

**ON UNIVERSAL INTERACTION.
AN EXTENSION OF CLASSICAL DYNAMICS.**

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A translation of:

**Über den Wirkungszusammenhang der Welt.
Eine Erweiterung der Klassischen Dynamic.**

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1. INTRODUCTION

Electrodynamics, as is well known, specifies that the [electrodynamic] force on an electron¹ (i.e., a charged point mass) is proportional to its charge and the electromagnetic fields to which it is exposed, as well as to its own velocity. On the other hand, the LORENTZ-MAXWELL differential equations alone are not sufficient to unambiguously determine the motion of electrons from field strengths, or alternately from potentials. General solutions for the potentials can be found quite simply, they consist of retarded integrals over the charges within a volume, or over the potentials and their differential quotients on the surface enclosing this volume. Likewise, in stead of retarded expressions, that is: potentials dependant on the source charges at past times equal to r/c , where r is the distance to the source charge, one could consider “advanced” expressions dependant on source charges as they will be a time r/c in the future. These two possibilities basically, must be mathematically equivalent. Only when the surface integrals [i.e., boundary conditions] are neglected will they differ in general.

The fact that electron motion—or the alterations thereof—is governed by fields, which in turn are not uniquely determined by electrons themselves, makes it clear why one might prefer to consider fields as ‘primary,’ in which electrons are high intensity singularity-deviations from solutions (either as fields or potentials) of the LORENTZ-MAXWELL equations. In fact, however, there is no reason fields should be determined fully by single points; it could well be that there exist fields derived from no charge at all, but rather that simply emerge from infinity. Such a theory for matter was proffered by MIE; it is subject to HAMILTON’s principle and fails to explain quantum effects, however. In fact, such quantum effects appear incompatible altogether with field concepts.

For these reasons I elect to take the opposite viewpoint by considering those features as primary for which there is direct empirical evidence, namely, positive and negative electrons [charges], as countable items with individual trajectories. I take it that detailed trajectory changes (i.e., accelerations) for individual charges are caused only by other charges and consider fields as just mathematical constructions which, wherever quantum effects can be neglected, assist in describing mutual interactions, but otherwise are immaterial.

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¹*Translator’s note:* At the time of writing, ‘electron’ appears to have been virtually a synonym for ‘charged, massive particle.’

2. CLASSICAL INTERACTION

Following this basic premise we must take it, that the potentials mentioned above give correct values whenever we take the space integrals so as to include all charges but ignore the surface integrals. Furthermore, we shall tentatively take both positive and negative time directions as being of equal significance. Thus, the effective potential shall be the mean of advanced and retarded potentials:

$$(2.1) \quad \Phi = \frac{1}{2} \int [P]_{t-\frac{\epsilon}{c}} \frac{dS}{r} + \frac{1}{2} \int [P]_{t+\frac{\epsilon}{c}} \frac{dS}{r},$$

where P is the 4-current.

This expression can also be written in the form:

$$(2.2) \quad \Phi = \int c dt' \iiint \frac{f(\sigma^2)}{\sigma^2} P' dx' dy' dz',$$

where σ is 4-distance:

$$\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2 - c^2(t-t')^2} = \sqrt{r^2 - c^2(t-t')^2},$$

from each source point (primed) to each point of application. The function f is to satisfy the following conditions for each positive, non null Δ :

$$f(\pm\Delta) = 0 \quad \text{and} \quad \int_{-\Delta}^{+\Delta} \frac{f(x)}{x} dx = 1$$

(for negative Δ , the integral equals -1 naturally). From this it follows that only over an infinitesimal region of the "light cone" (i.e., where $\sigma = 0$) does the integrand make a contribution.

The contribution of a single electron with charge e' and velocity v' , given that

$$e' = \int \frac{P'_l}{i} dS', \quad \text{and} \quad e' \frac{\mathbf{v}'}{c} = \int \mathbf{P} dS',$$

whenever the dimensions of an electron can be regarded as being very small, would be:

$$(2.3) \quad \Phi_1 = \int_{-\infty}^{+\infty} \left(ie', e' \frac{\mathbf{v}'}{c} \right) \frac{f(\sigma^2)}{\sigma^2} c dt' = e' \int \frac{f(\sigma^2)}{\sigma^2} d\rho',$$

where $d\rho'$ is the vector line element of the world-line of this electron with components $d\rho_l = ic dt'$, $d\rho_x = dx$, etc. Integration of the second integral also is to be carried out over the whole world-line. For evaluation of the first integral, it is to be kept in mind that a space distance r that goes into σ^2 , in this case varies with t' . So, one gets:

$$\Phi_1 = \frac{1}{2} \left[\frac{\left(ie', e' \frac{\mathbf{v}'}{c} \right)}{r \left(1 - \frac{v'_z}{c} \right)} \right]_{t-\frac{\epsilon}{c}} + \frac{1}{2} \left[\frac{\left(ie', e' \frac{\mathbf{v}'}{c} \right)}{r \left(1 - \frac{v'_z}{c} \right)} \right]_{t+\frac{\epsilon}{c}},$$

according to well known formulas.

