Appeared in: *Physics Essays* **5** (2) 226-234 (1992)

AN INTUITIVE PARADIGM FOR QUANTUM MECHANICS

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ABSTRACT. Use is made of a relativistic kinematic modulation effect to compliment imagery from Stochastic Electrodynamics to provide intuitive paradigms for Quantum Mechanics. Based on these paradigms, resolutions for epistemological problems vexing conventional interpretations of Quantum Mechanics are proposed and discussed.

1. INTRODUCTION

For lack of an intuitive paradigm or physical model, Quantum Mechanics (QM) remains an abstruse theory. Thus far all attempts to find underlying motivation for its fundamental precepts which is somehow compatible with intuition, have been found wanting. Moreover, some efforts to "understand" QM have led to results indicating that it has conceptual features which are fundamentally irreconcilable with macroscopic experience and classical physics.

Bell's Theorem, for example, is understood nowadays to mean that certain aspects of natural phenomena described by QM, are "nonlocal."[1] That is, events at a particular point with space-time coordinates x_μ, according to the principles of QM, depend on the values of functions evaluated at points outside the past light cone centered on x_μ. Because the classical theories of fundamental physical interactions are all "local" in this sense, this theorem appears to preclude the possibility that QM could be explained at a deeper level with principles from classical physics, for example by insinuating "hidden variables" which are implicitly averaged out at the level of QM as currently formulated.

This implies in particular, that in spite of the probabilistic interpretation given wave functions $\psi(x)$, QM can not be just an elaboration on classical Statistical Mechanics quantifying effects attributable to the accumulation of finer-grained, fundamental interactions which are somehow governed by principles from classical physics. Moreover, certain formalistic similarities to equations from statistical physics notwithstanding, several arguments show that time evolution in QM can not be described by a Markov process of the sort used in Stochastic Mechanics. Markov processes are those for which time evolution has no long term memory, as it were; i.e., trajectories can not be retrodicted. QM, to the contrary, *is* time-reversible—at least up to the point of measurement, an event which, in any case, is not encompassed by the QM formalism for time evolution. Thus, despite parallels with Statistical Mechanics, QM must encompass something essentially different from irreversible (Markovian) Brownian motion, or similar phenomena.

The purpose of this article is to propose a new conceptual paradigm which, nevertheless, does provide intuitive imagery from classical physics to motivate the basic precepts of QM. From the viewpoint of this new model, the apparent nonlocal aspects of QM are not objectionable because QM is reinterpreted, not as a fundamental theory of elementary

Date: 15 January 1991.

interactions, but as a theory of "many-body effects" attributable in a certain way to the accumulation of local, two-body, classical electromagnetic interactions with the rest of the universe.

2. Fundamental Concepts

QM is formulated on two levels. The basic theory, or "First Quantization," was conceived originally to describe the emission and absorption spectra of atoms and molecules, and certain other phenomena, all having features suggesting resonances in oscillatory systems or wave propagation. Thereafter, "Second Quantization" was devised to describe refinements to the basic theory such as the Lamb shift, the line width of atomic spectra, stimulated emission and the anomalous magnetic moment of the electron, etc. Later it was seen that effects described by Second Quantization often can be given imaginable, physical motivation in terms of a *Zitterbewegung*, or a fine-scale jitter similar to Brownian motion and attributable to what often is called the "quantized vacuum." This observation, in turn, motivated various attempts to derive the fundamental equations of QM or to describe quantum phenomena strictly on the basis of statistical physics. Some of these attempts, under the rubric of Stochastic Electrodynamics (SED), have successfully quantified certain phenomena usually described using Second Quantization techniques. These phenomena include the Van Der Waals Forces, the behavior of a charged nonrelativistic harmonic oscillator, and others, where, not coincidently, the imagery of *Zitterbewegung* appears germane.[2] However, the main corpus of QM, including atomic structure, spectral lines and most strikingly, the wave-like diffraction of particle beams, continues to elude all clarification in these terms.

