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ELECTRODYNAMICS: ACTION-AT-*NO*-DISTANCE?

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An action-at-a-distance (on the lightcone) theory of the electromagnetic interaction is described. It is argued that this theory is internally consistent and avoids several of the infamous pitfalls of field theories such as advanced interaction and divergent self energy of a single charge (or background radiation). In addition, a resolution for problems posed by asymmetric kinematical ageing is proposed.

1. Introduction

Common experience associates "action" with contact: pushing, pulling, lifting, etc. This seems to have deeply prejudiced understanding of action altogether; Newton even deemed the concept of action-at-a-distance as virtually oxymoronic. It was only with great struggle that the action of force which is due to gravity or electrodynamics was eventually equipped with the concept of "field," the function of which for intuition seems to have been in part to serve as an agent transmitting somehow 'contact,' the familiar agent for action.

Newton's theory, while it advanced the understanding of interaction immensely, opened up as many questions as it answered. Most such questions had to do with the 'instantaneousness' of the implied interaction, but others focused on the ethereal character of interaction without contact. The development of Maxwell's equations from Coulomb's Law, provided a partial resolution of the tension engendered by instantaneousness by introducing the speed of light. It addressed the other concern by introducing the concept of 'field.' This (electromagnetic) field is the contact agent. It is emitted by a source particle and then travels to another particle where it then influences it by direct contact, as it were, according to the Lorentz force law. Moreover, field propagation itself is often thought to be the result also of contact between infinitesimal elements of 'field' moving against each other. (Sometimes in the literature this self-contact-process is denoted the Faraday Induction Process.)

This imagery seems to have deep appeal to the human mind and indeed some practitioners of modern physics seek to banish totally the concept of particle in favor of 'fields' in which particles are singularities or resonances.

Nonetheless, from a mathematical perspective, fields are superfluous. Consider the situation with Maxwell field theory, the mother of all field theories. No matter what one wishes to consider as the most fundamental elements of a theory, either particles or fields, in the end experiments consist of measurements on particles. Sooner or later an experiment comes down to determining the world lines of the objects of study, i.e., particles. Thus, the most elementary problem of all would seem to be that of determining the motion of two interacting charged particles. To solve this problem using Maxwell field theory, first one particle is taken as a current, and its fields at the position of the second are found using Maxwell's equations. Then these field values are used in the Lorentz force law to determine the reaction of the second particle to the first. Now, the second particle is considered a current whose fields perturb the motion of the first. This perturbed motion is then used to recalculate more accurate field values at the position of the second. This process is continued back and forth until the desired degree of accuracy is obtained.

Obviously, this procedure is only an approximation. In reaction, one might seek a self consistent system of equations of motion for the particles and fields and seek to eliminate superfluous variables. When this is done, the variables that turn out to be superfluous are the *field* variables! In this sense, mathematics is pointing the way for the ontologist; fields do not posses '*onta*,' they are artifacts of convenience.

If this is so clear and fundamental, why has it not long ago become the standard approach? Why have so many overlooked it? One can not see into the minds of others, but the following issue surely has something to do with the answer to these questions. The main point is this: the equations of motion that result from eliminating the field variables are differential-delay equations. Such equations can not be solved for unique solutions with Cauchy initial data; i.e., the positions and velocities on the boundary of the region of interest. Instead, the world lines must be specified between the past and forward light cones centered at some point. Practical considerations make this data difficult to obtain. The field approach on the other hand, given this difficulty, permits one to attack practical applications with much less ado.

In addition, there were some technical problems with the fieldless equations of motion resulting from the existing versions of mechanics compatible with special relativity that were confusing. One of these has to do with the definition of a system-propertime, or independent variable conjugate to the Hamiltonian for the system. This problem had several manifestations, the most infamous of which leads to the "twin paradox." Another such problem was the introduction into mechanics of "advanced interaction." It is the purpose of this report to highlight the authors' resolutions of these issues. Our final conclusion is that while the field approach will remain the tool of preference for nearly all practical applications, the fieldless equations of motion are superior tools for analyzing certain fundamental issues.