The HAMILTONIAN for motion of an electron with mass m_1 and charge e_1 immersed in an electromagnetic field is known to be (with signs reversed with respect to the usual mechanics):

$$H_1 = m_1 \int \sqrt{-(d\vec{\rho}_1 \cdot d\vec{\rho}_1)} - \frac{e_1}{c} \int (\vec{\Phi}_1 \cdot d\vec{\rho}_1) = m_1 \int ds_1 - \frac{e_1}{c} \int (\vec{\Phi}_1 \cdot d\vec{\rho}_1),$$

where $\vec{\rho}_1$ are the space-time coordinates of the electron, and ds_1 is the infinitesimal arclength along its world-line. Integrations are understood to extend over the whole length of world-lines.

When the field is engendered by a second electron, Eq. (2.3) gives:

$$\Phi_1 = e_2 \int \frac{f(\sigma_{1,2}^2)}{\sigma_{1,2}^2} d\rho_2,$$

so that

$$H_1 = m_1 \int ds_1 - \frac{e_1 e_2}{c} \iint \frac{f(\sigma_{1,2}^2)}{\sigma_{1,2}^2} (d\rho_1 \cdot d\rho_2).$$

For infinitesimal variation of the world-line of the first electron, which vanishes for $t = \pm\infty$, and provides the stipulation: $\delta H_1 = 0$, one gets an equation of motion of the first charge in the field of the second. Here, H_1 can be replaced by

$$(2.4) \quad H_{12} = m_1 \int ds_1 + m_2 \int ds_2 - \frac{e_1 e_2}{c} \iint \frac{f(\sigma_{1,2}^2)}{\sigma_{1,2}^2} (d\rho_1 \cdot d\rho_2),$$

as variation of the first world-line is carried out ignoring the second. This expression, however, is symmetric such that for a variation of the second world-line it gives an equation of motion for the second charge under influence of the first. Thus, $H_{i,j}$ constitutes the mutual ‘influence function’ of each charge on the other. It is, however, no longer a valid HAMILTONIAN, as it is not an integral over a single, uniquely defined time parameter.

The expression

$$(2.5) \quad -\frac{e_1 e_2}{c} \iint \frac{f(\sigma_{1,2}^2)}{\sigma_{1,2}^2} (d\rho_1 \cdot d\rho_2),$$

is, except for the factor f , the 4-space analog to a NEWTONian, or 3-space, electrodynamic interaction potential of two electric current circuits, namely:

$$-i_1 i_2 \iint \frac{(\vec{dl}_1 \cdot \vec{dl}_2)}{r}.$$

The ‘influence function’ for the universe would be:

$$(2.6) \quad H = \sum_i m_i \int ds_i - \sum_{j \neq i} \frac{e_i e_j}{c} \iint \frac{f(\sigma_{i,j}^2)}{\sigma_{i,j}^2} (d\rho_i \cdot d\rho_j).$$

3. QUANTUM INTERACTION

We now wish to extend this structure, which we extracted from classical equations for electromagnetic interactions, so as to admit the possibility of covering quantum effects. In place of the function $f(\sigma^2)$, consider another, for the time being undetermined, function, w , of the 4-velocity and relative coordinates of interacting pairs of charges. That is, write:

$$(3.1) \quad H = \sum_i m_i \int ds_i - \sum_{j \neq i} \frac{e_i e_j}{c} \iint w_{i,j}(ds_i ds_j),$$

where $w_{i,j} = w_{j,i}$, which is to be a function of three variables $\mathbf{v}_i, \mathbf{v}_j$ and $\rho_i - \rho_j = \sigma_{i,j} = -\sigma_{j,i}$, and which for the three permutations, $++$, $--$ and $+-$, of charge gender, can be distinct, e.g., for electrons and protons, say.

