections, because its wave function should be “collapsed” (diffracted) in sympathy with its partner beam.

Observation shows that nothing of this sort happens.[20] The beam that does not pass through a slit, carries on as if *nothing* occurred, ignoring the evolution of its partner.

The conclusion from such observations must be: wave function collapse does not occur. In turn, entanglement beyond conventional correlation is an imaginary artifact. Whatever correlations exist among members of a system described by a wave function, they are identical to those among classical objects⁷; the projection hypothesis is groundless.

VIII. CONTINUOUS VARIABLE VERSIONS OF EPR TESTS

One of the difficulties mentioned above in experiments to test Bell inequalities is based on the fact that BOHM’S change of venue from phase space to qubit space (polarization or spin) introduces unintended, covert hypothesis.[22] The essence of this problem is that while phase space *can be* quantized, and thereby made non-commuting, polarization (qubit) space cannot be modified, i.e., “quantized;” it is already non commuting by virtue of its geometric structure. Qubit spaces are locked down; for them there is no option of having either classical (taken to be commuting) or quantum (non commuting) structure.

This objection might be evaded by suitable experiments in phase space. It appears that straight forward ‘EPR’ formulations, are not practical, however; clever designs are essential. In recent times such proposals have been made, usually described in terms of “continuous variables,” although, as such, *continuity* by itself is not really important; a ‘quantum’ venue is. On the other hand, continuity complicates the issue as the usual forms of Bell Inequalities are inadequate for non discrete variables; a new discriminator is required. [23]

The tactic taken most often to find a discrimination criterion is based on the observation that the correlation of the ordinary and extraordinary output signals from a PDC crystal implies that for the transversal components, the EPR stipulations hold; that is: $x_o + x_e = 0$ and $k_o - k_e = 0$. Thus, appropriate measurements of diffraction effects on such signals can be used to probe these features.

The logic of the discriminators proposed in the literature⁸ is based on the following assertion: whereas the variances of the outputs of a PDC are constrained by Heisenberg Uncertainty:

$$(\Delta(x_o + x_e))^2(\Delta(p_o - p_e))^2 \geq 1/4, \quad (14)$$

in fact, QM allows even perfect correlations:

$$(\Delta(x_o + x_e))^2(\Delta(p_o - p_e))^2 \geq 0. \quad (15)$$

Experiments then consist of measuring the dispersions seen in these signals to show that Eq. (14) is not satisfied, which

⁷ Indeed, EPR correlations have been observed to occur in an incontestably classical setting.[21]

⁸ Excepting discrimination criteria based directly on the validity of Bell inequalities, e.g., [24].

is taken to mean that an ineluctable quantum phenomenon, entanglement vice ‘classical correlation,’ has been observed.

In point of fact, however, Eq. (15) is *not* dictated by principles unique to QM, but by conservation principles which are just as valid classically. Moreover, Eq. (14), implies a questionable application of HEISENBERG’s Uncertainty Relation, which applies to Hamiltonian conjugate variables. Thus, it would apply only to the variables of each signal separately, because the sum (difference) of these variables for two such signals are not conjugates.

Most significantly, this claim makes no reference to encoding “locality,” the crux of the matter for BELL’s considerations.

On this basis, it appears that these arguments evade the issues addressed by EPR’s logic.

IX. LOOPHOLES

Quite independent of the defects in derivations of BELL’s Inequalities, there are obstacles that affect only their experimental tests or analysis of taken data. These include those considered in Ref. [5], such as the “detection loophole” which results from irregularities, e.g., nonlinearities, in either the polarizers or detectors, and the “conspiracy loophole” surmised to arise from some cooperative effect between the correlated particles effectively mimicking an intentional ‘conspiracy’ to mislead experimenters. In addition, there are other, often overlooked effects, such as “chronological disordering,” which could result from the overlap of the relevant wave functions between successive events.[25] Although, in all cases, such practical experimental considerations are preempted by fundamental defects, sometimes they are brought into theoretical analysis as if relevant there too.

X. CONCLUSIONS

The points made above offer several explanations for the observation noted in the introduction, that BELL’s Ansatz, Eq. (1), cannot be found in treatises on statistics and probability. To begin, there is misleading notation; BELL used a comma to separate the independent arguments, whereas ‘hidden’ variables, by definition would be conditioning parameters, and, as such, in the notation customary in works on probability, are separated from independent variables by a vertical bar. This malapropos turn of the pen appears to have been an important facilitating element in the general misconstrual of BELL’s analysis. Once this defect is corrected, it is a short leap to the understanding of the necessity for applying BAYES’ formula; a leap apparently made first by JAYNES. [7]

And, once it is clear that Bell Inequalities cannot be derived using BAYES’ formula, the issue of nonlocality is rendered moot. This, in turn, resolves one conflict between two fundamental theories of modern physics—a conflict that on the face of it has the character typical of small, technical misunderstandings. This is only reinforced by the observation that there is *no* empirical evidence for nonlocality; that which has

been taken as such is *not* derived from time- and span-of-flight data, but just an interpretation imposed indirectly on correlation statistics, and moreover, as argued herein, incorrectly.

The main conflict between QM and General Relativity remains, however. The energy density of the quantized ground state of the free electromagnetic field, i.e., the “quantum vacuum,” is, in view of physically admissible ‘cutoffs,’ at least 120 orders of magnitude larger than allowed by cosmological constant considerations!

A similarly perplexing philosophical issue brought to the discussion of the nature of the interpretation of QM is that concerning the ontological status of states, such as the singlet state, ostensibly comprised of the superposition of mutually exclusive entities. This particular issue, seemingly, has caused only mild discomfort among physical scientists, apparently since it is in conflict ‘only’ with philosophical considerations, not major physics theories. Nevertheless, it is auspicious, if only symbolically, that properly understood fundamental physics theories do not encompass gross conflict with the foundations of the enlightenment and the scientific revolution. Rejecting the ‘completeness hypothesis’ contributes to just that.

Finally, based on the analysis presented above, it is arguable that the reasoning behind ‘measurement theory’ and the ‘projection hypothesis’ is fully disputable, and dispensable; and, that there are coherent alternatives qualifying for OCCAM’s approval.

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