2. Universal or System Time

Special relativity considers the time of an event to be a fourth coordinate in Minkowski space, whereas in non relativistic mechanics, time is the independent variable parameterizing the dynamics. It can not play both roles at once, so a fifth variable must be brought into consideration: the "propertime." Use of propertime as the independent variable in a formulation of multi-particle mechanics compatible with special relativity is generally thought to be precluded by the *presumed* fact that the propertimes of multiple particles are not compatible. In other words, the propertime intervals on two separate world lines between two crossings of these world lines are thought not be identical. In its most simple rendition, this situation is known as the "twin paradox." We, however, consider this understanding of propertime simply a misconstrual [1], and that the situation can be clarified as follows:

Previous analysis of the twin paradox has not carefully considered the issue of the distance to the turn-around point (hereafter called the pylon) of the traveling twin. This distance is not a vector displacement between events on a Minkowski diagram, but in fact the space separation of two entire world lines, namely those of the terminus and pylon of the trip. The pylon, that is, its place in the world, is not an event but a location. The turn-around itself is, of course, an event in the usual meaning of that word for special relativity. For the traveling twin, however, the turn-around event is a secondary matter as far as his navigational needs are concerned. His primary concern is that he travel to the correct point in space, regardless of the time, before changing course. How can he do this? In the most natural way, he and his stay-at-home sibling chart a course before the beginning of the trip; they select an object in the world, a star say, and designate it as the pylon. From standard references they know that this star is located in a particular direction at a determined distance D. This distance is not the length of a Lorentz vector but the proper length of the displacement from the home location of the twins. Alternately, it can be taken as the length of a pure space-like vector between events on each world line. For parallel world lines, this value is invariant starting from any arbitrary point on either world line. With this in hand, the traveling twin then determines the speed capabilites of his craft and calculates the anticipated arrival time at the pylon.

The distance to the pylon star is not an apparent distance, the length of a moving rod, for example, but the proper length to the whole world line of the selected star. Such a length is a scaler and is invariant under Lorentz transformations. The location of the world line of the pylon on a Minkowski diagram depends on the axis to which it refers. That is, this world line with respect to the stationary twin passes through the space coordinate at 'D' on the abscissa. Likewise, this world line must pass through the traveler's abscissa also; but, because of the difference in the scale of the traveler's axis, this same world line, although still parallel to the stay-at-home's world line, will not be congruent to the line referred to the stay-at-home's axis but is displaced by the scale factor. (It is this displacement that has been overlooked in previous analysis and which distinguishes this approach.) The consequence of this displacement is that, the intersection of the traveler's world line with the world line of the pylon is found to be further out on the traveler's world line; i.e., the propertime taken to reach the pylon is seen to be greater than heretofore calculated. In fact, it is equal to the propertime of the stay-at-home as he himself computes it for the time taken by the traveler to reach the pylon. Thus, when the whole trip is completed, both twins agree that they have experienced equal portions of propertime since the start of the trip.[2] Their reports to each other via light signals on the passage of time, in the usual way do not agree, however. The final consequence of these considerations is that, contrary to oft expressed opinion, propertime can serve in a self consistent way as the independent variable for relativistic mechanics.

These points can be depicted graphically on Minkowski charts; see Fig. 1.

The same conclusion can be won also as follows: Let $\mathbf{x_j}$ be the Minkowski configuration four-vector with components x_j, y_j, z_j, ict of the j-th particle. Let $d\mathbf{x}_j$ be a differential displacement along the j-th particle's orbit; i.e., a differential of arc length. Two such differentials tangent to arbitrary points p and p' on orbits j and k are related to each other by the Lorentz transformation $L(p, p', j, k)$, between the instantaneous rest frames of j and k; i.e., given that $d\mathbf{x}_j|_p$, then $d\mathbf{x}_j|_{p'}$ is defined by