The condition $\delta H = 0$ yields, whenever the variation of the coordinates at infinity vanish, the equations of motion for each index i

$$(3.2) \quad \frac{d\pi_i}{ds_i} = - \sum_{j \neq i} \int \frac{\partial w_{i,j}}{\partial \rho_i} ds_j,$$

where

$$\pi_i := m_i v_i - \sum_{j \neq i} \left\{ \frac{\partial w_{i,j}}{\partial v_i} + v_i \left(\frac{\partial w_{i,j}}{\partial v_i} \cdot v_i - v_i w_{i,j} \right) ds_j \right\}.$$

Here d/ds_i indicates differentiation along the world-line. Quantities with subscript i pertain to particular points along the world-line of the i -th particle, whereas those with subscript j are covered by an integration along the entire length of the j -th particle.

For each particle there are four equations corresponding to the four components of Eq. (3.2). Among them, however, interdependencies must obtain, in so far as there is a certain arbitrariness, within limits, of the correlation of points on varied trajectories to those on un-varied trajectories, and because this correlation can have no significance as the world-line is mapped onto itself. In fact, by scalar multiplication of Eq. (3.2) with v_i , an identical equation is obtained, when account is taken of both the fact that $v_i \cdot v_i = v_i^2 = -1$, and that:

$$\frac{dw_{i,j}}{ds_i} = \left(\frac{\partial w_{i,j}}{\partial \rho_i} \cdot v_i \right) + \left(\frac{\partial w_{i,j}}{\partial v_i} \cdot \frac{dv_i}{ds_i} \right).$$

One observes, that in case the constant v_i^2 explicitly appears in $w_{i,j}$ (which can be arbitrarily introduced in fact), that this has no effect on the equations, as such alterations in $\frac{\partial w_{i,j}}{\partial v_i}$ and in $v_i \left(\frac{\partial w_{i,j}}{\partial v_i} \cdot v_i \right)$, mutually compensate each other.

Eq. (3.2) links motion of an electron to that of all other electrons for all times, although some limited time segments have greater influence, which is similar to the situation in classical mechanics in which motion of other electrons only at a specific time matters.

The functions $w_{i,j}$, naturally, will have to have certain convergence properties, otherwise divergencies will occur in calculating Eq. (3.2). We shall take it, that the quantity $\int w_{i,j} ds_j$ for large distances between electron i and the world-line of charge j fall off as $1/r$ in general, except, perhaps for a few exceptional points; the average of $\int w_{i,j} ds_j$ over a small segment of the i -th world-line should, however, closely approach $1/r$. Exceptions from this pattern, we shall interpret below as quantum energy transfers.

Perhaps the simple integral in Eq. (3.1) should be generalized to a double integral, traversing the same world-line twice. This would lead to:

$$(3.3) \quad H = \frac{1}{2} \sum_i \sum_j \iint w_{i,j} ds_i ds_j.$$

When indices are identical, it is necessary to make a distinction, however, as each pertains to different positions along the world-line; so that it might be better to write:

$$\frac{1}{2} \iint w_{i,i} ds_i ds_i.$$

For straight world-lines and for longer straight segments, this must reduce to $m_i \int ds_i$. Whenever, $w_{i,i}$ exhibits certain uniformity and convergence characteristics, may one write $\int m_i ds_i$ for it, as m_i is in general variable and depends on acceleration and higher accelerations of even order. We shall not make use of this generalization herein, as we further consider only motions for which they have no effect.

Let us consider now electron trajectories that come in from infinity, mutually influence one another, and then escape to infinity again. Here we shall take it that all other charges in the universe have insignificant influence during the mutual interaction. Let us denote this as an hyperbolic interaction, so that before and after interaction, the above discussed integrals with $w_{i,j}$, may be written: $\pi_i = m_i v_i$.

By multiplying Eq. (3.2) with ds_i , then integrating over interaction from $-\infty$ to $+\infty$ and summing for n electrons, gives for the change in the total momentum and total energy $\sum^n m_i v_i$, a sum of double integrals over $ds_i ds_j$ that cancel pairwise because:

$$\frac{\partial w_{i,j}}{\partial \rho_i} = -\frac{\partial w_{i,j}}{\partial \rho_j}.$$

This shows that after a hyperbolic interaction, momentum and energy are unchanged. That is, in this extended sense, the laws of conservation of momentum and energy are valid, although they do not hold as they do in classical mechanics for each infinitesimal moment.

