

On the nature of information-erasing*

A. F. Kracklauer,

Bauhaus Universität, Weimar, Germany

It is argued that “quantum effects” are those that necessarily depend on hypothetical input unique to quantum mechanics. In effect, they must be a consequence of the non-commutivity of the operators spanning phase space; i.e., Heisenberg uncertainty (HU). This fact implies that all effect described by the group $SU(2)$, which is isomorphic to the group $O(3)$, cannot be quantum effects, as their non-commutivity is due to geometry, not HU. The consequences of this deduction are applied to the “quantum eraser,” effect. The conclusion is that data from ‘quantum eraser’ experiments can be explained with Malus’ Law, i.e., a non quantum principle, and that information is just concealed, not erased.

I. PRELIMINARIES

The problem addressed here has a certain formalistic character benefiting, possibly, from a general, abstract clarification. Thus, first, I shall delineate this aspect based on an analogy with the foundations of geometry as a logical, mathematical structure.

To begin, recall that a logical structure has two categories of inputs: primitive objects and axioms. Primitive *objects* cannot be defined within the structure itself; they can only be understood from experience external to the final logical construct; in geometry, for example, they are the intuitive notions of a point and a line. Axioms also cannot be extracted from the structure, but must be input. They consist of statements specifying the fundamental relationships to be attributed to the primitive objects. For geometry in its most general and extensive form there are four such axioms. For Euclidean geometry an additional axiom is added, namely, that there is only one parallel to any line through a point not on the line.

Clearly, this additional axiom restricts the structure by removing what may be seen as freedom of choice in certain features. If one is thinking of building up a logical structure from the ground up through additions to the axiom set, then such an additional stipulation might be seen as an advancement. On the other hand, if one is striving to encompass more structural possibilities, then the additional axiom might be seen as a step backwards. With regard to geometry, because the features of Euclidian geometry are closer to common experience, the historical development of, and generally the psychological disposition to, non-Euclidean geometry is regarded typically as a progressive endeavour, when in terms of structure it was actually the discovery of more primitive (i.e. less constrained) structural features. The more sophisticated attitude is to consider Euclidean geometry the limit of non Euclidean geometry when certain fundamental parameters, e.g., the radius of curvature, go to some limit, infinity, say. In any case, any theorem not dependant on the additional axiom is properly not a theorem in Euclidean geometry.

The development of Quantum Mechanics (QM) from pre-

quantum mechanics is a very close historical and logical parallel to the development of geometry. Here the essential additional axiomatic input, taking the structure from less to more structural confinement, is a statement on the commutivity of Hamiltonian canonically conjugate variables. That is, whereas these variables may be non-commutative in the less restricted structure (QM), they are mandated to be commutative in the more restricted structure (Classical Mechanics, or pre-quantum mechanics—which from the hierarchical, but not historical, point of view should be thought of as post-quantum). Again, only structure not admitting the axiomatic input mandating commutative canonical variables, is properly described as ‘quantum.’

So much for abstract generalities; now we turn to specifics.

II. PRE- AND POST-QUANTUM (NON)COMMUTIVITY

Non-commutivity in various mathematical arenas arises in several different ways. Within the mathematics used for mechanics, there appear to be three distinct ‘causes.’ The first of these is that due to the non-commutivity of rotations on a sphere. The patterns of this feature are encoded in the group $O(3)$. Since the primitive objects being described in mechanics, point masses, move about in Euclidean 3-space, agglomerations of such point masses (e.g., solid bodies) can rotate, so that this non-commutative feature must arise somewhere in a theory of mechanics. Further, it is known that non parallel Lorentz boosts do not commute. Thus, a theory of mechanics taking electrodynamics with, loosely speaking, light-interaction into account, must also exhibit this second non-commutative property somewhere.

But neither of these causes pertain to the distinguishing feature between pre- and post-quantisation, namely, non-commutivity of *canonically conjugate variables*, which was historically introduced, essentially as an intuitive leap in the spirit of ‘trial-and-error,’ but with much motivational imagery and argumentation on the basis of uncertainty, imprecision and so on. In the literature, these motivation factors have been concentrated largely into statements known as ‘Heisenberg’s Uncertainty Principle’ and ‘Bohr’s Complementarity.’