$$d\mathbf{x}_k|_{p'} = L(p, p', j, k) d\mathbf{x}_j|_p. \tag{1}$$

Thus, the differential of arc length,$(d\mathbf{x}_j \cdot d\mathbf{x}_j)^{1/2}$ is invariant, because at each point it satisfies

$$\begin{aligned}(d\mathbf{x}_k|_{p'} \cdot d\mathbf{x}_k|_{p'})^{1/2} = & \quad (d\mathbf{x}_j|_p L^* \cdot L d\mathbf{x}_j|_p)^{1/2} \\ = & \quad (d\mathbf{x}_j|_p \cdot d\mathbf{x}_j|_p)^{1/2}.\end{aligned} \tag{2}$$

All such differentials may, therefore be set equal to the common differential $c\,d\tau$, where c is the speed of light and τ is the independent parameter which assumes the units of time; i.e.,

$$c\,d\tau = (d\mathbf{x}_j \cdot d\mathbf{x}_j)^{1/2} = (d\mathbf{x}_k \cdot d\mathbf{x}_k)^{1/2}, \tag{3}$$

dividing (2.3) by c and rewriting yields

$$d\tau = dt_j \gamma_j^{-1} = dt_k \gamma_k^{-1}, \tag{4}$$

where $\gamma_j^{-1} = \left(1 - (v_j/c)^2\right)^{1/2}$ in the customary notation.[3]

Experimental evidence purporting to establish the empirical validity of asymmetrical aging caused by relative motion is in fact much more dubious than generally realized.[4] (It is essential to distinguish between effects arising from frame-to-frame measurements,

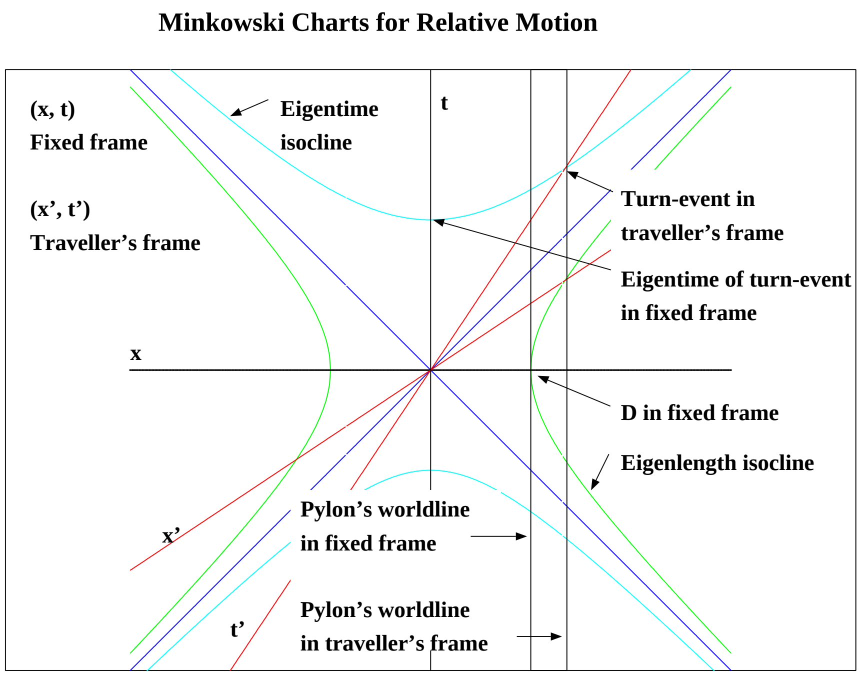

Figure 1: This figure is comprised of two Minkowski charts superimposed on each other. The world line of the pylon in the fixed frame passes through the point '*D*' on the x-axis. The corresponding point on the x'-axis is found by sliding up the *eigen*length isocline to the intersection with the x'-axis. The world line of the pylon passes through this point on the prime chart. The intersection of the pylon's world line with the t'-axis is the point on the traveler's chart representing the 'turn-around' event. The *eigen*time of the turn-around event in the fixed frame is found by sliding down that *eigen*time isocline which passes through the turn-around event to its intersection with the t-axis. It is clear that this value is identical with the time assigned by the fixed twin to the turn-around event as it may be projected horizontally over to the intersection of the Pylon's world line in the fixed-frame with the time axis of the traveler. The paradox arises by using, incorrectly, the particular *eigen*time isocline which passes through the intersection of the traveler's and the pylon's fixed-frame worldlines.