A distinct feature of this formulations is, that not only that retarded potentials from the past affect any particular electron, but also advanced potentials from the future. Nevertheless, the two time directions are not fully equivalent. For reasons derived from the theory of invariance, the interactions $w_{i,j}$ depend only on the scalars $\sigma_{i,j}^2, (\mathbf{v}_i \cdot \mathbf{v}_j), (\mathbf{v}_j \cdot \sigma_{i,j})$ and $(\mathbf{v}_j \cdot \sigma_{j,i})$. In those cases when the charges involved in an interaction are different types, the terms in $(\mathbf{v}_i \cdot \sigma_{i,j})$ and $(\mathbf{v}_j \cdot \sigma_{j,i})$ in $w_{i,j}$ can be non-symmetrical. In so far as the imaginary time component of \mathbf{v}_i and \mathbf{v}_j are always positive (imaginary), and $ic(t_i - t_j)$ in $\sigma_{i,j}$ has the opposite sign, i.e., $-ic(t_j - t_i)$, in $\sigma_{j,i}$, while the space components do not change sign, this asymmetry distinguishes time directions, such that negative charges are influenced largely, or even exclusively, by the past of positive charges, while the latter are influenced mostly by the future of the former.. This accords with the fact that the transformation $t \rightarrow -t$ is not in the LORENTZ group so that it must not leave the of laws of nature invariant. The difference in the masses of the two charge genders could be, together with this time asymmetry, very significant for the irreversibility of some phenomena. This contrasts with the usual assumption that an asymmetry in time arises, in spite of time symmetry of the basic laws of mechanics, because of a statistical principle of essential disorder at the molecular level. That such a principle follows from the classical laws of mechanics somehow, is, however, unproven and presumably false.

Let us examine the question regarding conservation of angular momentum in this formulation. It is easy to show, as we have for momentum and energy above, that if angular momentum is defined as a sum of cross products of the form $\pi_i \times \rho_i$, and considers that all terms with $\mathbf{v}_i \times \mathbf{v}_j$ vanish, one gets for the change in angular momentum a double integral with integrand

$$\sigma_{i,j} \times \frac{\partial w_{i,j}}{\partial \sigma_{i,j}} + v_i \times \frac{\partial w_{i,j}}{\partial v_i} + v_j \times \frac{\partial w_{i,j}}{\partial v_j}.$$

The sum of these three cross products vanish identically, as can be easily seen by direct calculation if one takes $w_{i,j}$ as function of the four scalars as given above. Angular momentum is usually defined as $\sum_i \rho_i \times m_i v_i$ such that although at large distances up to terms of the order $1/r$, one may write: $m_i v_i = \pi_i$, only the factors ρ_i increase proportional to r , so that the identity of $\sum \rho_i \times \pi_i$ with $\sum \rho_i \times m_i v_i$ is not assured. Although I have found that a sufficient condition, is the symmetry of $w_{i,j}$ in $v_i \cdot \sigma_{i,k}$ and $v_j \cdot \sigma_{j,i}$, as far as I can see, it is not necessary.

4. SPECULATIONS AND CONCLUSIONS

Thus far I have not been able to determine the functions $w_{i,j}$ (nor even to prove their existence) that lead to the actual laws of nature, namely those covering quantum phenomena. Nevertheless overarching considerations based on these ideas seem to support their general rectitude. Moreover, there is considerable latitude for the possible generalization of $w_{i,j}$ as

functional relationships of a very general nature between world-lines, so that the following considerations seem secure. Perhaps even, such a generalization will be unnecessary.

In classical dynamics one gets one and only one possible motion for a system given coordinates and momenta as initial conditions at a specific time, which is a consequence of the fact that time differentials in HAMILTON's equations of motion are dependant on each other simultaneously. According to Eq. (3.2) in this formulations, there are contributions to each point on each trajectory from earlier and later times and so it is not at all clear if at each point there is a unique orbit in phase space satisfying this equation. For hyperbolic orbits this may well be the case as the orbits come in from infinity and it would seem that there is sufficient latitude there to allow accommodations. For quasiperiodic motion, however, it seems possible that the functions $w_{i,j}$ may accommodate a solution to Eq. (3.2) only at a limited number of points of phase space; for example, at points corresponding to stable BOHR orbits, where SOMMERFELD's quantization conditions are satisfied. If this is the case, it might be that the confluence of orbits for two negative charges and one positive charge orbit structurally can accommodate just a hydrogen atom and a free electron.

