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EPR-B correlations: quantum mechanics, or just geometry?

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Abstract
Based on the observation that polarization phenomena of EM waves are geometric rather than quantum mechanical in nature, it is argued that experiments involving 'entangled polarization' do not address the issues brought up by EPR. A fully classical explanation is offered for a recent experiment of this type, and a fully classical (local and realistic) photoelectron-by-photoelectron simulation is described of ordinary two-fold experiments thought to prove Bell's 'theorem'.

Keywords: polarization entanglement, Bell's theorem, GHZ correlations, quantum mechanics, non-locality

(Some figures in this article are in colour only in the electronic version)

1. Background facts

Any object can be viewed from various angles. When such changes in view-point are restricted to the surface of a sphere centred on the object of interest, then these changes in appearance can be systematized in terms of the change of viewing angle as represented by a vector from the centre of the sphere to its surface. In mathematics this structure is captured in terms of the group $SO(3)$. The generators of this group do not commute.

This structure is not limited to describing just changes in view-point, of course. It would also describe, for example, changes of a **k**-vector specifying a plane wave. However, in so far as electromagnetic plane waves are comprised of transverse oscillations, there are for each vector direction on the sphere two transverse directions, i.e., there are also two 'polarizations'. As with the wavevector, these polarization subdirections can also be viewed from various angles. If the wavevector is fixed, then the change in view-point is restricted to rotations, not on the sphere, but on the circle. In mathematics this substructure is rendered in terms of the group $SO(2)$. Its generators, unlike those for rotations on the sphere, do commute.

Now however, when the wavevector is allowed to wander over the sphere, it drags along with it the two orthogonal 'polarization' vectors, so that for different wavevector directions, they too inherit the noncommutativity of the wavevector. It is easy to see that this noncommutativity is not due to anything except the rotation on the sphere, *as passed along*. In mathematics all this structure too is codified, in terms of the group $SU(2)$. From these considerations it is absolutely clear why the groups $SO(3)$ and $SU(2)$ are homomorphic: they encode the very same structure as inherent in rotation on the sphere, once for a vector and once for its two orthogonal partners [1].

With respect to polarization, as physics, all this was worked out long ago by Stokes before Planck, Pauli, etc were even born. This is the reason that the so-called 'Pauli spin matrices', which in the $SU(2)$ structure turn out to have the properties (under rotation on the sphere) of basis elements for $SU(2)$, were in use as 'Stoke's operators' decades before spin was ever conceived. Their noncommutativity is a purely geometrical fact, whether in connection with polarization or spin.

All of the above is beyond any dispute; it is in no way wild, iconoclastic speculation by this writer. Nevertheless, it has serious iconoclastic consequences for the *interpretation* of some contemporary physics.

2. Main issues

Einstein, Podolsky and Rosen (EPR) tried to reveal the incompleteness of Quantum Mechanics (QM) by employing a gedanken experiment in phase space. According to a fundamental premise of QM, the operators spanning phase space, $\hat{x}$ and $\hat{p}$, do not commute because of Heisenberg uncertainty. Since the EPR experiment as proposed was not realizable, Bohm suggested an alternative employing spin. He did so, without providing extensive justification, apparently solely on the grounds that because the Pauli spin operators do not commute, as the operators spanning quantized phase space do not commute, Pauli spin operators, too, must be noncommuting for the same basic reason. From the considerations delineated above, however, it can be seen that this is not correct. Noncommutativity of all operators having to do with spin, the homeomorphic partner of polarization, does not arise from QM, but just from geometry. A homeomorphism assures that both of these groups have the very same character, either quantum or not. As one, for polarization, is obviously non-quantum, both must be, no matter how first conceived. This is so even while the existence of the vector quantity, spin, is a quantum phenomenon. No matter how it originates, its vector nature dictates is geometric transformation properties.

What this means as a practical matter is that all experiments exploiting polarization 'entanglement' do not plumb QM. They do not address the issues raised by EPR as formulated in their gedanken experiment. Thus, experiments to date credited with establishing empirical verification of the existence of non-locality in the natural world, are being misinterpreted. Bell's so-called 'theorem' has not been verified empirically; all data taken in efforts to test it can be explained using just classical physics (Euclidian geometry, really), for the very reason that all polarization phenomena can be so explained.

