

The improbability of non-locality

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Abstract: Both philosophical and mathematical considerations argue against the existence of ‘non-locality,’ or faster than light interaction. Nevertheless, it is widely accepted that non-locality has been established by Bell’s Theorems as an intrinsic feature of the natural world. Herein it is shown, to the contrary, that the analysis supporting Bell’s Theorems contains implicit hypotheses that are fully disputable and unacceptable on the basis of logic, probability theory and basic physics. Finally, the consequences of the rejection of non-locality for the interpretation of Quantum Mechanics are discussed.

Résumé: Les considérations philosophiques aussi bien que mathématiques semblent contredire l’existence de la non-localité, l’interaction plus rapide que la vitesse lumière. Néanmoins, il est généralement reconnue que la non-localité est démontrée par le Théorème de Bell comme une qualité intrinsèque du monde naturel. Nous essaierons de montrer ici, au contraire, que l’analyse sur laquelle s’appuie le Théorème de Bell comporte des hypothèses implicites qui sont entièrement contestables et inacceptables en termes de logique, de théorie de la probabilité et de physique élémentaire. Enfin nous parlerons des conséquences du rejet de la non-localité sur l’interprétation de Mécanique Quantique.

Key words: Non-locality, Bell’s Theorem, Quantum Mechanics, Entanglement, Pilot Wave, Bohmian Mechanics

I. INTRODUCTION

Non-locality—faster than light interaction—has been widely accepted as an intrinsic property of the natural world as described by Quantum Mechanics (QM). The source of this conviction, “Bell’s Theorem,” was found in re-analysis of the Einstein, Podolsky and Rosen (EPR) paper from 1935 in which it was argued that the wave function of QM is not complete, i.e., that there should exist a deeper theory giving a more extensive specification of atomic scale phenomena. EPR’s point seems nearly incontrovertible given that wave functions yield ‘probability densities,’ which naturally implies incomplete knowledge. However, for various reasons ultimately having to do with measurement and bandwidth considerations for matter waves (i.e., the uncertainty principle), Bohr, Heisenberg and most others have rejected EPR’s notion. In pursuit of what might be found underlying this dispute, Bell deduced his ‘theorem,’ showing, seemingly, that a deeper theory must incorporate non-local interaction. Nevertheless, no use is made of non-locality in any area of Physics besides those based on Bell’s analysis of the EPR experiment. Herein, this bizarre is clarified by showing that Bell’s analysis employs a misdirected codification of locality. When corrected, the consequence is that non-locality has no place in Physics, and QM may indeed accept a local, realistic completion.

II. BELL’S ANALYSIS.

EPR proposed the following ‘thought experiment.’ Consider the disintegration of a stationary particle into twin daughters, racing off in opposite directions. QM asserts that it is impossible to simultaneously measure with arbitrary precision the momentary position and momentum of any particle, specifically here to include either daughter in an EPR experiment. Based on this principle it is taken that ontically a particle does not in fact even have these qualities simultaneously. To this, EPR rebutted (*in our interpreted version*) by pointing out, that after measuring the location of one daughter and the momentum of the other to arbitrary precision, and then calling on symmetry, one can specify to the same precision the unmeasured first twin’s momentum and the second’s location. On the intuitively reasonable grounds that what can be specified exactly should enjoy status as *ontic*, EPR suggested that missing structure could well be found in a deeper theory.[1]

Historically, this dispute raged with passion and vigor. Von Neumann contributed a theorem to the debate showing ostensibly that such a deeper theory must be inconsistent with the empirically verified principles of QM.[2] It was quickly found, however, (although not so quickly widely recognized) that von Neumann’s theorem contained inappropriate hypothetical presuppositions; that it was actually irrelevant.[3] John Bell subsequently re-found the lacuna in von Neumann’s

arguments and proceeded to reformulate the analysis with the intention to bring the issue to an empirically testable proposition.[4]

To this end he considered a variation of the EPR experiment, as amended by David Bohm, exploiting polarized photons. Certain atomic transitions generate ‘twin’ photons meeting the EPR suppositions in that if one is polarized parallel to an arbitrary direction but perpendicular to the line of flight, then with respect to the same direction, the other must be perpendicularly polarized. When the measuring polarizers, with settings a and b respectively, are not orthogonal to one another but make an angle θ between a and b , then for this phenomena QM gives a coincidence probability, $\rho_{(1,1)}(a, b)$ as a function of θ of observing both photons of the form: $(1/2)\sin^2(\theta)$. If now a deeper theory exists, in order to be consistent with QM and empirical truth, its formula for this coincidence rate must reduce to this same expression when its additional, deeper variables (“hidden” at the quantum level) are averaged out.

