

PILOT WAVE STEERAGE: A MECHANISM AND TEST

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An intuitive, generic, physical model, or conceptual paradigm for pilot wave steering of particle beams based on Stochastic Electrodynamics is presented. The utility of this model for understanding the Pauli Exclusion Principle is briefly considered, and a possible experimental verification for the underlying concepts is proposed.

Key words: Quantum Mechanics, Pilot Wave, Pauli Exclusion Principle, Stochastic Electrodynamics

1. PROBLEM DEFINITION AND BACKGROUND

Feynman said of the diffraction of particle beams: “In reality, it contains the only mystery” [in Quantum Mechanics].⁽¹⁾ This is so for lack of an explanation of how individual particles ‘know’ about the geometry of the objects which cause beam diffraction; for example, whether there is one or two slits in Young’s experiment. Two paradigms dominate theorizing on this question. The prevailing orthodox view is ‘dualism,’ or ‘complementarity,’ which holds that while in transit, the ‘wave’ nature ‘feels’ the boundaries and determines behavior, but that at the instant of measurement, the wave collapses to its complementary ‘particle’ nature. The main, perhaps only, alternatives are variations on de Broglie’s pilot wave notion. Historically, de Broglie’s core idea was that the ontological essence of a particle is in fact an object consisting of a particulate kernel embedded in a wave which serves as a scout, guiding the kernel.

As they are, neither of these concepts is natural. The orthodox idea suffers profound problems for lack of a fundamental distinction between those interactions which are ‘measurements’ and therefore cause collapse, and those that do not. This is a deep problem

in view of the fact that most measurements are made by capturing radiation, which at the moment of emission, could just as well never end in a measurement, astronomical observations, say. The pilot wave theory, on the other hand, lacks a plausible mechanism for describing just how the wave arises and does its guiding. All obvious explanations, to the extent any has been proposed, lead to the expectation of high particle densities where the wave has nodes, the opposite of what is observed. ⁽²⁾

This work is a contribution to the theory of pilot wave guidance. Its goal is only to cobble together a paradigm of components from classical physics to rationalize this element of Quantum Mechanics (QM), not to further analyze the foundations for deeper consequences of these components. Such studies are left for the future; in the first instance, any classical rationalization of QM is by itself a breakthrough.

As an aside at this point, note that de Broglie's pilot wave theory was inspiration for what has become known as the de Broglie-Bohm alternate interpretation. ⁽³⁾ In this Bohm variant, the 'scouting' function is attributed to an additional 'quantum' potential (in some formulations, implicitly) for which the theory offers no further motivation, in particular, none with respect to classical wave phenomenon. With this in mind, it therefore seems that Bohm's Mechanics is for the purposes of ontology equivalent to 'Copenhagen' QM. Certain other alternate interpretations; e.g., Consistent-Histories, Many-Worlds and others, also seek to invest QM with an internally consistent interpretation without reference to wave or other concepts from classical physics. As such, they belong to intellectually separate streams for which this study has no relevance, and of which it offers no evaluation, rather just competition on the field of intuitive appeal.

2. FUNDAMENTAL CONCEPTS

The foundation of the model or conceptual paradigm for the mechanism of particle diffraction proposed herein is Stochastic Electrodynamics (SED). Most of SED, for which there exists a substantial literature, is not crucial for the issue at hand. ⁽⁴⁾ The *nux* of SED can be characterized as the logical inversion of QM in the following sense. If QM is taken as a valid theory, then ultimately one concludes that there exists a finite ground state for the free electromagnetic field with energy per mode given by $\hbar\omega/2$. SED, on the other hand, inverts this logic and axiomatically posits the existence of a random electromagnetic background field with this same spectral energy distribution, and then endeavors to show that ultimately, a consequence of the existence of such a background is that physical systems exhibit the behavior otherwise codified by QM. The motivation for SED proponents is to find an intuitive local realistic interpretation for QM, hopefully to resolve the well known philosophical and lexical problems as well as to inspire new attacks on other problems.