that can be altered by perspective, and those that can only result from a closed circuit, e.g., asymmetric ageing, which modify material. The former are really no more puzzling than the fact that one's hand ($\sim 10^2$cm^2) can shade the sun ($\sim 10^{40}$cm^2.)) Reanalysis of the raw data (long held confidential!) from the renowned clocks-around-the-world experiment of Hafele and Keating — widely cited as clinching empirical verification for asymmetric ageing — convincingly supports the conclusion that the instrumental precision of that experiment was two orders of magnitude short of being adequate.[5] To date, this critical analysis remains unchallenged. Full, or even cursory technical data on alleged subsequent similar experiments of greater precision appear never to have been reported in conventional literature. In sum, we suggest that there is more than sufficient grounds to allow challenge of the conclusions attributed to the results of this experiment.

Experiments exploiting muon decay have been explained in [2] and criticized in [4]. We add the observation that the usual interpretation does not take into account that the population density of decaying muons is transformation dependant. Once again there is plenty of room to legitimately question the popular conclusions drawn from such experiments.

In sum, asymmetric ageing need *not* be considered an empirically established fact.

Here, we note in passing, that the two most widely promulgated interpretations of Special Relativity, namely Einstein's and Lorentz's versions, each challenges comprehension. For Einstein, spacial dimensions contract (*à la* Fitzgerald) and time slows while objects in space-time are invariant (if both change, measurements could show no contraction or slowing). For Lorentz, on the other hand, it is just the meter sticks that contract, and clocks that go slow while space and time as such are held to remain Euclidian (as if meters sticks were not of the same material as invariant 'objects'); see, e.g.:[6]. Both of these schemes[a] seem to treat space and time as objects, as *onta*, rather that Kantian categories of thought or interrelationships, thereby invoking much further reflection.[7] The modification proposed herein, on the other hand, in accord with Occam's principle, leaves both of these "sets" unaffected, and considers a contracted meter stick, for example, as the hypothetical meter stick that an observer would have to have *in his frame* to yield the same measurements for him as those mediated by electromagnetic waves of a meter stick from a relatively moving frame. That is, contracted meter sticks and slow clocks are fictions, just as the hypothetical 2 cm diameter circular object at arm's length that appears to be the same size as the sun, is a fictional portrayal in perspective. These fictions express the effects of the four-space geometry of signal transmission with finite speed (perspective), not mechanical alterations.

3. Advanced Interaction

Fokker developed a closed formulation for the electromagnetic force by incorporating lightcones into direct action mechanics. Essentially he found a Lagrangian which is not

[a]Because of complexities with relatively rotating or more than two frames, neither scheme, it seems to us, is consistently applied; our characterization of them is, therefore, just schematic.

merely the sum of of individual Lagrangians whose variation yields coupled equations of motion.[8] This Lagrangian, however, produced yet another perplexity: It led to simultaneous advanced and retarded interaction for each particle. This feature is problematic on two levels. First, it raises questions of causality because it would mean that the present is always partially conditioned by all of the future, contrary to observation. Secondly, it introduces the calculational complication of precluding the known methods of integrating the equations of motion (this point will be discussed below).

No resolution for the causality difficulties of the *pure* two-particle problem appear to have been proposed; in fact, apparently the only attempt at resolution immerses the problem in a many body universe by invoking radiation absorbers at infinity.[9] Even this solution is dubious. If a model of the universe is built up inductively starting with two particles, then adding more one-by-one, it would appear that the absorbers should be just ordinary charges, devoid of special properties.