One of the main motivations for a search for a deeper theory is the hope that it will not be plagued by the well known weirdness in the interpretation of QM. In this spirit, Bell sought to enforce locality, a.k.a. ‘Einstein causality,’ namely that all interaction mediated by a force must propagate at speeds equal or lower to that of light. To encode this stipulation in the relevant formulas, Bell supposed that the QM correlation of EPR measurements, $P(a, b)$, in terms of the individual event detection probabilities of a deeper theory should be rendered as follows:

$$P(a, b) = \int A(a, \lambda)B(b, \lambda)\rho(\lambda)d\lambda \quad (1)$$

where $\rho(\lambda)$ represents some possible density over the variable set λ from the deeper, ‘hidden,’ theory. Bell described $A(a, \lambda)$ and $B(b, \lambda)$ as “the result of measuring $\sigma_1 \cdot a$ and $\sigma_2 \cdot b$ ” and $P(a, b)$ as the “expectation of the value of the product of $\sigma_1 \cdot a$ and $\sigma_2 \cdot b$,” i.e., their correlation. If these definitions are to be mutually consistent among themselves and with their QM correspondents, then we may write: $A(a, \lambda) = a\rho_A(a, \lambda)$ and $B(b, \lambda) = b\rho_B(b, \lambda)$ so that Bell’s joint probability would therefore necessarily have to be of the form:

$$\rho(a, b, \lambda) = \rho_A(a, \lambda)\rho_B(b, \lambda)\rho(\lambda), \quad (2)$$

where all ρ ’s are probability densities over the appropriate parameter spaces.

The crucial feature incorporated in Eq. (2) is Bell’s encoding of locality, namely, that the probability of a photon detection at station A, $\rho_A(a, \lambda)$, must be independent of the settings of the measuring apparatus at station B, i.e., that it does not depend on the variable b , and visa versa. Using Eq. (1) then, Bell and others derived inequalities for sets of correlations, P ’s, which are intended to be empirically testable; e.g.:

$$P(a, b) - P(a, b') + |P(a', b') + P(a', b)| \leq 2. \quad (3)$$

Virtually all experimental tests of these inequalities show violations, indicating that at least one of the assumptions that went into their derivation is not satisfied in reality.[5] This has lead Bell and many others to deduce, inasmuch as the only

explicitly insinuated supposition in the derivation of these inequalities was ‘locality,’ that locality is the culprit. From here it is a small leap to conclude that QM can accommodate this result only by being intrinsically non-local.

Such is the state of quantum dogma at present. But is it really so?

III. THE ENCODIFICATION OF LOCALITY.

For optical versions of EPR experiments, every detection necessarily comprises two elements: an anticorrelated pair of ‘photons’ and suitable detectors. A set of ontic variables, labeled λ , specify the state of the photons, i.e., they should specify ‘*what is.*’ In addition, there is a set of epistemic variables, a and b , specifying the measuring devices; i.e, they convey to us what can be ‘*known*’ about that which ‘*is.*’ A measurement is then a process to convert the inaccessible ontic specifications into available epistemic ‘information.’ This process is successful to the extent that whatever structure is encoded by unavailable but presumably existent expressions of the dependant ontic variables is faithfully converted into expressions of the accessible detector parameters. Here the property of interest is solely the polarization, thus λ gives the polarization of the photons, and a and b give the polarization angle of the detectors. “Measurement” then consists in setting values for a and b and then counting mutual clicks within a time window, δt , for a given time interval T , where $\delta t \ll T$; changing a or b and counting again for the same interval, and then repeating this until the ranges of a and b have been adequately sampled. Of course, for each individual event, λ has been chosen by ‘nature,’ an experimenter has only indirect influence; e.g., he can turn the source off, but not fix it exactly. In an EPR experiment, for each set of values of a and b , the detectors’ polarizers filter out a subset of photons for which some value of λ correlates with or overlaps with some unknowable degree of exactitude the value of the detector parameter, a , say. Here $\rho(a, b, \lambda)$ then, by definition, is the frequency of occurrence of *mutual* detections of photons when the detectors at stations A and B have the settings a and b respectively. In other words, it is the ratio of the number of occurrences in restricted sets of events over the number of all possible correlated pairs (often denoted the ‘universe of discourse’ or ‘sample space’ in probability theory) emitted in the time interval T . $\rho(a, b, \lambda)$ can be obtained in practice, obviously, only by multiple repetitions of the experiment as described above. However, after all is done, the λ dependence remains implicit and essentially unknown except insofar as it is revealed by explicit dependence on the detector parameters in their role as epistemic variables..

According to basic probability theory, joint probabilities, when expressed in terms of the probabilities of *individual* detections at stations A and B, are encoded according to Bayes’ formula:

$$\rho(a, b, \lambda) \equiv \rho_A(a|b, \lambda)\rho_B(b|\lambda)\rho(\lambda), \quad (4)$$

where $\rho(c|d)$ denotes a *conditional* probability; i.e., the probability of the occurrence of an event parameterized by c given that a *condition* specified by d is met.[6] In application to the

EPR experiment, for example, $\rho_A(a|b, \lambda)$ is the probability of a detection of a photon at station A when its polarizer is in the a direction, given that its companion photon is detected coincidentally at station B when its polarizer is in the b direction. The probability of a detection of the companion at station B , on the other hand, is taken to occur earlier and considered to be independent. The form of this equation is somewhat arbitrary; the interdependencies of ρ_A and ρ_B , in other words the correlations, can, with no untoward consequence, be built into either ρ_A or ρ_B . In addition, there is no implication of non-locality in such a condition; the correlation is imbued by a “common cause” in the past light cones of both entities. Eq. (4) is, it must be stressed, the lodestar for use of coincident probabilities; it provides the correct initial tact for determination of $\rho(a, b) = \int \rho(a, b, \lambda) d\lambda$ no matter what the details of the application.