The question of the origin of this electromagnetic background is, of course, fundamental. In the historical development of SED, its existence has been posited as an operational hypothesis whose justification rests *a posteriori* on results. Nevertheless, lurking on the fringes from the beginning, has been the idea that this background is the result of

self consistent interaction; i.e., the background arises out of interactions from all other electromagnetic charges in the universe. ⁽⁵⁾

For present purposes, all that is needed is the *Ansatz* that particles, as systems with charge structure (not necessarily with a net charge), are in equilibrium with electromagnetic signals in the background. Consider, for example, as a prototype system a dipole with characteristic frequency ω_0 . Equilibrium for such a system can be expressed as

$$m_0c^2 = \hbar\omega_0. \quad (1)$$

This statement is actually tautological, as it just defines ω_0 for which an exact numerical value will turn out to be practically immaterial.

This equilibrium in each degree of freedom is achieved in the particle's rest frame by interaction with counterpropagating electromagnetic background signals in both polarization modes separately which, on the average, add to give a standing wave with antinode at the particle's position:

$$2\cos(k_0x)\sin(\omega_0t). \quad (2)$$

Again, this is essentially a tautological statement as a particle doesn't 'see' signals with nodes at its location, thereby leaving only the others. Of course, everything is to be understood in an on-the-average, statistical sense.

Now consider Eq. (2) in a translating frame, in particular the rest frame of a slit through which the particle as a member of a beam ensemble passes. In such a frame the component signals under a Lorentz transform are Doppler shifted and then add together to give what appears as modulated waves,

$$2\cos(k_0\gamma(x - c\beta t))\sin(\omega_0\gamma(t - c^{-1}\beta x)), \quad (3)$$

for which the second, the modulation factor, has wave length $\lambda = (\gamma\beta k_0)^{-1}$.

From the Lorentz transformation of Eq. (1),

$$P' = \hbar\gamma\beta k_0, \quad (4)$$

$\gamma\beta k_0$ can be identified as the de Broglie wave vector from QM as expressed in the slit frame.

In short, it is seen that a particle's de Broglie wave is modulation on what the orthodox theory designates Zitterbewegung. The modulation wave functions as a pilot wave. Unlike de Broglie's original conception in which the pilot wave emanates from the kernel, here this pilot wave is a kinematic effect of the particle interacting with the SED Background. Because this SED Background is classical electromagnetic radiation, it will diffract according to the usual laws of optics and thereafter, modify the trajectory of the particle with which it is in equilibrium. ⁽⁶⁾ (See Ref. 4., Section 12.3, for a didactical elaboration of these concepts.)

3. PILOT WAVE STEERAGE

All the above is a brief review of concepts to be found in the literature, in part for up to 70 years. What remains unanswered, however, is the question of just how a pilot wave steers a particle. This question is made particularly vexing in that obvious mechanisms seem to lead to a close, but still wrong answer. Specifically, if it is imagined that particles are nudged by the radiation pressure of pilot waves, then particles should be found preferentially at the nodes of these waves where pressure is lowest. But this is not so. Neutron diffraction experiments, and others, yield classical Fraunhofer single slit patterns with a distinct central hump—if radiation pressure from the pilot wave is the steering mechanism, there should not be a central hump but twin humps located at the flanking nodes. ⁽⁷⁾ Clearly, something is missing.

It is the purpose herein to suggest additions to this model to amend this deficiency. The basic concept exploited to achieve this end is to take the modulation function of pilot waves seriously, and to observe that the energy pattern of the actual signal that pilot waves are modulating, and to which a particle tunes, comprises a fence or rake-like structure with prongs of varying average heights specified by the pilot wave modulation. These prongs in turn can be considered as forming the boundaries of energy wells in which particles are trapped. Intuitively, it is clear that where such wells are deepest, particles will tend to be trapped and dwell the longest. The exact mechanism moving and restraining particles is radiation pressure, but not as given by the modulation, rather by the carrier signal itself. Of course, because these signals are stochastic, well boundaries are bobbing up and down somewhat so that any given particle with whatever energy it has will tend to migrate back and forth into neighboring cells as boundary fluctuations permit. Where the wells are very shallow, however, particles are laterally (in a diffraction setup, say) unconstrained; they tend to vacate such regions, and therefore have a low probability of being found there.