Suppose two atoms in different states of excitation are located near each other, normally it is to be expected that they would have little influence on each other; however, under special conditions with respect to positions and velocities, possibly also in the vicinity of a third atom, it might be that strong interactions occur, similar to the way in which the average of $\cos(nt) \cos(mt)$ is zero except when $n = m$. Such a situation could well lead to an energy transfer between atoms such that their excited states are exchanged. The energy loss of one and the gain of the other could occur in a time interval corresponding to their separation; that is, we would have an instance of emission from one atom and absorption by the other. While according to classical understanding, emission is a random event leading to radiation that also randomly might somewhere at sometime be adsorbed; here in this theory, the source and sink of a radiative interaction are virtually predetermined paired events.

That is, in effect, the sun would not shine at all were there no other charged bodies in the universe to adsorb its radiation. Between heat radiation and heat conduction there is no principle difference in this theory, because atoms on the surface of a warm body are continuously in interaction with those of the absorbing body, the only difference with radiation is that the separation distance for the latter is much larger.

The fundamental equations even permit absorption to precede emission, however, it seems very improbable that this occurs. Absorption can also naturally occur when an atom undergoes changes, for example, via ionization or other reactions.

It is also possible that between source and sink, additional complex systems of charges could get involved in the interaction and cause refraction, reflection or similar deviations. Light that is emitted as a quantum, is also absorbed as a quantum, although in the interim, it may exhibit the character of an extended object, as seems to be indicated by interference effects. According to conventional theory, which overlooks the essential pairwise unitarity of the emission:absorption process, this is a logical contradiction.

This new formulation elucidates another mystery. To date, it is considered that emission of a quantum of light is often a random event, with no identifiable cause. According to this new conception, however, emission occurs when conditions for subsequent absorption at a later time are suitable. One might tempted to imagine that the large quantity of mass in the universe would accommodate excessive emission. However, this is not inevitable, as two absorbing centers need not reinforce each other, rather they might suppress each other. Were the mass of the universe large enough and distributed evenly, its size would then lose

all effect. At the moment, however, failing mathematical development for these issues, further discussion is pointless.

If I, for example, using a telescope observe a star 100 light years away, not I alone shall have known that light seen at that moment was emitted 100 years ago; as before I even existed at least atoms on the star must have been predestined and informed of my future existence to include even the design and construction of my telescope, and so on. Such an idea sounds quite paradoxical, as it is at odds with normal understanding. Contemporary science teaches the primacy of proximate causes and the dominance of a unidirectional, but possibly statistically modified, causality principle. It is known, however, that such notions were not the original spiritual inclination of mankind; in earlier times they were not universally considered valid, and the existence of other, now exotic, realities was conjectured, although often in a rather naive manner.

Thus, purely on the basis of logic, there can be, in my view, no objection to this new formulation. It is actually no more than an extension of classical dynamics to relativistic interaction of point masses, as made necessary by LORENTZ transformations.

Moreover, there seems to be no contrary empirical evidence, as least as far as this initial analysis indicates. When solar radiation is emitted and then eight minutes later absorbed on Earth, in the intervening time interval, according to classical physics, it is to be found each moment at a distinct location as field energy. This new theory, however, does not recognize fields, so in its terms in the interval the energy is nonexistent, although it will reappear at the moment of absorption. This leads to no observable differences. Of course, a third body could be placed between the Sun and Earth that itself absorbed a portion of the radiation so as to substantiate existence and reality of the interaction in the interval, but this intervening matter is unrelated to the surface of the Earth, leaving the issue of the Sun-Earth interaction still open in principle. A diffracting object could also be inserted in the path of Sun-Earth interaction, but again without absorption at some place and time, this exercise too, would be empty. Although classical conceptions of interaction seem more natural and satisfying to our ontological understanding, quantum phenomena force us, in my view, to abandon classical principles, even while they, for purposes of setting up calculations in many cases (diffraction, dispersion, etc.), practically are indispensable, especially as wave optics in this new formulation would be, presumably, quite intractable.

The speculations in this section have exceeded rather grossly the mathematical results that we have obtained. Our dynamics contains, as does classical dynamics, an undetermined force function. Since determining the precise form of this function will be in all probability very difficult, and therefore require considerable time, it seemed appropriate to speculate on further developments even while having an eye on the fact that these very developments might necessitate mathematical modifications. However, all this seems the only way to suggest the possibility that this new formulation can explicate quantum phenomena and resolve conflicts between them and classical physics.