While there are ideas in the literature suggesting a possible deep relationship among pairs (maybe even all at once) of

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these three “causes” of non-commutivity ¹(1), at present such a connection is unaccepted, incompletely worked out, and mostly regarded as the result of ill understood coincidences. For the purposes of present analysis, it is taken that whatever structural similarities these three seem to have, they are distinct in terms of their fundamental causes and significance for theories of mechanics.

A particular illustration relevant to what follows of this situation with respect to QM, is the appearance of the group structure $SU(2)$. It is at the core of vast amounts of cotemporary analysis usually involving what has become known as “q-bit” space, for example. This structure is widely thought to be an essential and unique feature available for exotic new applications of QM in computation, communication, etc.

However, it is just a straightforward mathematical fact from Group Theory, that the group $SU(2)$ is homeomorphic to the group $O(3)$. The latter group encodes the structure of rotations on a sphere so that obviously the non-commutivity involved is just a geometric effect in absolutely no way derived from the distinguishing axiomatic input into QM. Further, because of, or better put: in accord with, the homeomorphism between these groups, any non-commutivity in q-bit space, having nothing to do with pairs of canonical variables, also must be exclusively geometrical, not quantum, in nature.

So much for theory, we now turn to an application.

III. QUANTUM ERASER EXPERIMENTS

Here focus is directed to two particularly conceptually clean optical experiments that are credited with exhibiting the so-called ‘quantum eraser’ effect.(2; 3) The crux of this effect is considered a demonstration of quantum complementarity.

In both cases a signal and idler is generated by parametric down conversion (PDC) in a crystal; see: Fig. 1.

The signal is directed to a Young double slit setup such that the crystal axis and slits are parallel. The idler is separately detected with or without a polariser. The tactic is to mark the sub signals passing through slits so as to obtain knowledge of which slit a “photon” passed through. This is done by placing orthogonal polarisers at $\pm 45^\circ$ with respect to the slits’ axis before each slit respectively. Thus, if without the polarisers a diffraction pattern was seen, with them it vanishes. This difference is said to represent “erasure of information.” But, if the input signal sent through the Young setup was generated by PDC, then it is possible to examine coincidences between the signal passing through the slits with the so-called idler signal on an independent optical path passing through a separate polariser. Since the signal and idler from PDC are strictly (anti)correlated (depending on crystal type), these correlations can be used to filter the data stream into distinct subsets.

Without going into the currently favoured ‘quantum’ interpretation,² which I dispute in any case, the data taken in such experiments consists of three sets, one without signal-idler coincidences, and two more when the idler is first passed through a polariser, either parallel or orthogonal to the axis of the crystal; see: Fig. 2.

The purely geometrical explanation for the nature of the data sets is depicted in Fig. 3. There it is seen that the signature phenomenon for ‘quantum erasing,’ namely the extraction of a subset of data exhibiting either fringe or antifringer patterns, is just a consequence of what can be called “coincidence filtering.” In no case is any data erased or restored in retrospect—as is sometimes claimed on the basis of the customary ‘quantum’ analysis of these experiments.(5)

IV. IMPLICATIONS

In short, the conclusion from these considerations is that the signature effect seen in these experiments, namely that the fringe and anti-fringe patterns combine to form a fringe-free total, has no uniquely ‘quantum’ interpretation; it is a simple consequence of Malus’ Law and geometry. Further, there is no erasure of information; at most, it can be said only that information is concealed or encrypted; and, thereafter it can be revealed or decrypted using ‘coincidence filtering.’ From the vantage of these arguments, it is not a simple matter of taste or interpretation; the essence of these conclusions is largely a question of the consistency of the mathematics involved, and then of the syntax of the language used to discuss them.

This has, as pointed out by Scully(6), *among others*, immediate consequences for the analysis of any other quantum phenomena described using the group $SU(2)$.

The most celebrated example is Bohm’s version of analysis of Einstein-Podolsky-Rosen (EPR) correlations. The original Gedanken-experiment proposed by EPR was set in phase space where canonical variables span the space and can, therefore, be non commutative for dynamical reasons, i.e., by cause of the axiomatic foundations of QM. However, because of practical limitations, an experiment as EPR proposed it, is not feasible in phase space, Bohm suggested a change of venue, to q-bit space. This space is known to be adequate for describing polarisation of transverse electromagnetic waves, as was discovered first by Stokes in 1852(7), nearly 50 years before the need for QM was appreciated. Again, q-bit space structure is encoded in $SU(2)$ and is fundamentally *not* quantum in character, so that all analysis depending on it is also not quantum, including all analysis formulated in the tradition of Bell on EPR correlations. In other words, Bohm’s modification was not legitimate because he did not distinguish between the various causes of non commutivity.