The observable consequences of the constraints imposed on the motion of particles is a microscopic effect which can be made manifest only in the observation of many similar systems. For illustration, consider an ensemble of similar particles comprising a beam passing through a slit. Let us assume that these particles are very close to equilibrium with the background, that is, that any effects due to the slit can be considered as slight perturbations on the systematic motion of the beam members.

Given this assumption, each member of the ensemble with index n , say, will with a certain probability have a given amount of kinetic energy, E_n , associated with each degree of freedom. Of special interest here is the beam direction perpendicular to both the beam and the slit in which, by virtue of the assumed state of near equilibrium with the background, we can take the distribution with respect to energy of the members of the ensemble to be given in the usual way by the Boltzmann Factor: $e^{-\beta E_n}$, where β is the reciprocal product of the Boltzmann Constant k and the temperature, T , in degrees Kelvin. The temperature in this case is that of the electromagnetic background serving as a thermal bath for the beam particles with which it is in near equilibrium.

Now, the relative probability of finding any given particle; i.e., with energy $E_{n,j}$ or $E_{n,k}$ or \dots , trapped in a particular well will be, according to elementary probability,

proportional to the sum of the probabilities of finding particles with energy less than the well depth, d , say:

$$\sum_{\{l|E_{n,l} \leq d\}} e^{-\beta E_{n,l}} \cong \int_0^d d(E_n/\mathcal{E}_0) e^{-\beta \mathcal{E}_n} = \frac{1}{\beta \mathcal{E}_0} (1 - e^{-\beta d}), \quad (5)$$

where approximating the sum with an integral is tantamount to the recognition that the number of energy levels, if not *a priori* continuous, is large with respect to the well depth.

If now d in this equation is expressed as a function of position, we get the probability density as a function of position. For example, for a diffraction pattern from a single slit of width a at distance D , the intensity (essentially the energy density) as a function of lateral position is: $\mathcal{E}_0 \sin^2(\theta)/\theta^2$, where $\theta = k_{pilotwave}(\frac{2a}{D})y$, and the probability of occurrence, $P(\theta(y))$, as a function of position, would be

$$P(y) \propto \left(1 - e^{\frac{-\beta \mathcal{E}_0 \sin^2(\theta)}{\theta^2}} \right). \quad (6)$$

Whenever the exponent in Eq. (6) is significantly less than one, its r.h.s. is very accurately approximated by the exponent itself; so that one obtains the standard and verified result that the probability of occurrence, $\psi^* \psi$ in conventional QM, is proportional to the intensity of a particle's deBroglie (pilot) wave. (See Ref. 6. for an account relating $\psi^* \psi$ to a probability and ψ to a pilot wave on the basis of SED.)

For more complex particles which have more than just a dipole interaction, the carrier wave becomes more complex. In turn, the spike structure becomes more complex, but the general considerations above remain valid. In any case, the spike structure is on a scale much finer (at Zitter frequencies and Compton-like wave lengths) than the modulation, and would therefore remain essentially unobservable so that only modulation patterns are manifest.

The condition that the exponent in Eq.(6) is to be less than one, depends on contributions from two factors, β , and $d(y)$. The first of these reflects the thermodynamic environment of the ensemble member in contact with the background as a heat bath. Elementary fundamental considerations set a limit on the term $\beta \mathcal{E}_0$. If a particle trapped in an energy well as described above is regarded in its rest frame as exposed to an harmonic oscillator potential (as a first order approximation to the energy well given by $\sin^2(\frac{m_0 c}{\hbar} y)$), then the mean total energy equals kT while the mean kinetic energy is $kT/2$, and this implies that the exponent in Eq. (6) is less than $1/2$.⁽⁸⁾

The second factor determining the magnitude of the exponent in Eq. (6) is the factor $d(y)$. Above, the expression used for single slit diffraction is the idealized Fraunhofer amplitude which ignores the r^{-1} fall-off of the intensity. In more accurate calculations, this fall-off contributes to a reduction of the exponent, thereby further improving the approximation.