Moreover, although integration of the pure two-particle equations has been attempted, thus far the proposed schemes are clearly approximation techniques or useful in severely restricted circumstances.[10]

The essence of our theory[b] is that it has a single independent parameter: the system time; its function is analogous to that of a step counter in a numerical calculation. The only objects with physical significance in this formulation are the world lines; everything else, including the independent parameter, is a mathematical aid to their calculation.

Continuing, let four-velocities be defined as

$$\mathbf{v}_j := d\mathbf{x}_j/d\tau = \gamma_j(\mathbf{v}_j, ic) := \dot{\mathbf{x}}_j, \tag{5}$$

and momenta as $m_j\mathbf{v}_j$, where m_j is the j-th particle's rest mass. With these definitions, the four-vector version of Hamilton's principle

$$\delta \int_{\tau_1}^{\tau_2} L(\mathbf{x}_j, \mathbf{v}_j, \tau)\, d\tau = 0, \tag{6}$$

where (for N (number of particles) $= 2$)

$$\begin{aligned} L = &\sum_{j=1}^{2} m_j\,(\mathbf{v}_j\cdot\mathbf{v}_j)^{1/2} \\ &-2\sum_{k\neq j}^{2} e_j e_k \int_{-\infty}^{\tau} \mathbf{v}_j(\tau)\cdot\mathbf{v}_k(\tau')\,\delta\left(\left(\mathbf{x}_j(\tau)-\mathbf{x}_k(\tau')\right)^2\right) d\tau' \end{aligned} \tag{7}$$

[b] We seek only a few-body covariant formulation. Efforts to develop *instantaneous* direct interaction (also nearly always many-body) theories; e.g., such as those described in: [11], seem to us "challenged," as the interaction terms can not be covariant. Note also that our formulation appears not to provide a vantage from which to evaluate issues regarding the "reality" of longitudinal fields, or pseudo-instantaneous interaction (i.e., delayed fields pointing at the instantaneous position of the source charge)[12], cosmic background fields and the like, in that such matters relate in various ways to the structure of Maxwell's equations, their solutions and (Cauchy) boundary conditions, all of which are avoided *ab initio* in a direct action formulation. Testing our version's validity would consist solely, in the end, of comparing computed and observed worldlines.

yields equations of motion coupled by only two interactions (Because of the upper bound on the integral, not all possible interactions are included.):

$$m_j \ddot{\mathbf{x}}_j^{\mu} = \frac{e_j}{c} \left(\sum_{k \neq j} F_k|_{\text{ret}} \right)^{\mu\upsilon} (\dot{\mathbf{x}}_j)_{\upsilon}, \quad j = 1, 2, \tag{8}$$

where

$$F_k^{\mu\upsilon} = 2c \int_{-\infty}^{\tau} \left(\dot{\mathbf{x}}_k^{\upsilon} \partial_{\upsilon} - \dot{\mathbf{x}}_k^{\mu} \partial_{\mu} \right) \delta \left((\dot{\mathbf{x}}_j(\tau) - \dot{\mathbf{x}}_k(\tau))^2 \right) d\tau'. \tag{9}$$

By virtue of the Dirac delta function, all "interaction" is restricted to the lightcone, on which, of course, all distance vanishes. In this sense, this interaction is abstractly similar to action by contact and is effectively "action-at-*no* -distance."

The special features of this formulation can best be delineated by comparison with Fokker's version. The most outstanding difference is that Fokker's formulation does not exploit (3) and therefore employs a separate independent parameter for each particle. Fokker's Lagrangian, however, is not simply the sum of individual Lagrangians added together in an *ad hoc* manner. He argued that a truly fundamental formulation should proceed from the variation of a *single* system Lagrangian to a set of coupled equations of motion. The Lagrangian L_F,

$$\begin{aligned} L_F = \sum_j^N L_j = \sum_j^N m_j (\mathbf{v}_j \cdot \mathbf{v}_j)^{1/2} \\ -2 \sum_{k \neq j}^2 e_j e_k \int_{-\infty}^{+\infty} \mathbf{v}_j(\tau_j) \cdot \mathbf{v}_k(\tau_k) \delta \left((\mathbf{x}_j(\tau_j) - \mathbf{x}_k(\tau_k))^2 \right) d\tau_k, \end{aligned} \tag{10}$$