With respect to Bell’s analysis, the critical point here is: the rhs of Eq. (4) reduces to the integrand of Eq. (1), if and only if:

$$\rho_A(a|b, \lambda) \equiv \rho_A(a|\lambda) \quad \forall b. \quad (5)$$

This expression, however, can be given meaning (generically, without regard to its implicit specific role in Bell’s analysis) in different ways.

1.) It can be interpreted to mean that the universe of discourse is a set for which there is no correlation between the *measurement* results taken at stations A and B for all settings b ; i.e., when they are *statistically independent* with respect to whatever b concerns (or, by symmetry, a). In other words, in the EPR experiment, although the measuring stations are setup to be sensitive to photons polarized in the directions labeled a and b respectively—that’s what the presence of a and b in Eq. (1) means—as a result of imposing this interpretation on Eq. (5), it is actually mandated that no *measurable correlations* exist. In turn, as the measuring stations are taken, nevertheless, to have polarizers with the orientations a and b , and therefore would detect correlated photons if present, this must imply that under these conditions no correlation could have been invested in the polarization of the photons in the first place. But this is explicitly contrary to the hypothesis of the EPR ‘thought experiment,’ and *a fortiori* contrary to all performed experiments where phenomena involving measurable correlated events are intentionally selected for investigation. Thus, in this case, Eq. (1), is valid only for uncorrelated events; Bell inequalities derived using it can pertain only to uncorrelated photon pairs, thereby rendering such inequalities meaningless for the issue at contest. In other words, Eq. (5) effectively respecified the universe of discourse so that it in fact excludes the very type of phenomena that is the object of interest.

Although discussions on exactly this point are not explicit, it is highly unlikely that this is what is generally intended; rather more likely is the following option:

2.) The implicit meta-logic of Bell’s analysis is based on the hypothesis (common to virtually all classical physics) that at the ultimate depth, everything is deterministic. It is at this level that a complete set of “hidden variables” λ would specify everything; there is then no longer need for probability densi-

ties, conditional or otherwise, with values other than zero and one. This might be called a counter-factual or ‘what-if’ world for which, it might be asked: does this world admit statements which cast a ‘shadow’ into the factual, laboratory world of incomplete knowledge in which experiments are done? Bell inequalities are intended to be exactly such ‘shadows’ cast by the procedure of extracting a marginal correlation. As a matter of logic, this tactic can be legitimate to the extent the statements made at the counter-factual level are correct and significant. Thus, the question becomes: is Eq. (5) correct at the ultimate depth? Do all *conditional* probabilities inexorably become factorable (in other words, independent probabilities or a functional equivalent), at this depth just *because* there is no instantaneous communication? There appears to be no justification or need for this additional assumption so long as the sample space is that of *pairs (anti)correlated intrinsically (presumably at inception)*; and therefore no foundation for the logic of Bell’s Theorem. That is, if Eq. (5) is even just functionally correct, then it is so on the basis of cosmological determinism without dependence on additional assumptions regarding ‘locality.’ In other words, if it is a valid statement because the correlation is encoded by λ , then the b in $\rho_A(a|b, \lambda)$ is superfluous. However, in that case there would be hidden variables, λ_b carrying the dependence formerly expressed by b so that in effect all that is achieved is a change in notation. On the other hand, if Eq. (5) is inappropriate—because, say, absolute determinism with respect to ‘ λ -qualities’ for as yet undiscovered reasons does not reign at a fundamental level—then, as argued above, Eq. (1) does not follow and, as it is easy to show, the extraction of a Bell inequality does not go through. Either way, the logic behind Eq. (5) is porous. In fact, Eq. (4) is insensitive to the source of correlation, it can be anything, even nonlocal, but it *need not be*, and that is the point.¹

IV. CRITIQUE OF BELL’S ANALYSIS

This actual significance of Bell inequalities for dichotomic sequences can be revealed by elementary analysis. Consider four dichotomic sequences comprised of ± 1 ’s with zero mean

¹ Subsequent to submitting this manuscript, it was brought to our attention that Jaynes deduced our main argument, namely that Bayes’ formula was incorrectly applied, no later than 1988.[20] We reasoned backwards from the physical situation to the underlying probability theory, whereas Jaynes first refined reasoning by inference based on Bayes’ formula, Eq. (4), and then applied his insight to various physical situations including Bell’s analysis of EPR experiments. In addition, we also learned that Perdijon independently obtained this result.[28] Recently, Hess and Phillip have criticized Bell’s analysis for failing to consider time dependant correlations.[29] We consider their basic point correct but point out that the only feature necessary to invalidate Bell’s analysis is that the correlations are invested in the EPR pairs at an event in the past light cones of both detection events, not that the correlations need vary in time. Others also have independently re-discovered Jaynes’ criticism[30; 31]. Finally, we direct attention to G. Adenier’s original and insightful analysis which shows that from within QM consistent interpretation of Bell inequalities demands that the rhs be 4, a value that is never violated in experiments.[32]