Exploitation of this deviation to experimentally verify the model would be probably very difficult. Even in the limiting case when $\beta \mathcal{E}_0 = 1/2$, the deviation of, for example, a

single slit diffraction pattern is slight. Figure 1 compares the curve derived from Eq. (6) for a particle beam with that for radiation (i.e., the exponent in Eq. (6)) where the curves are normalized so that their peaks coincide. The geometry here mimics closely that of the single slit neutron diffraction experiments described in Ref. 7. The deviation is startlingly small.

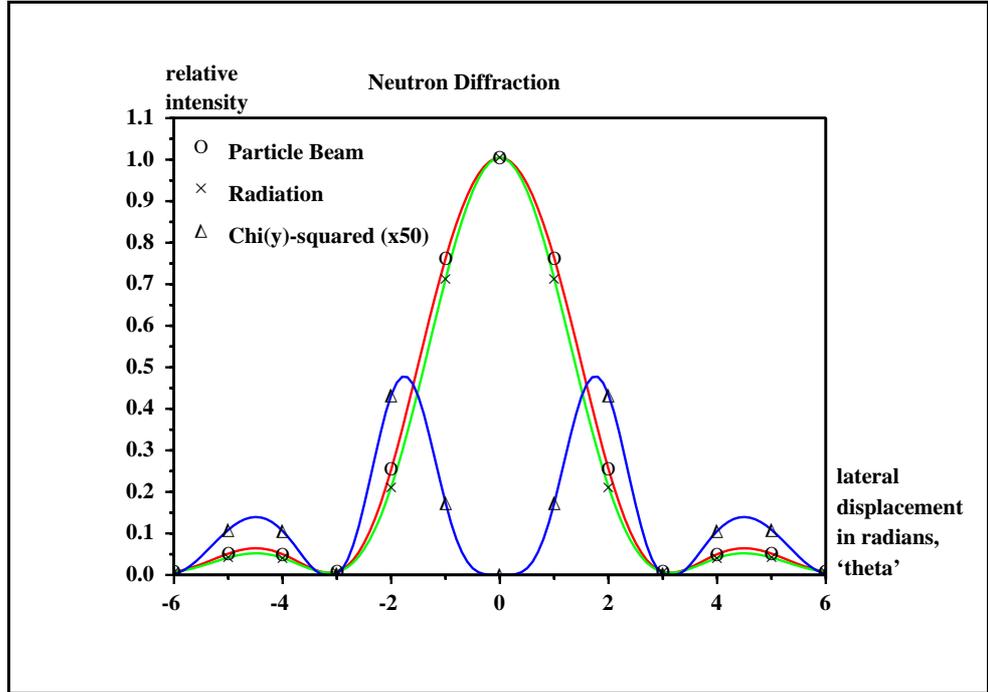


Figure 1: Comparison of the single slit radiation diffraction pattern to that for a particle beam as given by Eq. (6). The slit geometry and beam characteristics (i.e., background) correspond to those described in Ref. 7. for an experiment with neutrons. The χ^2 -curve shows the contribution of the deviation in each displacement bin to χ^2 as used in regression analysis. It is clear that an attempt to fit data described in nature by Eq. (6) with $\sin^2(x)/x^2$ would fall well within the statistical significance of current experiments. As the curves in this figure are computed with $\beta E_0 = 1/2$, when in fact for neutrons this factor would be significantly less, the fit in fact is much better than shown here.