Indeed, elsewhere this writer has shown calculations based on the classical formula for correlation of arbitrarily high order that accurately yield results encompassing all taken data [2]. In that this formula is the same one that elucidated the Hanbury-Brown and Twiss experiment, which in its day was widely misunderstood as a violation of fundamental concepts, the current misunderstanding of the EPR/Bell tests has an historical precedent.

Herein, this technique is applied to a recent experiment credited with exhibiting 'entanglement purification' [3]. I aim to show that whatever nomenclature is used for the observed phenomena, they can be accounted for, excluding the vagaries of laboratory reality (noise), using non-quantum, classical reasoning. In addition, I shall exhibit a photoelectron-by-photoelectron simulation of the simplest version of an EPR-B experiment. The resulting sequences are precisely those that Bell tells us cannot be generated without somehow calling on non-locality. The 'hidden' variables in this simulation turn out to be random, but, after-the-fact, their values can be displayed. The protocol used to do the simulation can be broken into parts and run on three separate machines in such a way as to exclude beyond dispute non-locality from the logical flow of the setup. This simulation constitutes a direct, unambiguous counterexample to Bell's 'theorem', thus, arguably, proving it to be false.

Bell in fact never formulated his arguments as a 'theorem'. Nevertheless, they are built on hypotheses, some covert. His conclusion is clearly stated, however:

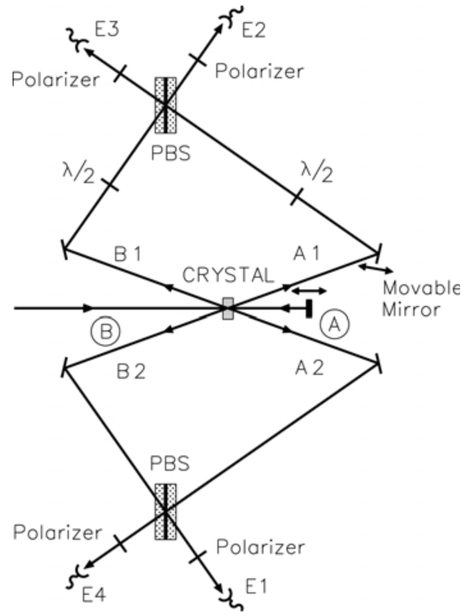

Figure 1. A schematic representation of a four-fold experiment. Differences in the visibility of the intensity of four-fold coincidences between selected polarizer setting regimes when the PBSs are absent or present is said to show 'entanglement purification'. However, all coincidence phenomena observed with this setup are fully explained by non-quantum theory.

> In a theory in which parameters are added to QM to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the settings of one measuring device can influence the readings of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant [4, p 14].

As the simulation described below shows by counterexample, this statement is misguided; it must rest on an inappropriate hypothesis, the nature of which, however, is an independent issue analysed elsewhere [2, 5, 6].

3. GHZ correlations

High order, multiple field point correlations are not always intuitively understandable. Even though in effect they are nothing other than a manifestation of a generalized Malus' law, their complexity encumbers insight. This difficulty led to acrimonious debate concerning the Hanbury-Brown and Twiss experiment, and it continues to challenge contemporary researchers.

An example is provided by a recent experiment said to exhibit 'entanglement purification'. This experiment, which as a laboratory exercise appears to be a *tour de force*, is schematically represented in figure 1.

A stimulus signal is sent through a crystal in which, by action of parametric down conversion, a pair of pulses (in the vernacular: 'photons') with anti-correlated polarization is generated. This same stimulus signal is then reflected by a mirror so as to repass through the crystal in the opposite direction, where it again generates a second pair of anti-correlated pulses. As shown on the diagram, the pulses are