and length N : a, a', b and b' . Now compose the following two quantities $a_i b_i + a_i b'_i = a_i(b_i + b'_i)$ and $a'_i b_i - a'_i b'_i = a'_i(b_i - b'_i)$, sum them over i , divide by N , and take absolute values before adding together to get:

$$\begin{aligned} & \left| \frac{1}{N} \sum_i a_i b_i + \frac{1}{N} \sum_i a_i b'_i \right| + \left| \frac{1}{N} \sum_i a'_i b_i - \frac{1}{N} \sum_i a'_i b'_i \right| \\ & \leq \frac{1}{N} \sum_i |a_i| |b_i + b'_i| + \frac{1}{N} \sum_i |a'_i| |b_i - b'_i|. \end{aligned} \quad (6)$$

The r.h.s. equals 2, so this equation is in fact a Bell inequality equivalent to Eq. (3). This derivation demonstrates that this Bell inequality can be simply an arithmetic tautology. Thus, all dichotomic sequences comprised of ± 1 's, including those generated empirically, identically satisfy Bell inequalities.²

In addition, so called 'quantum;' i.e., 'non-local' correlations can be and have been both simulated from classical physics[7] and reproduced empirically with fully local, realistic and classical apparatus.[8] In the light of this demonstration and this experimental confirmation, there should be no residual of doubt that the association of 'non-locality' with Bell inequalities is an artifact of miscomprehension.

Thus, all such sets of *four* sequences, even those generated empirically, must by the facts of arithmetic, satisfy Bell inequalities. It is well known, however, that optical experiments to test Bell inequalities with respect to EPR experiments show an incontestable violation. This can be reconciled, however, with the following observations.

In the derivation of a Bell inequality the events considered comprise a single pair of photons. All experiments to test Bell inequalities measure, however, the *density* of pairs per unit time, T , given settings a and b . The later 'events' constitute the natural outcome of Malus' Law, which is what is implicitly called upon when reducing data obtained from these experiments. It yields a correlation that in fact is fundamentally incompatible with the dichotomic sequences originally considered by Bell.[9]

Moreover, in the extraction of Bell inequalities, a simplified form of the correlation of two random variables was used. The full form is:

$$Cor(A, B) = \frac{\langle |AB| \rangle - \langle A \rangle \langle B \rangle}{\sqrt{\langle A^2 \rangle \langle B^2 \rangle}}. \quad (7)$$

² This argument was inspired by Ref. [33] Note that it is impossible to generate *four* dichotomic correlated sequences using an EPR type setup. One gets five distinct sequences so that to meet the conditions of Eq. (6), an additional assumption is needed, namely that two of the sequences so generated, are identical. It is, however, always possible to satisfy Eq. (6) with any four arbitrary sequences of ± 1 's with equal length and zero mean [After-the-fact note: Stipulating zero mean is an error; however, as this is not used in fact, the argument is unaffected.], however generated. This supports our contention that Bell's Inequalities are devoid of meaning with respect to the question of the existence of non-locality. Moreover, this complication does not invalidate our point below that the QM correlation is in fact just Malus' Law which pertains to the *density* of outcomes per detector settings (continuous variables), not to the dichotomic values of the outcome.

where $\langle \cdot \rangle$ indicates an ensemble average. For symmetric dichotomic random variables having the values ± 1 ; i.e., those which Bell consciously considered, $\langle A \rangle = \langle B \rangle = 0$ and $\langle A^2 \rangle = \langle B^2 \rangle = 1$, which simplifies Eq. (7) to the expression, Eq. (1), used in Bell's analysis. When the full form of the Correlation is taken into account, however, the rhs of Eq. (3) becomes:

$$2 + \frac{2 \langle A \rangle \langle B \rangle}{\sqrt{\langle A^2 \rangle \langle B^2 \rangle}}, \quad (8)$$

which, as can be easily shown for continuous random variables representing electric field intensities responsible for photoelectrons, exceeds the value, $2\sqrt{2}$, observed in optical experiments.

In sum, Bell inequalities have no relation to locality. At best they are arithmetic tautologies pertaining only to uncorrelated sequences, with no relevance to EPR correlations; as such, they are meaningless. To the extent that QM violates them, is the result of the fact that Eq. (7) (or the actual, continuous nature of the appropriate random variables) has been ignored.

V. FLAWS OF CALCULATION

The above analysis implies that there must be a fundamental flaw in the extraction of Bell inequalities which has gone unnoticed. This is indeed true; it has to do with the nature of deterministic probability densities. In the deterministic limit, all probability densities become Dirac delta functions with a peak at that value of λ which labels an event. At the deterministic limit, each event has a unique value of λ at which it is non-zero. This circumstance works its way into the derivation of Bell inequalities as follows:

The derivation of a Bell Inequality starts from the fundamental *Ansatz*, Eq. (1):

$$P(a, b) = \int d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda), \quad (9)$$

where, per *explicit assumption*: A is not a function of b ; nor B of a . This is motivated on the grounds that a measurement at station A , if it respects 'locality,' can not depend on remote conditions, such as the settings of distant measuring devices. In addition, each, by definition, satisfies

$$|A| \leq 1, \quad |B| \leq 1. \quad (10)$$

Eq (9) expresses the fact that when the hidden variables are integrated out, the usual results from QM are recovered.