Furthermore, if data described by Eq. (6) taken in a particle beam diffraction experiment is fitted using χ^2 -regression techniques to the radiation diffraction pattern, the fit can be improved by adjusting the assumed slit width. When done, the result is approximately a 1.5% reduction in the slit width. It should be noted here that in Ref. 7. it was reported that for neutron single slit diffraction, the fit to the pure radiation pattern was improved by assuming an approximately 6% *increase* in the slit width. On the basis that an

essentially identical result was observed for laser beam diffraction through the identical optics chain, this was attributed to an artifact of the optical geometry of the experiment . Thus, in combination, the effect discussed herein, seemingly would be observable only as a slight reduction of the increase and would be below the statistical significance of their experiments. In particular, because the neutron is a complex particle for which $\beta\mathcal{E}_0$ can be expected to be less than $1/2$, the optimal slit size reduction would be less than 1.5%; e.g., for $\beta\mathcal{E}_0 = 1/5$, the reduction is 1%. (The curves in figure 1 address only the issue of the suitability of the radiation diffraction pattern for fitting data which is in fact described by Eq. (6) and not the specific geometry of the experiment described in Ref. (7) for beam generation and measurement.)

Electron diffraction patterns, as determined by the exigencies of experimental setups, are typically multi-peak patterns for which errors and tolerances overwhelm deviations attributable to the essential difference in particle beam and radiation diffraction patterns. (9) See Figure 2.

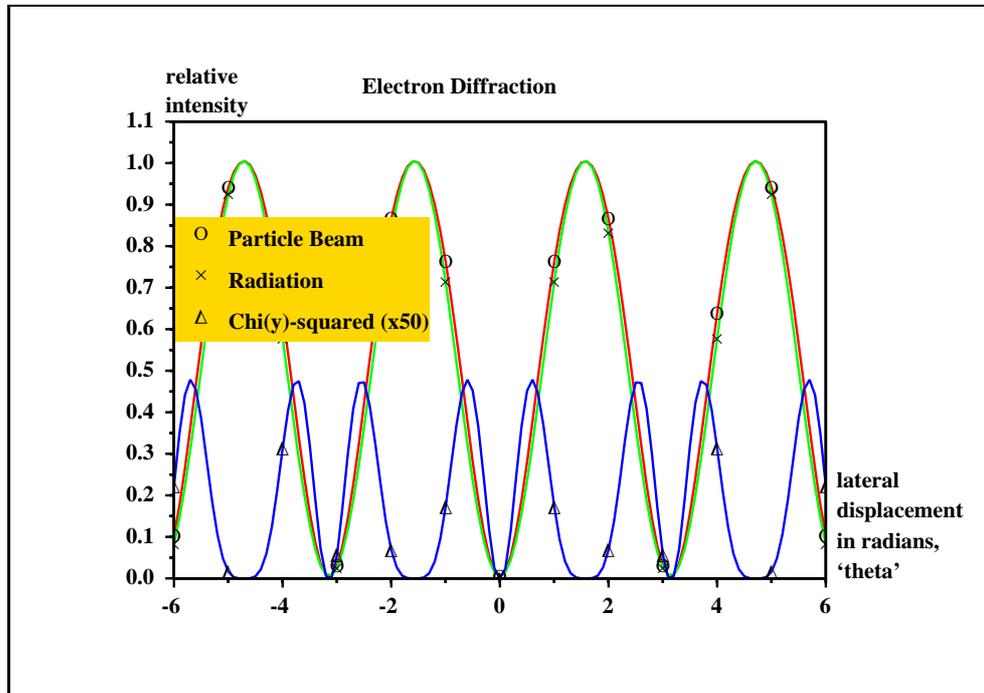


Figure 2: Comparison of particle beam and radiation multislit diffraction patterns corresponding to the experiment described in Ref. 9. for electron beams. Here, although the value of $\beta\mathcal{E}_l = 1/2$ is fully appropriate, the deviation of the particle beam pattern from the radiation pattern is still well below the limit set by the statistical significance of current experiments.