With regard to Bell’s analysis this conclusion seems to be in conflict with a ‘proven theorem.’ However, while the math-

¹ For example, see (1) for analysis exploiting a formal connection between $SU(2)$ and the Lorentz group. It is still unclear to this writer if this is just a geometrical congruence or or the consequence of fundamental physics.

² See:(4) for a more extensive discussion of the subtleties of the nowadays conventional understanding of this effect and of ancillary issues of interpretation.



Figure 1 The experimental setups to observe the ‘quantum erasure’ effect. The one depicted on the left, by Walborn et al., uses polarisers (or quarter wave plates) to distinguish the signals passing through each slit. If no polarisers are present, then the usual diffraction pattern is observed. If orthogonal polarisers are placed before the slits, no diffraction pattern appears. But, when a polariser is placed before the idler signal and *coincidences* as a function of displacement are counted, they show either a fringe or anti fringe diffraction pattern. The setup on the right, by Kim et al., uses beam splitters to randomly select the various subsets.

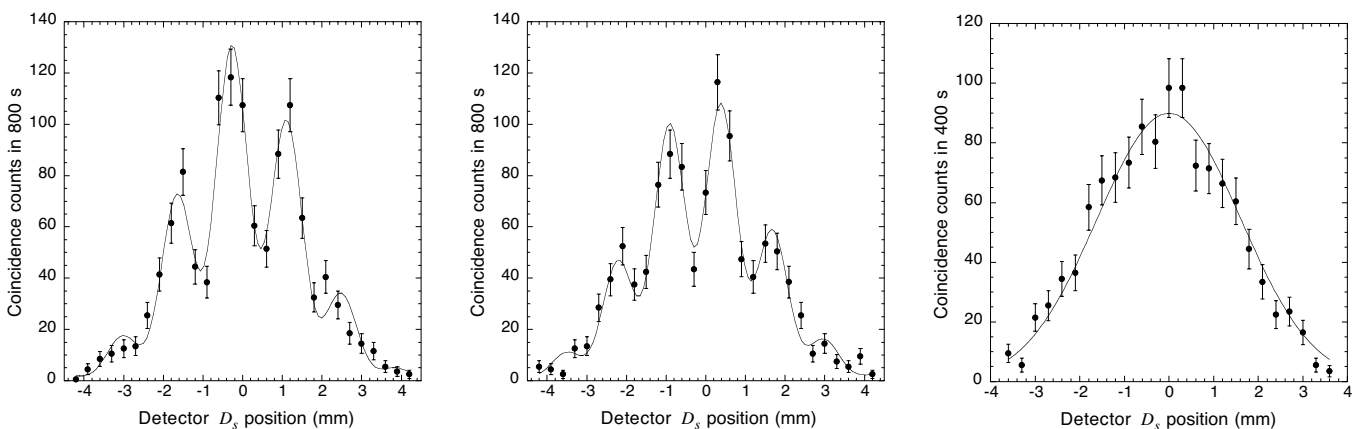


Figure 2 The fringe pattern depicted on the left was observed in the *coincidences* between signal and idler when the polariser in the idler beam is vertical. The middle pattern occurs when this polariser is horizontal. The pattern on the right is the sum of the two and is seen when no coincidence filtering is done.

emathical manipulations employed by Bell, and subsequently others in various elaborations of his argument, are, as algebra alone, correct, their physical significance cannot be a matter of formal logic. This follows from the fact that no physics argument can be a ‘theorem,’ i.e., a syllogism based ultimately on given primitive elements and an axiom set because, *inter alia*, the axioms would be the fundamental theories that are still largely unknown! Moreover, there is also a fatal misstep.