The extraction proceeds by considering the difference of two such correlations where the parameters of one measuring station differ:

$$\begin{aligned} P(a, b) - P(a, b') = \\ \int d\lambda \rho(\lambda) [A(a, \lambda) B(b, \lambda) - A(a, \lambda) B(b', \lambda)], \end{aligned} \quad (11)$$

to which zero is added in the form:

$$\begin{aligned} A(a, \lambda) B(b, \lambda) A(a', \lambda) B(b', \lambda) \\ - A(a, \lambda) B(b', \lambda) A(a', \lambda) B(b, \lambda) = 0, \end{aligned} \quad (12)$$

to get:

$$P(a, b) - P(a, b') = \int d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda) [1 \pm A(a', \lambda) B(b', \lambda)] + \int d\lambda \rho(\lambda) A(a, \lambda) B(b', \lambda) [1 \pm A(a', \lambda) B(b, \lambda)], \quad (13)$$

which, upon taking absolute values, Bell wrote as:

$$|P(a, b) - P(a, b')| \leq \int d\lambda \rho(\lambda) [1 \pm A(a', \lambda) B(b', \lambda)] + \int d\lambda \rho(\lambda) [1 \pm A(a', \lambda) B(b, \lambda)]. \quad (14)$$

Then, using Eq. (9), Bell's "Ansatz," and normalization $\int d\lambda \rho(\lambda) = 1$, one gets

$$|P(a, b) - P(a, b')| + |P(a', b') + P(a', b)| \leq 2, \quad (15)$$

a Bell inequality.[10]

However, if the λ are a complete set thereby rendering everything deterministic, then the A 's and B 's in Eq. (13) are pair-wise (as individual events) non-zero for distinct values of λ , which do not coincide for distinct events. That is, for each pair of settings (a, b) , there exists a unique value of $\lambda_{(a,b)}$ for which $A(a|\lambda_{(a,b)})B(b|\lambda_{(a,b)})$ is non-zero (+1 in the discrete case, ∞ in the continuous case). Therefore, in the extraction of a Bell inequality, all quadruple products of the A 's and B 's with pair-wise different values of λ in Eq. (13) are identically zero under the integration over λ so that the final form of a Bell inequality is actually the trivial identity:

$$|P(a, b) - P(a, b')| \leq 2. \quad (16)$$

This identity is, like the inequalities for symmetric dichotomic sequences discussed above, a tautology. In particular it admits Barut's model for particles with spin. Consider, following Barut, that the spin axis of pairs of particles have random but totally anticorrelated instantaneous orientation: $\mathbf{S}_1 = -\mathbf{S}_2$. Each particle then is directed through a Stern-Gerlach magnetic field with orientation a and b respectively. The observable in each case then would be $A := \mathbf{S}_1 \cdot a$ and $B := \mathbf{S}_2 \cdot b$. Now by standard theory,

$$Cor(A, B) = \frac{\langle |AB| \rangle - \langle A \rangle \langle B \rangle}{\sqrt{\langle A^2 \rangle \langle B^2 \rangle}}, \quad (17)$$

where the angle brackets indicate averages over the range of the hidden variables, here the spherical coordinates γ and φ . This becomes:

$$: Cor(A, B) = \frac{\int d\gamma \sin(\gamma) d\varphi \cos(\gamma - \theta) \cos(\gamma)}{\sqrt{(\int d\gamma \sin(\gamma) \cos^2(\gamma))^2}}, \quad (18)$$

which evaluates to $-\cos(\theta)$; i.e., the QM result for spin state correlation.[11] This model, essentially by itself alone constitutes a counterexample to Bell's analysis, and shows that continuous functions (vice dichotomic) satisfy all relevant requirements, in particular Eq. (16).

VI. BELL-KOCHEN-SPECKER THEOREMS

There are two basic formulations of Bell's hidden-variable no-go theorems: a version found first by Bell [10], but shortly thereafter refined and generalized by Kochen and Specker [12]; and a second version also found by Bell [13] and subsequently modified to accommodate experimental realities by various investigators.[14] The second version, discussed above, is the one most often encountered in physics literature. Both versions are false—as physics; as mathematics they are true but irrelevant to the actual application. The error in Bell-Kochen-Specker versions can be seen as follows:

First, recall some elementary facts concerning spin, which although well known, are essential below, and for convenience are repeated.[15]

Given a uniform static magnetic field, B , in the z -direction, the Hamiltonian is:

$$H = \frac{e}{mc} B \sigma_z, \quad (19)$$

for which the time-dependent solution of the Schrödinger equation is:

$$\Psi(t) = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{-i\omega t} \\ e^{+i\omega t} \end{bmatrix}, \quad (20)$$

and this in turn gives time-dependent expectation values for spin values in the x, y directions:

$$\langle \sigma_x \rangle = \frac{\hbar}{2} \cos(\omega t); \quad \langle \sigma_y \rangle = \frac{\hbar}{2} \sin(\omega t), \quad (21)$$

where $\omega = eB/mc$.