Also of interest is the question regarding the coherence length of guiding waves. According to standard theory, the coherence length of a signal, Δl , equals $c/\Delta\nu$, where c is the speed of light and $\Delta\nu$ is the bandwidth of the signal. In this application, the bandwidth of the background is undefined as all frequencies are present in the background. Nevertheless, the effective acceptance function of the particle, arising, *inter alia*, as inverse line broadening from random motion will result in the same thing.

The lateral coherence area of background signals, also according to standard theory, is $\Delta A \sim R^2\lambda^2/S$ where R is the distance to the source, λ is the wave length of the signal and S is the surface area of the source. Common astronomical sizes and distances attributed to specific sources of a particular background signal, e.g., $R \sim 10^9$ lightyears, $\lambda \sim 5 \times 10^{-12}m$ (typical for electron beams), and $S \sim 10^{-20}m^2$ (atoms) to $S \sim 10^8m^2$ (stars), lead to coherence widths circa 10^2 to $10^{25}m$. While these results must not be taken too seriously, they do confirm that straight forward estimates do not render the underlying concepts improbable.

4. PAULI EXCLUSION PRINCIPLE

In these considerations, the outline of a qualitative rationalization of the Pauli Exclusion Principle from an SED viewpoint may be emerging. In SED, Spin can be seen as a manifestation of the vector character of electromagnetic background signals. In a magnetic, ‘ B ,’ field, particle motion in a plane perpendicular to the this B field can be resolved in terms of clock- and counterclockwise motion each separately in interaction with likewise polarized background-signals. ^(4,6) Such average circular motion gives rise to a magnetic dipole. The energy difference between alignment and antialignment of these magnetic dipoles resulting from this background-driven gyration in typical laboratory B fields is circa 10^{-8} that of the rest energy of the particle. Thus, per the Boltzmann factor, the populations in an ensemble of ‘spin up and down’ particles are virtually equal when they effectively do not interact, for example, when they are distant, independent beam particles. However, in an atom, where because of proximity strong interaction is inevitable, the energy difference between aligned and antialigned dipoles, being proportional to r^{-3} where r is the separation, is large and implies in turn that the equilibrium population distribution difference as given by the Boltzmann factor is large and in favor of the antialigned state. In effect, aligned dipoles preferentially escape from the constraining wells envisioned above leaving only those antialigned states ‘permitted’ by the Pauli Principle in the same diffraction pattern— customarily denoted as a ‘quantum state.’ In an atom, of course, cyclicity, rather than geometrical boundaries such as slits determines standing wave patterns.

In this paradigm, interaction is a mechanism to foster energetic differences in ‘states,’ which then, according to the Boltzmann factor, result in population differences between these states. Likewise, monopole interaction, in this paradigm, would cause one or the other particle of a pair to experience energy excursions, the effect of which would cause it to exceed the retaining energy of the wells in which it is trapped. Also, with respect to

dipole interaction, an essential conceptual element of this model is that spin is engendered by a magnetic field which means, that because two electrons in close proximity (where r^{-3} is large) are exposed to essentially the same magnetic field, the geometry of the spin interaction is restricted to being parallel and antiparallel only. This feature, as is well appreciated, distinguishes particles with ‘spin’ from classical dipoles.

In summary, these considerations begin to render the Pauli Exclusion Principle intuitively consistent with classical thermodynamics given an SED background. That it is a rigorous necessity in detail remains to be shown.

5. POSSIBLE EXPERIMENTAL TEST

A possible test of the above concepts might be made by so arranging that both openings of a Young double slit experiment are transparent to pilot wave radiation, while only one of them is transparent to particles. With electron beams this might be achieved, for example by applying a transverse electric field to slit A, say, while leaving slit B in its innate state. If set up propitiously, particles passing through slit A will be forced away from the registration zone of the observation screen. A particle passing through slit B, however, will remain in equilibrium with the double slit pattern as its pilot wave passes unaltered through both slits. The consequent effect then, will be simply to reduce by half the intensity of the pattern seen on the observation screen.