The exact misconstrual in Bell’s theorem, first discussed by Jaynes(8), occurs in the encodification of the concept of non-locality. He pointed out that Bell wrote for the joint probability of the EPR correlations

$$P(a,b) = \int d\lambda \rho(\lambda) A(a,\lambda) B(b,\lambda), \quad (1)$$

whereas he should have written

$$P(a,b) = \int d\lambda \rho(\lambda) A(a|b,\lambda) B(b|\lambda). \quad (2)$$

This is most easily seen if the functions $A(a,\lambda)$ and $B(b,\lambda)$ are identified as ‘expectation functions.’ i.e., as the integrands for calculating ‘expectation values,’ or

$$\langle a \rangle = \int_D d\lambda A(a,\lambda) \rho(\lambda), \quad (3)$$

where D is the complete domain of whatever variables specify the outcomes. From probability theory it is known that such ‘expectation functions’ have the generic form $x\rho(x)$ so that $\langle x \rangle = \int dx x\rho(x)$ where $\rho(x)$ is the probability distribution of the variable x . Now, for calculating correlations in situations where there are two or more probabilistically distributed variables, such probability distributions must obey Bayes’ formula, namely, the joint distribution $\rho(x,y)$ in terms of the independent probabilities is given by $\rho(x,y) = \rho(x|y)\rho(y)$, where $\rho(x|y)$ is a *conditional probability*. When taking this structure into account, it is seen that the correct version of Eq. (1) is of the form of Eq. (2), which precludes derivation of any form of a ‘Bell inequality.’ In effect, Bell misencoded

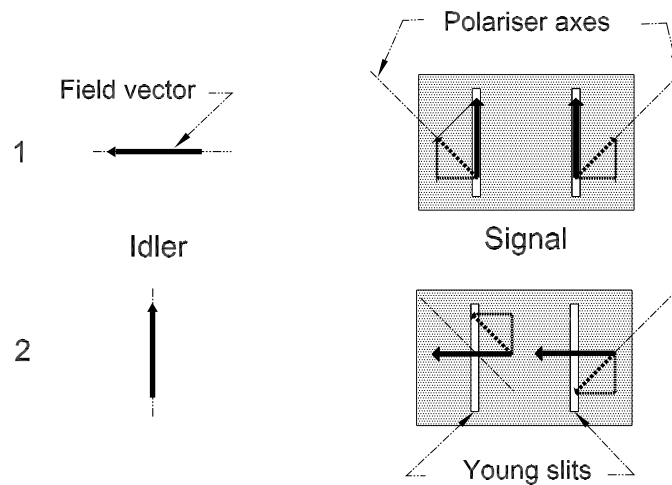


Figure 3 This diagram depicts the geometrical relationships among the various components of the output signals from PDC after having been directed through polarisers oriented as described in the text. If the crystal is chosen so that the signal and idler are anticorrelated, then the total data distribution seen on the right in Fig. 2 can be resolved as follows. Coincidences with horizontal idler signal (row 1) result from pairs for which the signal was vertical. This implies that the vertical components of the polariser outputs add while the horizontal cancel, thereby selecting the points which exhibit a fringe pattern. If the idler polariser is vertical (row 2), then complimentary situation selects the antifringe subset. From these relationships it is evident that the observed phenomena labelled ‘quantum erasing,’ is, in fact, just ‘coincidence filtering’ on the basis of geometrical resolution of the PDC outputs after passing the various polariser setting regimes.

‘locality’ as statistical independence, contrary to the initial assumption of *correlated outcomes*.

In other words, this latter form explicitly incorporates *conditional probabilities* which need to be employed to encode the correlations between the two EPR daughter outcomes—the experiments actually consist of measuring just such correlations; if correlation is attributed exclusively to hidden variables, without manifestation in terms of measurable quantities, then no experiment is possible. A direct consequence of correcting this misstep is that the derivation of all versions of ‘Bell inequalities’ becomes impossible. The consequences of this have been extensively analysed by this writer elsewhere(9–13), to include even a data point by data point simulation free of non local interaction, and so will not be elaborated here. The crucial point for the analysis herein is simply that the belief that Bell’s ‘theorem’ is in conflict with conclusions drawn above on the basis of the non quantum character of the structure of the group $SU(2)$, is fully disputable.

Dedication

This work is dedicated to the memory of Willis Lamb, for his judgement that coherent philosophy must accompany coherent physics, as exemplified in his publication with Marlon Scully providing a semiclassical model for the photoelectric effect.(14)

Note: Preprints of references (4; 9–13) can be downloaded from the writer’s web-page: <http://www.nonlocalphysics.000freehosting.com>

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