satisfies these criteria and leads, by means of the variation

$$\delta \int \sum_j^N L_j d\tau_j = 0, \quad j = 1, 2, \ldots, N \tag{11}$$

to the equations of motion

$$\begin{aligned} m_j \ddot{\mathbf{x}}_j^{\mu}(\tau_j) &= \frac{e_j}{2c} \sum_{k \neq j}^N (F_k|_{\text{ret}} + F_k|_{\text{adv}})^{\mu\upsilon} (\dot{\mathbf{x}}_a(\tau_j))_{\upsilon}, \\ j &= 1, 2, \ldots N. \end{aligned} \tag{12}$$

These equations, however, cannot be integrated by a local procedure, as is obvious if one imagines attempting a machine integration of the j-th equation at a given value of τ_j. Such an integration; i.e., a calculation of an incremental extention of the world line for an incremental increase of τ_j, requires knowledge of the i-th world line on the forward light cone of the j-th particle, which, in order to be computed, required knowledge of the j-th world line on the forward lightcone centered at this point of the i-th particle itself; but, this portion of this orbit is yet to be computed, etc., *ad infinitum.* In effect, the solution is needed as initial data in order to compute the solution in this way.

Of course, advanced interaction could be precluded by changing the upper limit of integration in (10) to τ_{ij}, where τ_{ij} is that value of τ_j which includes only the retarded interaction from the j-th particle; however, as τ_{ij} would then also appear in (10), it could be written as the sum of individual Lagrangians and therefore would not qualify as a true system Lagrangian.[13]

Schemes can be imagined which circumvent this problem by some sort of global approach; i.e., by seeking the whole solution at once. For example, perhaps the solution could be found as the limit of a technique, each successive step of which gave a closer approximation to the entire world line. At present, however, such techniques appear to have not been developed—Eqs. (12) are in general numerically and analytically unsolvable.

Eq. (8), on the other hand, can always be integrated by machine, because the information needed to compute each incremental increase of any world line has already been computed. Also by imagining a machine calculation, it is clear that if each particle's world line between the past and the future with respect to the same, but otherwise arbitrary light-cone, is given as initial data, then the system of world lines can be extended by calculations indefinitely into the future or the past. Although this type of initial data is greater that the customary Cauchy data $\{\mathbf{x}(\tau_\mathbf{a}), \dot{\mathbf{x}}(\tau_\mathbf{a})\}$, it is a general characteristic of differential-delay equations that Cauchy data are insufficient to determine a particular solution, as enough initial data must be given to span the delay.[14]

4. RADIATION REACTION

Because the classical derivation of the mathematical expressions for radiation reaction employs advanced potentials[15], which this formulation excludes, a new physical model of radiation reaction is needed.

Assuming that the universe as a whole is electrically neutral, a particular charge will induce among all other charges a coincident virtual negative image charge. Radiation reaction is assumed to be the interaction of a charge with its own induced image. The equations of motion for this system are (8), where particle 1 is the charge and particle 2 is its image. Solving this system is made easier by the following: One, to first order, $\mathbf{x_1}$ equals $\mathbf{x_2}$ (modulo effects of radiation lag). Two, the interaction from the induced image implodes on the charge as if from an oppositely charged concentric spherical shell. To an accelerated charge, in its own frame, this interaction is identical to that of a pre-counter-accelerated shell, which in turn, is identical to the sign-changed, time-reversed effect of the charge itself; i.e., $F_2|_{ret}$ equals $F_1|_{adv}$. With this substitution, Eqs. (8) can be added to give (note $e_2 = -e_1$)

$$m_a \ddot{\mathbf{x}}_a^\mu = \frac{e_a}{2c} \left(F_a|_{\mathrm{ret}} - F_a|_{\mathrm{adv}}\right)^{\mu\upsilon} \left(\dot{\mathbf{x}}_a\right)_\upsilon . \tag{13}$$