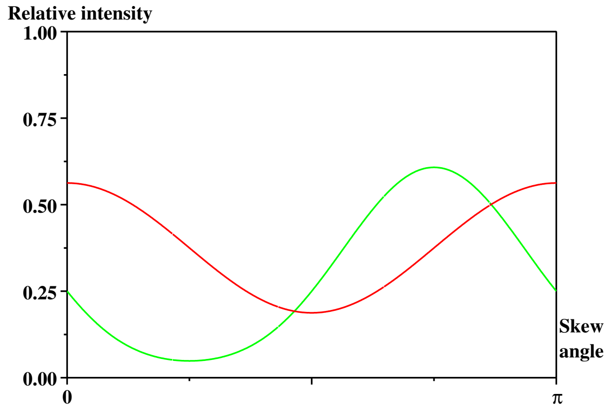

Figure 2. Results from a non-quantum model of a GHZ setup. The upper curve on the left gives the four-fold coincidence intensity as a function of skew for one of the polarizers on a photodetector when the PBSs are absent. The other curve pertains when the PBSs are present for a particular set of polarizer settings. That its visibility is greater is said to indicate that the procedure 'purified' the entanglement. Both curves were normalized by their maximum when the $1/2$-waveplates are absent. On this chart, the phase difference is an artefact; actual data was reported only at the maxima and minima.

propagated along equal length paths such that each pulse meets a partner pulse from the other pair in a polarizing beam splitter (PBS, a device which transmits horizontally polarized and reflects vertically polarized signals), after which the outputs of the two PBSs are fed into photodetectors, each with an adjustable polarizing filter. The experiment consists of studying the intensity of four-fold coincident generation of photoelectrons as a function of the four filter settings. Some polarizer settings regimes result in no coincidences and certain others exhibit peculiar variations when one member from each pair is subject to a degree of mismatch by the insertion of half-waveplates (in current jargon: generate a mixed state with less than full 'entanglement').

Detailed analysis of the angular dependence of such four-fold coincidences when, say, one of the filters is rotated, is tedious. Nevertheless, the essential features of the core phenomena can be seen in figure 2.

The curves were calculated using the *classical* formula for high order correlations in those settings regimes reported for the experiment. The uppermost curve on the left is the intensity variation as a function of a filter axis angle when three others are fixed for the case when the PBSs are removed; i.e., the pre-purified state. The lower curve on the left shows the intensity variation with the PBSs present, again as a function of angular variation of one filter when the starting point is a particular set of filter settings used in the experiment. Because the latter curve shows a higher visibility (its minimum value is about 60% less than that for the other curve), it is said to have been 'purified' by employing the protocol involving PBSs. What this calculation shows is that, whatever terminology is preferred, the essential elements of these phenomena are to be found in classical physics. This point is, of course, entirely consistent with that raised above, namely, that polarization phenomena are geometrical, not quantum mechanical, in nature.

Experiments, such as this one, cannot, therefore, address any issue raised by EPR.

4. An EPR-B simulation

Elsewhere I have given calculations for the correlations seen in generic experiments which are considered to demonstrate Bell's result [2]. For these calculations use is made of the classical definition of correlation among electromagnetic fields; no structure from Quantum Mechanics is exploited in any way. They are, therefore, fully 'local' and 'realistic', and as such serve as counterexamples to Bell's statement. Nevertheless, these calculations are somewhat like a 'black-box': what goes in and comes out is clear, but what happens inside is mysterious.

Below I will describe a photoelectron-by-photoelectron simulation, ultimately based on the above calculation, of an EPR experiment. While this simulation adds nothing to the rigour of the analysis, it does foster much deeper insight into exactly how EPR correlations arise.

For the simulation of a simple Clauser-Aspect type setup with two arms, five elements are considered: the signal source, $\boldsymbol{S}$, two experimenter inputs, i.e., the polarizer settings, $\boldsymbol{A}$ and $\boldsymbol{B}$, and finally two analyser stations, one on each arm, $\boldsymbol{X}$ and $\boldsymbol{Y}$.

$\boldsymbol{S}$ is implemented as an equal, flat, random selection of one of two possible signal pairs. One is comprised of a vertically polarized pulse to the left, say, and a horizontally polarized pulse to the right; the second signal pair has exchanged polarizations.

In this local protocol, $\boldsymbol{A}$ and $\boldsymbol{B}$ are implemented as simple random number generators with a flat distribution between 1 and 0, such that if a random number is $\leqslant 0.5$, one polarizer setting is used, otherwise the other setting is used, etc.