By way of contrast, if the current orthodox interpretation of QM is correct, blocking the particles in slit A in any way should result in the interference pattern changing to that of a single slit as well as a reduction in the intensity. A particle passing through a double slit is put into a ‘cat’ state, $1/\sqrt{2}(|A\rangle + |B\rangle)$, which is then to interfere with itself to yield the double slit pattern. If particles are prevented from passing through slit A with certainty, then the subsequent state can only have the $|B\rangle$ component, so that the wave function can exhibit only the single slit interference.

As is usual with Young’s double slit experiment on a microphysics scale, realizations are not unproblematic. In this case, an additional crucial factor arises; namely, whatever is done to or in slit A, must not spill over into slit B and destroy the coherence of the beam passing through it by introducing dispersion in velocity. Such spurious intervention, to first order at least, would destroy completely the diffraction pattern rather than transform it from the double to single slit pattern.

From the imagery afforded by the SED model of particle diffraction, it can be seen also that ‘which-way’ identifier operations in two-slit experiments that seek to tag particles passing one slit must do so such that phase shifts are not induced. It is not sufficient that the input and output wave vectors of the tagging operation are identical. In order to test complementarity, it is also necessary that the tagging operation does not introduce a random phase shift with respect to that portion of the pilot wave that passes through the other slit. If such a phase shift is randomly distributed, ensemble uniformity is lost, diffraction patterns vanish, but principles remain untested. In particular, this means that tagging operations in which polarization is affected would be disallowed as the two modes

are independent and therefore, the phases are randomly distributed from particle to particle even when no (net) work is done in the propagating direction.

6. ANCILLARY COMMENTS

In many particle beam experiments, the optical elements are not passive but actively introduce an intervention which is functionally equivalent to a measurement. For electron beams, for example, two slits can be simulated with a so called biprism that consists of a charged wire parallel to a transverse direction of the beam. ⁽¹⁰⁾ As the beam passes on each side of the wire, it is deflected away somewhat from the longitudinal direction of the beam so as to form two slightly diverging beams. A second such wire parallel to, oppositely charged and downstream from the first, then serves to draw the diverging half beams together again so that they converge and interfere on the observing screen. In this arrangement the beam particles (electrons) are deflected by work done by imposed electric fields and not by virtue of diffraction of matter waves (or by energy pattern wrinkling in SED induced pilot waves). Since the beams, after passing a biprism can reconstitute matter waves (i.e., reequilibrate with new signals in the background), experiments of this type seemingly can not reveal particle/pilot-wave feed-back or self- interference, but rather interbeam interference.

On the other hand, the fact that a biprism works at all, provides backhanded evidence that local hidden variables exist. In conventional QM, the wave function is considered complete and uniform; there are no separate particle and wave aspects; the two qualities are totally intermingled. Thus, when a single particle wave function is divided at a biprism, both the wave and particle aspects must be similarly divided. However, when a single particle wave is divided and measurements are made only on a portion of the beam, either nothing at all or the whole particle is observed. Collapse of the wave packet at the moment of observation can be evoked to explain the appearance of the whole particle. But this explanation runs amok when it is recalled that the division of the wave function in a biprism in the first place occurred by virtue of deliberate intervention, (i.e., by consciously evoked fields whose effect is recordable by observing the current in the prism wire—whether in fact done or not), which is equivalent to a measurement. Then, if the wave is collapsed at the prism, there should be no wave thereafter to interfere at the observing screen. Moreover, if this intervention is admitted into the class of agents provoking collapse, then, as these intervention fields are not localized; i.e, $1/r^n$ vanishes only at ∞ , the Zeno effect should prevent collapse altogether! In short, Occam's razor points, inexorably, to rejecting the concepts of distributed 'particleness,' as well as wave collapse, and supports instead admitting the image of concentrated particles at distinct locations, which implies that they have preexisting, local configuration coordinates—a.k.a.: 'hidden variables'—imbedded somehow in a separate (pilot) wave aspect. ⁽¹¹⁾ In the imagery supported by SED the wave aspect is engendered by the background.

Hopefully the above inspires an illuminating experiment.

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