What is here to be seen is that in the x - y plane, in contrast to the z (magnetic field) direction, expectation values for spin can not be made up of summed eigenvalues, as certain formal dicta would have it. Furthermore, what is usually meant by measuring spin in these directions requires re-orienting the magnetic field so that in effect one is measuring σ_z in the new B -field direction.

Following Ref. [14], consider now the following version of the Bell-Kochen-Specker Theorem. At the onset, the system of interest is presumed prepared in a 'state' $|\psi\rangle$ and described by observables $A, B, C \dots$. A specific hidden variable theory, then, is a mapping algorithm v of the observables to numerical values: $v(A), v(B), v(C) \dots$ so that if any observable or mutually commuting subset of observables is measured on that system, the results of the measurement on it will be the appropriate values.

If this theory is to be compatible with quantum mechanics, the observables must be then operators on a Hilbert space (an erudite way to say that the solutions of the Schrödinger Equation have the properties of hyperbolic differential equation), and the measured values $v(A)$, etc. are the eigenvalues of these operators. It is simply a fact that if a set of operators all commute, then any function of these operators $f(A, B, C \dots) = 0$ will also be satisfied by their eigenvalues: $f(v(A), v(B), v(C) \dots) = 0$.

From this point, the proof of a Kochen-Specker Theorem proceeds by displaying a contradiction. Surely the least complicated rendition of the proof considers two ‘spin- $\frac{1}{2}$ ’ particles. For these two particles, nine separate mutually commuting operators can be arranged in the following 3 by 3 matrix:

$$\begin{pmatrix} \sigma_x^1 & \sigma_x^2 & \sigma_x^1 \sigma_x^2 \\ \sigma_y^2 & \sigma_y^1 & \sigma_y^1 \sigma_y^2 \\ \sigma_x^1 \sigma_y^2 & \sigma_x^2 \sigma_y^1 & \sigma_z^1 \sigma_z^2 \end{pmatrix}. \quad (22)$$

It is then a little exercise in bookkeeping to verify that any assignment of plus and minus 1’s for each of the factors in each element of this matrix results in a contradiction, namely, the product of all these operators formed row-wise is plus one and the same product formed column-wise is minus one.[14]

As for errors, it can be seen immediately that the proof depends on simultaneously assigning the [eigen]values ± 1 to σ_x , σ_y and σ_z as measurables for each particle. (With some effort, for all other proofs of this theorem, one can find an equivalent assumption.) Above, however, it was seen that if the eigenvalues ± 1 are tenable measurement results in the “ B - field” direction, then in the other two directions the expectation values oscillate out of phase and therefore, can not be simultaneously equal to ± 1 , an observation first made by Barut.[16] Thus, this variation of a Bell theorem also is defective physics.

Moreover, note that Kochen-Specker no-go Theorems for hidden variable extensions of QM do not pivot on locality, but rather on the non commutivity of operators for angular momentum (spin). And, as has been observed elsewhere with respect to this issue [17], the components of angular momentum do not commute even in classical physics, so that the import of these theorems as a constraint on possible extensions to QM is questionable from the start.

VII. A LOCAL ACCOUNTING OF EPR CORRELATIONS

Consider the following model using continuous variables to model ‘Clauser-Aspect’ type experiments of EPR polarization correlations.

In these experiments the source of radiation is a vapor, typically of mercury or calcium in which a cascade transition is excited by an electron beam or intense radiation of fixed orientation. Each stage of the cascade results in emission of radiation (a “photon”) that is polarized orthogonally to that of the other stage. In so far as the sum of the emissions can carry off no net angular momentum, the separate emissions are antisymmetric in space. The intensity of the emission is maintained sufficiently low that at any instant the likelihood is that emission from only one atom is visible. Photodetectors are placed at opposite sides of the source, each behind a polarizer with a given setting. The experiment consists of measuring the coincidence count rate as a function of the polarizer settings.

The model consists of simply rendering the source mathematically, and then of computing the coincidence rate. Photodetectors are assumed to convert continuous radiation into an electron current at random times with a Poisson distribution but in proportion to the intensity of the radiation. The

coincidence count rate is taken to be proportional to the second order coherence function.

