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DICHOTOMIC FUNCTIONS AND BELL'S THEOREMS

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ABSTRACT. It is shown that correlations of dichotomic functions can not conform to results from Quantum Mechanics. Also, it is seen that the assumptions attendant to optical tests of Bell's Inequalities actually are consistent with classical physics so that in conclusion, Bell's Theorems do not preclude hidden variable interpretations of Quantum Mechanics.

The analysis attending Bell's Theorems imposes on hidden variable formulations of Quantum Mechanics (QM) the requirement that they be able to correlate dichotomic functions (representing spin) so as to duplicate the result from QM:

$$P_{\mathbf{a},\mathbf{b}} = \int \rho(\lambda) A(\mathbf{a},\lambda) B(\mathbf{b},\lambda) d\lambda = -\cos(\mathbf{a},\mathbf{b}), \tag{1}$$

where content and notation are taken from Bell.[1]

Note, however, that, *inter alia*, only the right side of Eq. (1) is a harmonic function. This means that the equation itself is an absurdity. To see this, consider the derivative of the QM expectation expressed as a correlation of dichotomic functions, $D_{\mathbf{a},\mathbf{b}}$, which change sign at the points x_j, (δ is the Dirac delta function):

$$\begin{aligned} -\frac{\partial \cos(\tau)}{\partial \tau} &\propto \frac{\partial}{\partial \tau}\frac{1}{T}\int_0^T D_{\mathbf{a}}(x-\tau) D_{\mathbf{b}}(x) dx, \\ \sin(\tau) &\propto \frac{1}{T}\int_0^T \pm\delta(x_n) D_{\mathbf{b}}(x) dx, \\ &\propto \frac{1}{T}\sum_n^N (\pm) D_{\mathbf{b}}(x_n) = \frac{\text{integer}}{T}. \end{aligned} \tag{2}$$

(The sign of individual terms in the above series depends on the specific form of $D_{\mathbf{a},\mathbf{b}}$, which, for present purposes, is not required.) A further differentiation by τ yields:

$$\cos(\tau) \equiv 0, \tag{3}$$

a false equation.

Nothing was in play here but the dichotomic nature of the functions D; it is clear that such functions simply can not be correlated to yield harmonic functions.[2] Obviously, if dichotomic functions cannot yield the QM result, then QM is doing something other than correlating them. If this be the case, then a hidden variable alternative to QM need not be required to so correlate them and the logic of Bell's Theorem is broken.

Likewise, the famous result from Greenburger et al. [3] showing that the correlation of three or more dichotomic functions representing three or more particles is never; i. e., even for perfect correlations, compatible with the harmonic functions given by QM, is attributable to the same structural cause. This is most easily seen by expressing multiple correlations (for three particles, say) in terms of dichotomic sequences $A, B, C\ldots$ (i. e., data points) as follows

$$-\cos(\tau+\theta) = \frac{1}{NM}\sum_{j,k}^{N,M} A_{j,k}B_{j,k}(\tau)C_{j,k}(\theta), \tag{4}$$

where (ignoring for the moment obvious analytic absurdities) it is clear that even for $\tau+\theta = 0$ or π; i. e., for perfect (anti)correlations, the equation can not hold because odd (even) multiples of -1 do not (do) cancel. This exposes an internal contradiction in no need of empirical verification. (Degenerate cases occurring for even multiples of dichotomic functions do not invalidate the principle. Moreover, for those with strong intuition, it is clear that "*n*-chotomic" functions of any rank will result in analytic problems.)

The question arises: if QM is not doing the mathematically impossible, what is it doing? For the case of polarized "photons," used virtually exclusively for testing Bell's Inequalities, QM is in fact giving a classical result. The analysis of optical analogues of the EPR event artificially associates an unpolarized with an anticorrelated state by *defining* [4] the "coefficient of correlation" to be

$$E(\mathbf{a},\mathbf{b}) \equiv P_{(++)} + P_{(--)} - P_{(+-)} - P_{(-+)}, \tag{5}$$

where the P's are the "quantum mechanical predictions" for detection coincidences; i.e.:

$$\begin{aligned} P_{(++)} = P_{(--)} &= \frac{1}{2}\cos^2(\mathbf{a},\mathbf{b}), \\ P_{(-+)} = P_{(+-)} &= \frac{1}{2}\sin^2(\mathbf{a},\mathbf{b}), \end{aligned} \tag{6}$$

so that Eq. (5) becomes:

$$E_{\text{optical}} = -\cos 2(\mathbf{a},\mathbf{b}). \tag{6}$$

Note, however, that the P's just give the intensity of polarized light as measured with respect to an axis different from the axis of polarization according to Malus' Law—a non quantum rule. Of course, the P's must be suitably interpreted to correspond ultimately to "click" probabilities in experiments performed at minimal intensity, where the statistics within each polarization state may require more than traditional physics.

This is in total accord with the fact that creation/annihilation operators for photons of different states of polarization commute. As is well known, QM differs from classical physics where and only where conjugate operators do not commute. (This observation conforms with work reported elsewhere in these proceedings [5] wherein supposed QM polarization (or spin) correlations were obtained from a model involving rubber bands or "random gun" and no Planck's constant!) Thus, tests to plumb the ontological implications of QM; e. g., nonlocality, can not be made on the basis of optical experiments involving polarization states. As the creation/annihilation operators for photons of different colors also commute, these arguments extend directly to experiments exploiting parametric down conversion and so on.

QM multiparticle correlations, such as Eq. (4) or its *n*-particle generalizations, as noted elsewhere [6], make contact with the Bell-Kochen-Specker Theorem. Although analysis of such equations, restricted as it is to considering perfect (anti-)correlations, is less general than the full Theorem, it is more transparent because it is just tracking factors of -1. In addition, it is here very clear that dependence on variables in addition to τ and θ; e. g., by investing A, B, C with nonlocality, can not circumvent inconsistency which results only from their dichotomic values. Again, however, Eq. (4) can be rendered rigorously correct and physically sensible in any n-particle generalization if dichotomic sequences A,B,C etc. are replaced by cosines and sums converted to integrals; i.e., by considering classical polarization correlations.

In conclusion, it is seen that a demand that hidden variable theories should correlate dichotomic functions to get a harmonic result is ill founded. When this stipulation is relaxed, then the extraction of Bell's inequalities does not go through and therefore hidden variable theories duplicating QM polarization correlations; e. g., [7], are not precluded in principle. Moreover, because the mathematics describing spin correlations; i. e., their interstate structure, is isomorphic to that for classical polarization, the underlying phenomena also must be classical in essence.

In other words, it is clear that Bell's Theorems establish beyond dispute that self consistent, intuitively clear hidden variable theories can not duplicate all of QM. Indeed such alternatives will be (and should be) unable to replicate those very aspects of QM that have been the source of confusion and contest from the beginning, namely the measurement projection hypothesis and an under restrictive identification of vectors in relevant Hilbert spaces as physically realizable states; e. g., pure and "cat" states and especially "photons."[8] These two features of orthodox QM lead not only to the logical contradictions exposed by Bell's Theorems but appear essentially untestable in all but certain "*n*-chotomic" circumstances where a classical explanation fits anyway. These observations are not here unique. Many roads lead away from "Copenhagen," another such, for example, ends in LA.[9]

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