This equation is precisely the starting point of the derivation of an explicit form for the derivation of an explicit form for the force of radiation reaction, which is not herein reiterated.[16]

The Lagrangians (7) and (10) both employ a notational gimmick that can lead to confusion. The problem is that in both formulations two types of integrations appear, each with a distinct function. In (6) the integration on τ and in (11) the integration on τ_j belong to the variational principle; whereas, the remaining integrations really are superfluous. They are part of a notational gimmick used to express Liénard-Wiechert potentials in an elegant form by exploiting the properties of the Dirac delta function.[17] In fact, the delta function can be expanded and the integrations over the dummy variables τ_k in (10) and τ' in (7) executed, so as to write these Lagrangians in a more exposed form before executing the variation. This form would preclude confusion regarding the distinct roles of the various τ's and integrations, albeit at a cost in elegance.

The essential difference between various formulations of the electromagnetic two-body problem is the selection of interactions. Any formulation in which the the interactions are derived from Liénard-Wiechert potentials is consistent with Maxwell's equations. In this formulation the mathematical formalism selects only retarded interaction.

5. Divergencies and Renormalization

Certain questions which arise naturally in field theory, are ill posed when viewed in the context of an interaction theory. A very troublesome problem of this nature is that concerning the quantized ground state of the free electromagnetic field. According to quantum theory, this field has an energy spectrum equal to $\hbar\omega/2$ per normal mode. Obviously, if integrated over all frequencies, this diverges. Even if a cut-off near the Compton wave length of the electron is introduced, still as a consequence of general relativity, this has lead some to estimate that there is such an huge reservoir of background energy that the 'big bang' should have collapsed back on itself within $\sim 10^{-30}$seconds! Lower cut-offs in the optical range still lead to discernable (but not seen) effects in the orbits of the planets, not to mention that they are incompatible with 'quantum' phenomena.

Rejecting the concept of free fields altogether, on the other hand, induces one to formulate this particular matter differently. In the context of this interaction formulation, a corresponding question might be how much work has been done since the big bang via all the interactions with the rest of the universe for a particular charge of special interest? Because there are two charge genders, most of this work is canceled out. All that remains is that due to the slight rearrangement of the exterior charges as caused by the formation of a Debye shield. This effect might be modeled as follows. To begin we suppose that the charges forming the sheath arrange themselves so as to form a cloud of ficticious net charge which may be considered to be composed of mini charges distributed in an energetically parsimonious manner — a stipulation that seems equivalent to distributing the "mini" charges in accord with Boltzmann's factor:

$$P(r) \propto e^{-(\varepsilon_p/\varepsilon_b)}, \tag{14}$$

where ε_p is the 'mini particle's' energy distribution proportional to e/r where r is the distance from the central charge and ε_b is the background heat bath energy, which, for the moment we take to be a constant, a. This ficticious charge distribution now can be considered a 'polarization' charge induced by the central charge. Thus, for such a polarization charge there will be an electric displacement field, D, induced around the central charge according to

$$D = P(r)E, \tag{15}$$

where $E = e/r^2$ is the electric field of the central charge. (In the last sentence the term "field" is used as a convenience to denote the continuous approximation to the totality of "individual interactions.") According to standard theory then, the total energy in the 'field,' i.e., $\mathcal{E} = \int E \cdot D dV$, or in our paradigm, in the totality of interactions, would be

$$\mathcal{E} = \frac{1}{8\pi}\int_0^\infty e^{-e/ar}\frac{e^2}{r^4}4\pi r^2\,dr = \frac{ea}{2}. \tag{16}$$

Setting $a = e/2mc^2$, i.e., half the traditional "radius of the electron," then implies that the total energy of interaction *vis-à-vis* remaining charges in the universe equals the 'rest energy' of the electron.[18] This step can be motivated as setting the scale empirically (although, we appreciate, it is also an invitation for deeper analysis). Furthermore, it can be interpreted to mean that the background energy available at any point in space where there is no charge is zero, thereby proposing a resolution for a fundamental conflict between electrodynamics (and ultimately quantum mechanics) and general relativity as mentioned above. In addition, this model and calculation may provide a physical motivation for the otherwise completely formalistic 'renormalization' procedure as a means to take into account the existence of two charge genders, and therefore, two energy contributions of opposite sign.