$\boldsymbol{X}$ and $\boldsymbol{Y}$ are implemented as models of polarizer filters for which the axis of the left (right) one is at an angle $\theta_{l\,(r)}$; each filter feeds a photodetector. These photodetectors are taken to adhere to Malus' law from classical electrodynamics; that is, they produce photoelectron streams for which the intensity is proportional to the incoming field intensity, and the number of photoelectrons is a random variable, usually zero or one, described by a Poisson process. In so far as these photodetectors are all independent, each Poisson process is event-wise uncorrelated with respect to all others. It is taken that the energy in each pulse from the source is such that when the pulse polarization axis is aligned with the filter axis, one photoelectron is elicited. As the angle between these two axes increases, then, in accord with Malus' law, the probability of the elicitation of a photoelectron is diminished. As a matter of detail, the photoelectron generation process is modelled by comparing a random number with the field such that if the random number is less than the field intensity, it is taken that a photoelectron was generated; if greater than the incoming intensity, none was generated. In other words, N_{hl} is increased by 1 when the random number is $\leqslant \cos^2(\theta_l)$; and N_{vl} is increased by 1 when another random number is $\leqslant \sin^2(\theta_l)$, etc.

To implement the effect of $\boldsymbol{A}$ and $\boldsymbol{B}$ for each run, i.e., each pulse pair individually, the random value of each is taken to specify which of the two orientation angles is to be implemented by the filters. This is done to explicitly cater to the notion seen often in the literature that the 'free will' of the experimenter to choose a measurement setting after the pair is emitted by the source is considered a vital element of the experiment[1]. Whether true of not, this slight elaboration is easily included in the simulation.

The final step of the simulation is simply to register the 'creation' of photoelectrons.

All information flow is from $\boldsymbol{S}$, $\boldsymbol{A}$, and $\boldsymbol{B}$ to $\boldsymbol{X}$ and $\boldsymbol{Y}$. There is no information flow between $\boldsymbol{X}$ and $\boldsymbol{Y}$, so that there is no non-local interaction. At this point the natural inclination concerning data analysis would be to count the total of the number of times the outputs are equal, i.e., $N_{ab}^{=}$, for the polarizer settings a and b, as well as the number of times they are unequal, $N_{ab}^{\neq}$, and the total number of trials, N_{ab}, and then with these numbers, compute the correlation, κ, for each setting pair, ab, using

$$\kappa_{ab} = \frac{N_{ab}^{=} - N_{ab}^{\neq}}{N_{ab}}. \tag{1}$$

These correlations, in turn, would then be used to compute the CHSH contrast

$$S = k_{12} + k_{11} + k_{21} - k_{22}, \tag{2}$$

which is to be tested for violation of Bell's limit, $|S| \leqslant 2$, as is well known.

This procedure, however, will lead to a dead-end, i.e., to results not conforming to calculations using QM, nor those observed in experiments. The reason for this problem is analysed in [6]; it can be summarized as follows: there is a widespread fundamental misconstruction of exactly what is calculated with QM and measured in experiments; it is not as tacitly taken, the correlations of events of pair-wise correlations, but rather the density of pairs per unit time and angular settings manifested as a photocurrent. The analysis procedure described above concerns the event correlations, and does not lead to a violation of Bell inequalities; it is a mathematical truth, but not an explanation of EPR-B phenomena.

In this simulation, therefore, the data analysis proceeds differently. To model the current density, which is what is in fact always measured in experiments, it is taken that the relative intensity between sides is given purely by geometrical considerations according to Malus' law by $\cos^2(\theta_\mathrm{r} - \theta_\mathrm{l})$ for like events, and $\sin^2(\theta_\mathrm{r} - \theta_\mathrm{l})$ for unlike events. What is correlated then is current densities, not individual photoelectron pairs *per se*. This is essentially equivalent to requiring internal self-consistency. That is, if the axis of one filter is parallel to the axis of the source, $\theta_\mathrm{r} = 0$, say, then it is quite obvious that the relative intensity as measured by photodetectors on the outputs of the other filter must follow Malus' law by virtue of the photocurrent generation mechanism. Further, since this geometric fact must be independent of the choice of coordinate system, it follows that a transformation of coordinates can be effected using

$$\begin{aligned}\cos(\theta_\mathrm{r} - \theta_\mathrm{l}) &= \cos(\theta_\mathrm{r})\cos(\theta_\mathrm{l}) + \sin(\theta_\mathrm{r})\sin(\theta_\mathrm{l}),\\ \sin(\theta_\mathrm{r} - \theta_\mathrm{l}) &= \sin(\theta_\mathrm{r})\cos(\theta_\mathrm{l}) - \cos(\theta_\mathrm{r})\sin(\theta_\mathrm{l}),\end{aligned} \tag{3}$$