The source is assumed to emit a double signal for which individual signal components are anticorrelated and confined to the vertical and horizontal polarization modes; i.e.

$$\begin{aligned} S_1 &= (\cos(n\frac{\pi}{2}), \sin(n\frac{\pi}{2})) \\ S_2 &= (\sin(n\frac{\pi}{2}), -\cos(n\frac{\pi}{2})) \end{aligned} \quad (23)$$

where n takes on the values 0 and 1 with an even random distribution. The transition matrix, χ , for a polarizer is given by,

$$\chi(\theta) = \begin{bmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) \\ \sin(\theta)\cos(\theta) & \sin^2(\theta) \end{bmatrix}, \quad (24)$$

so the fields entering the photodetectors are given by:

$$\begin{aligned} E_1 &= \chi(\theta_1)S_1 \\ E_2 &= \chi(\theta_2)S_2 \end{aligned}. \quad (25)$$

Coincidence detections among N photodetectors, γ , (here $N = 2$) are proportional to the single time, multiple location second order cross correlation, i.e.:

$$\gamma(r_1, r_2, \dots, r_N) = \frac{\langle \prod_{n=1}^N E^*(r_n, t) \prod_{n=N}^1 E(r_n, t) \rangle}{\prod_{n=1}^N \langle E_n^* E_n \rangle}. \quad (26)$$

It is shown in Coherence theory that the numerator of Eq. (26) reduces to the trace of \mathbf{J} , the system coherence or “polarization” tensor.[18] It is easy to see that for this model the denominator usually consists of factors of 1. The final result of the above is:

$$\rho(\theta_1, \theta_2) = \frac{1}{2} \sin^2(\theta_1 - \theta_2). \quad (27)$$

This is immediately recognized as the so-called ‘quantum’ answer. (Of course, it is also Malus’ Law.) Eq. (27) is the result for correlated outcomes. A similar expression with the sine replace by cosine pertains to anticorrelated outcomes. The total correlations is then

$$\frac{P(+, +) + P(-, -) - P(+, -) - P(-, +)}{P(+, +) + P(-, -) + P(+, -) + P(-, +)}, \quad (28)$$

for which the result here is $-\cos(\theta_1 - \theta_2)$, as is familiar from QM. This model is directly extendable to other tests of EPR correlations including multi-particle “GHZ” experiments. See [19].

VIII. ‘NONLOCALITY’ FROM ‘DUALITY’

There is no essential need for non-locality to explain EPR correlations. In contrast, there is such a need to explain particle beam diffraction. The dual particle-wave character of beam particles must be maintained until the particles are finally detected because the wave character is required while the particles are underway in order to explain the wave-like

navigation of beams. Then, at the moment of detection, the wave nature must be destroyed to accommodate the pure particle nature as it is seen by detectors and this process necessarily transpires non-locally in that the whole matter-wavefront collapses at once and that of a correlated particle at a space-like distance must do likewise simultaneously. Particles emitted by an EPR source, however, can have their final character crystallized at the moment of inception because there is no subsequent need (at least in the original experimental configuration envisioned by EPR) for wave-like navigation. It appears that in the literature the dual character or ontic ambiguity of EPR emissions is maintained only for the sake of consistency with the rest of QM. Similarly for ‘photons,’ where, in this case, the ‘particle’ character is maintained for the sake of consistency with the total theory, not because the explanation of phenomena in an EPR experiment, or anywhere else actually, demand it.

IX. ‘MATTER WAVE’ AS LORELEI.

While we can not now know Bell’s actual train of thought, it is tempting to speculate on how this misconstrual occurred. Possibly it was facilitated by the fact that a wave function, ψ is often considered a ‘field’ (since it diffracts) over the manifold of Euclidian parameter space, i.e., $x, y, z, (t)$. One naturally expects that a field intensity of a presumably ontically-physical ‘matter wave,’ at any point, as a function of parameters not at that point, is non-zero only when the external parameters fall in the past light cone. That is, ψ at A can not depend on b ‘instantaneously.’ With respect to quantum wave functions, however, such an expectation is somewhat misguided. ψ for multiple particles is actually defined over configuration space (multiple copies of Euclidian parameter space, one for each particle), and a Lorentzian displacement is not meaningful between two points when each is in a separate parameter space; locality in this case is actually undefined. Dependence of $P_A(a|b, \lambda)$ on its arguments implies only the appropriate ‘likelihood of registry of one of a pair of photons at station A given that its partner is *coincidentally* detected at station B . The characteristics of the photon pair are specified by λ and they were determined within the past light cones of both A and B . Clearly, b , representing an apparatus preparation, need not be set in A ’s past light cone to respect locality as it does not imply the transmission of anything from B to A , and its ontic correspondent among the λ was fixed in A ’s past light cone. In short, wave functions are simply not *only* ‘physical fields,’ although the tacit assumption that they are has lured many to a disaster on the shoals of broken logic.

Jaynes has given an alternate rendition of this error best illustrated by noting the difference between inferences and causality.[20] As he points out, if statement \mathcal{A} implies \mathcal{B} , then it follows rigorously that: $\neg\mathcal{B}$ implies $\neg\mathcal{A}$; on the other hand, the inverse of the statement: A causes \mathcal{B} , namely: $\neg\mathcal{B}$ causes $\neg\mathcal{A}$ is patent nonsense. With this insight, it might be taken that Bell was seduced by the need to regard ‘wave functions’ as something ontically substantive into also impugning ontic meaning to the essentially epistemic character of prob-

abilities; in this case, conditional probabilities in his *Ansatz*, Eq. (1). This is not only not needed, but impossible for a satisfactory completion of QM. See Ref. ([21]) for an account of a possible ‘local,’ realistic origin of the simultaneous character of solutions to the Schrödinger Equation as both wave and probability amplitudes.