6. Conclusions

In view of the fact that the interactions in the above formulation are encoded in terms of Liénard-Wiechert potentials, it is completely compatible with Maxwell field theory. Likewise, our formulation with a system propertime differs from orthodox Special Relativity only with respect to closed circuit effects (i.e., only when the reversal event is crucial). Therefore, it is not in conflict with results well verified empirically. In addition, it has the conceptual feature of admitting a Minkowski-space variant of Lagrangian and therefore Hamiltonian Mechanics *with interaction*. This is not the case with *instantaneous* direct action formulations with interaction terms off the lightcone. Although not established beyond dispute yet, we expect that this will prove crucial. It is in any case important for formal compatability with Quantum Mechanics.[19]

On the other hand, it is just as clear that for the purposes of designing and describing the workings of a radio, radar and perhaps even most physics experiments, the methods based on Maxwell's field equations are more tractable. Nevertheless, for a certain small number of fundamental questions, this formulation may lie closer to the structure actually employed by nature; and, it is for this reason that we study it.

No theory, in physics, mathematics, wherever, starts from nothing and deduces truth. In all cases a set of primitive elements must be given and also some axiomatic statements about these elements. Thereafter, with a chain of syllogisms, additional *true* statements about the primitive elements can be deduced — that is, *true* with respect to the original axiom set. In Physics the situation is complicated by the fact that the basic axiom set is unknown, it should be the basic theories that the whole enterprise is seeking to divine. Physics can be seen as a grand undertaking to work backwards to uncover the basic axioms and even primitive elements. In this context, science is a vast leap of faith that these primitive elements and axioms exist; faith because there is no means to know if the objects considered at any given stage of scientific evolution, are the final and fundamental "stuff" of reality. In this spirit we choose to take those things as the primitive elements without which mathematical encodification becomes hopelessly vague or for which there are "operational recipes" for observation. The concept of field fails these tests. They are in principle and fact unobservable except for their effect on particles, and they are mathematically superfluous. Discussions of the nature of free fields, therefore, seem fruitless exercises in academism. The raw issue is: how is the mutual interaction of systems of particles most cogently and efficiently encoded? What part of this encoding is dictated by logic (in the form of consistent mathematics) and what part by properties inherent to the particles? An interaction theory, action-at-no -distance as it were, for electrodynamics seems to best admit addressing these questions directly.

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18. S. M. Blinder, "Classical electrodynamics with Vacuum Polarization: Electron Self-Energy and Radiation reaction," *Reports on Math. Phys.* **47** (2), 269 (2001) inspired these considerations with a theory in which he supposes that the "vacuum" itself can be polarized. Herein only the rearrangement of actual charges is considered to lead to a polarization charge density.
19. V. Arunasalam, "Lorentz Covariance Versus Invariance: Deeper Insights," *Phys. Essays* **14**(4), 329-340 (2001), carefully delineates the distinction between Lorentz *covariant* physics laws and Lorentz *invariant* results from even non covariant laws. He points out that, except for Maxwell theory, contemporary physics laws are *not*, with rare exception, formulated so that their basic equations are covariant, even while they deliver invariant results. We note, in addition, that the condition that reconciles this matter is the subject of the "no-go" theorems studied in Ref. [3]; i.e.: without a fifth parameter conjugate to the Hamiltonian, self consistent multi-body direct interaction is precluded. The source of this restraint is the use of coordinate time, rather than system proper-time, as the variable conjugate to the system Hamiltonian. Thus, it seems to us, a successful covariant multi-body formulation of physics laws in general awaits the internally consistent formulation of covariant interaction, such as, we believe, that proposed herein.