which are just trigonometric identities. Note that although (3) are not in general factorable into the form $f(\theta_\mathrm{l})g(\theta_\mathrm{r})$ (sometimes said to be a criterion for 'non-locality'), all the information required for each factor is available on-the-spot from both sides independently, i.e., no violation of locality occurs. Then use is made of the fact that the individual terms on the right-hand side of (3) are, by virtue of photodetector physics, proportional to the square root of the number of counts in the regime, e.g.,

$$\begin{aligned}\cos(\theta_\mathrm{l}) &= \lim_{N\to\infty} \sqrt{N_\mathrm{hl}/N},\\ \sin(\theta_\mathrm{l}) &= \lim_{N\to\infty} \sqrt{N - N_\mathrm{vl}/N},\end{aligned} \tag{4}$$

where N is the total number of signals pairs generated by the source in each settings regime, which is approximately $1/8$ of the total number of pairs[2]. The exact form of (4) depends on the geometry of the detection setup and the type of event, i.e., hit or non-hit, sought by the coincidence count logic needed by the particular Bell inequality being simulated. In other words, the relative frequency of coincidences is determined as a function of the intensity of the photocurrent, or lack thereof, on-the-spot. No communication between right and left sides is involved; what correlation there is is there on account of the equality of the amplitudes (and therefore *intensities*) at the source of the signals making up the singlet state. *The* crucial feature here is that the correlations simulated are of the current intensities as a function of angle and time-window, not correlations of individual pair-events *per se*. Both the QM calculation and the experiments concern the former, whereas the latter, properly analysed, tautologically satisfy a Bell inequality; see [6].

Of course, in the simulation, an individual signal is a random selection between, instead of a superposition of, two options. In doing a simulation, provision also must be made to resolve the sign ambiguity; doing so, however, also does not violate locality as the required information is all available on-the-spot. In sum, instead of (1), I use

$$\kappa_\mathrm{rl}^{*} = \frac{2\cos^2(\theta_\mathrm{r} - \theta_\mathrm{l}) - 2\sin^2(\theta_\mathrm{r} - \theta_\mathrm{l})}{2\cos^2(\theta_\mathrm{r} - \theta_\mathrm{l}) + 2\sin^2(\theta_\mathrm{r} - \theta_\mathrm{l})}, \tag{5}$$

as required by Malus' law, but with the terms expressed using (3) and (4) to get results for the accumulation of 'photoelectrons'.

A somewhat more intuitive view of this simulation is gained by considering the flow in reverse. The final step, of course, is to compute the CHSH contrast, (2). The individual terms in (2) are now given by (5), which seem to require information from both sides, thereby introducing non-locality. This is not so, however, because they can be expanded using (3) in terms of the sum of products of terms each depending on information from one side only. These latter terms now are determined using (4) which are fully specified by a *delayed* signal from the source, a local polarizer-filter setting and local noise. Moreover, source signals are not superpositions of exclusive outcomes, but a random selection of viable outcomes

[1] This tactic originally was intended to foreclose the so-called 'conspiracy loophole' for which it is imagined that via communication between the pulses after emission, a skewed and misleading distribution of signals might be insinuated. Selecting the filter angles after the pulses are underway such that sufficient time for them to collude by light signals is not available would preclude this scenario. Nowadays, this matter is sometimes conflated with other considerations, making its discussion turbid [5].

[2] In laboratory experiments, for lack of an observable distinction between failed detection and non-emission, N is not measurable. For a simulation, obviously, this difficulty does not arise.