The consequence of all this is that either 1.) Eq. (1) is not valid for correlated events, and Bell inequalities do not pertain to correlated events of the sort actually employed in the many experiments to test the inequalities, or 2.) Eq. (1) is an inviolable arithmetic tautology. In effect, these experiments verify Eq. (4), as they should by their very design. At the root of the issue is not locality, but simply hereditary correlation. Thus in the end, Bell’s Theorem is seen to be irrelevant; it does not rule out speculation entertained by Einstein, Schrödinger, de Broglie and many others, that there may exist a local, realistic, deeper theory, a completion for QM.

X. CONCLUSION

With the single exception of discussions of EPR’s thought experiment using Bell’s analysis (nowadays in many various guises), rejecting Bell’s Theorem’s interpretation will have scant impact on the practice of QM. Quite the opposite, it brings this whole matter in harmony with quantum field theory in which one of the main canons, securing logical and correct results, is the demand that certain expressions be ‘analytic.’ This demand in effect banishes non-local interaction in such calculations.

Moreover, on the basis of the above argumentation, it can be taken that non-locality has not been shown to be an intrinsic characteristic of nature as described by QM. This can be seen better perhaps from the following point of view.

Using some innocuous transformations, David Bohm recast the Schrödinger Equation so as to suggest a radically different interpretation.[22] The essential novelty he introduced is, that whereas in orthodox QM the wave function seems to deny atomic scale entities the properties of arbitrarily precise simultaneous location and momentum, in Bohm’s mechanics they have exactly these properties (as in classical physics), but are destined to follow an un-specifiable but exact particular trajectory. In Bohm’s mechanics, it seems particles are put on rails. The rail system, however, is complex. For a beam passing through a slit, for example, the rail system is very dense in the slit, but then fans out in a variable manner so that the density of tracks downstream from the slit mimics a diffraction-like pattern. It is denser where particles are likely, according to QM, to be found, and less so elsewhere; see [23] for numerically calculated examples. In short, each particle is considered at any given instant to be exactly located with a specific momentum on a particular track; just which one it happens to be is an unknowable probabilistic matter, however.

Bohmian mechanics attributes the design of the rail system to a so-called ‘quantum potential’ comprising just those terms for which, under Bohm’s transformation, no classically interpretable correspondent emerges. Just how this potential arises and physically sets up its rail system before a particle even

comes in the vicinity of a slit, or other boundary, or measuring device, is unexplained. The equations seem to say that it is just there—instantaneously; that is, non-locally. The description of two interacting particles in Bohmian mechanics seemingly exposes non-locality even more distinctly. As Bohm's mechanics is equivalent in all operational aspects to QM, it is a widely held view, shared by Bell, that Bohm's theory is just exposing features common to both theories, essentially showing in another, independent way, that QM is ineluctably non-local. (In reality, however, none of this is so clear, because Bohmian mechanics is also cast over configuration space.)

It is also widely claimed that if QM, or the quantum potential, is non-local, then an extension as envisioned by EPR, must also be non-local. However, any local interaction can exhibit *apparent* non-locality; in fact, a steady state solution to the wave equation exhibits this form of non-locality. A standing wave on a string, for example, is such that the intensity at any point along the string is dependent on the length, that is, on the end points. If we ignore how a steady state is obtained, then it appears that the amplitude at a point in the middle of the string is 'instantaneously' dependent on the end points. But of course, the wave equation is not non-local. A steady state, a standing wave, for example, is preceded by transients that set it up. The propagator describing the transmission of energy by such transient waves is local, or in the jargon of applied mathematics, the Green's Function is analytic. Moreover, even a steady state is really a dynamic situation in equilibrium by virtue of constant regeneration by 'new' wavelets, each behaving locally.

These same principles could well apply to QM. The underlying theory sought by EPR could provide, for example, a fully local realistic description of how the 'quantum potential' establishes Bohm's rail system. Indeed, a theory of this sort has an historical tradition: de Broglie's 'Pilot Wave.' [24] The core of his theory is the notion that the ontic essence of a particle is an entity comprising a particulate kernel embedded in a 'piloting' wave whose rays correspond to Bohm's rails. Unfortunately, de Broglie never developed his theory to the point of providing a physical rationalization for the pilot wave. Recently, however, de Broglie's basic conceptions have been extended and amended with encouraging results. The basic concept used was the assumption that the effect of a certain sort of stochastic background radiation effectively engenders waves that can account for the undulatory behavior of particles—thereby providing a local, realistic rationalization for pilot waves. [25; 26]

In any case, regardless of how one appraises such exploratory efforts, it is high time to liberate imagination from the shackles of dogma and misguided prohibition. To quote Bell himself:

...long may Louis de Broglie continue to inspire those who suspect that what is proved by impossibility proofs is lack of imagination. [27]

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that these papers propose a remedy which is equivalent to retracting Malus' Law as the analysis therein fails to take into account that experiments measure the density of pairs per unit time and angular settings, quantities which are not dichotomic.