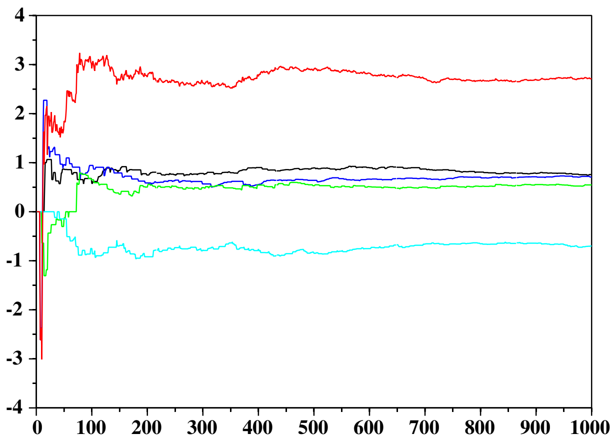

Figure 3. Results of a non-quantum, detailed simulation of EPR-B correlations. The upper curve shows the CHSH contrast as a function of the number of trials. The lower curves give the individual correlations. That the CHSH contrast is a sequence of values computed without recourse to non-locality constitutes a counterexample to Bell's 'theorem'.

comprising pairs correlated by common cause. Only after-the-fact calculation of the correlations blends information from both sides; non-local interaction or communication has no role in any physical process, just the analysis.

An example of the results from the simulation is presented in figure 3. The top curve is the CHSH contrast; the lower four curves are individual correlations for the four combinations of filter settings. The statistics stabilize after circa 700 trials and exhibit clear violation of Bell's limit of 2, and then by virtually exactly the amount calculated for singlet states and observing angles $\theta_l = 0, \pi/4$ and $\theta_r = \pm\pi/8$ using Bell inequalities: $2\sqrt{2}$. That these statistics can be found in a sequence of individual events, each of which is calculated without recourse to non-locality, constitutes a direct unambiguous counterexample to Bell-type 'theorems'. This simulation is not just an abstract academic exercise; laboratory confirmation can be found in [7].

5. Final comments

A few proponents of non-locality, Bell's analysis and the like, have criticized the arguments presented in [2] for surreptitiously containing quantum mechanical aspects. This charge can be rebutted on the following grounds.

The key concept used in all the calculations and simulations is the definition of high level correlations; i.e., for single-time, multiple-location, second-order cross correlation, i.e.:

$$P(r_1, r_2, \ldots, r_N) = \frac{\langle \prod_{n=1}^{N} E^*(r_n, t) \prod_{n=N}^{1} E(r_n, t)\rangle}{\prod_{n=1}^{N} \langle E_n^* E_n \rangle}. \qquad (6)$$

As can be seen in any text on Quantum Optics, this formula has two renditions: quantum mechanical and classical. In the genuinely quantum mechanical rendition, the E_j^* are operators, e.g., annihilation operators, while the E_j are creation operators. Since such operators do not commute (because they encode Heisenberg uncertainty) it is absolutely essential that the ordering of the products in the numerator of (6) be taken into account in order to obtain valid results. In the cases at hand, however, not only is there no noncommutativity, because there is no Quantum Mechanics involved ever for polarization, but there is also no noncommutativity for geometric reasons—all the variation of the polarizer filters settings on the photodetectors leaves the k-vectors for impinging signals fixed. Thus, the E_j are just real functions for which ordering is utterly inconsequential; i.e., for the calculations described in [2] and used to obtain the curves in figure 2, the purely classical variant of (6) was fully sufficient.

Complicating this particular matter in addition to consideration of commutativity, however, is the fact that the notion of 'quantum correlation' is only formally defined. In principle, correlation is always a mathematical concept with little room for modification for an application. In QM, the special nature of *some* correlations is just that they are vested with ontic ambiguity by fiat, i.e., by von Neumann's measurement theory which demands that correlated objects be superpositions of mutually exclusive possible outcomes, instead of simply states that are unknown but fully determined in fact by a common cause. Again, these complications, while vital to the overarching issues brought up by EPR, do not provide fatal obstacles to the construction of a local-realistic simulation. This whole matter is discussed more fully in [5, 6].

Finally, note that the conclusions herein do not support any effort to debunk or denigrate Quantum Mechanics, rather just to precisely define its domain of relevance. Although polarization entanglement, being a geometrical phenomenon, cannot be used to plumb the nature of quantum mysteries, experiments involving beam splitters, e.g., [8, 9] and those for which the statistics within a polarization mode involve more that a single photoelectron excitation at detection, e.g., [8], appear to exhibit some quantum mechanical aspects. Of course, $\hat{x}$ and $\hat{p}$ and the 'squeeze' variables, phase and amplitude, explicitly suffer Heisenberg uncertainty, and incontestably always exhibit a quantum character.

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