The basic, axiomatic premise underlying SED is that the universe is permeated with random electromagnetic background radiation in thermodynamic equilibrium with all charged particles. (For present purposes, a particle need not carry a net charge, any multipole alone is sufficient.)[1]The energy in this background radiation is envisioned to be a residue devolving from past interactions or emissions from all charged particles in the universe. For most derivations in classical physics this residue and its source are ignored; for example, the derivations of the Thermodynamic formulae for an ideal gas ignore radiation caused by acceleration incidental to collisions. Regardless of the physical model ascribed to the generation of this background, however, for the purposes of the formal theory or logical foundation of SED, its existence is asserted as an axiomatic proposition.[2]

The essential feature of background radiation is determined by the requirement that its spectral energy density $E(\omega)$ be invariant under Lorentz transformations in the sense that the total energy between two fixed numerical values of ω, a and b, be identical in all inertial frames; it i.e.,

$$\int_a^b E(\omega) d^3k = \int_a^b E'(\omega)\gamma(1-v/c) d^3k, \quad [\gamma \equiv (1-v^2/c^2)^{-1/2}] \tag{1}$$

As Physics, this stipulation is tantamount to the requirement that there be no distinguishable frames; and, it is based on the fact that, were it not true, the background would engender certain anisotropisms that in fact are not observed. Eq. (1) is satisfied by a linear spectral energy density $E(\omega) = constant \times \omega$ where the constant scale factor is determined empirically to be Planck's constant/4π—$\hbar/2$. [2, 3]

[1]On the other hand, particles with no charge structure, including no net charge, no multipole moments, etc., are "invisible" to all forms of electromagnetic radiation and are largely unmanipulatable by interaction with the material laboratory world comprised of atoms and, as such, are inaccessible to direct experiment. Their behavior remains, at best, a matter for indirect inference and is not governed by the considerations presented herein.

Taking this Lorentz invariant background radiation into account leads directly to the Planck blackbody spectrum and an understanding of "photon statistics."[3] This has been shown most simply by manipulation the following four equations involving the mean energy density $\overline{E_i}$, the mean square energy density $\overline{E_i^2}$, and the mean square deviation of the energy density $\overline{(\delta E_i)^2}$ of any two mutually incoherent radiation fields:

$$\overline{E_{sum}} = \overline{E_1} + \overline{E_2}, \tag{2}$$

$$\overline{E_1 E_2} = \overline{E_1}\ \overline{E_2}, \tag{3}$$

$$\overline{(\delta E)^2} = \overline{(\delta E_1)^2} + \overline{(\delta E_2)^2}, \tag{4}$$

and

$$\overline{(\delta E_i)^2} = \overline{E_i^2} - \overline{E_i}^2 = \overline{E_i}^2; \quad i = 1,2 \tag{5}$$

to obtain

$$\overline{(\delta E_T)^2} = \overline{E_T^2} + 2\overline{E_T}\ \overline{E_B}, \tag{6}$$

where, in the case at hand, $\overline{E_B}$ is the mean energy density solely of the background radiation, and $\overline{E_T}$ is the same for a temperature dependant radiation field. This latter field coexists with the background and is modified by it via the mutual interference terms which are included in $\overline{E_T}$. Invoking the Fluctuation Theorem :

$$\frac{\partial \overline{E_T}}{\partial \mu} = \overline{(\delta E_T)^2}, \quad [\mu \equiv -1/\kappa T] \tag{7}$$

for the thermal field at temperature T where κ is Boltzmann's constant, yields the differential equation:

$$\frac{\partial \overline{E_T}}{\partial \mu} = \overline{E_T}^2 + 2\overline{E_T}\,\overline{E_B}, \tag{8}$$

whose solution, with $\overline{E_B} = \hbar\omega/2$, is the Planck black body spectral energy distribution

$$\overline{E_T} = \frac{\hbar\omega}{e^{\hbar\omega/\kappa T} - 1}. \tag{9}$$

The Fluctuation theorem is applicable if the thermal field exchanges only energy with the background—a stipulation met here *ab initio*.

Furthermore, Eq. (6) written in the form:

$$\frac{\overline{(\delta E_T)^2}}{\overline{E_T}^2} = 1 + 2\frac{\overline{E_B}}{\overline{E_T}} \tag{10}$$

elucidates the source of the dualistic nature of radiation.[3] Were the first term on the right to stand alone, Eq. (10) would characterize intensity fluctuations of a classical radiation field while the second term alone, being proportional to $1/\overline{E_T}$, gives an equation characterizing density fluctuations of a particle ensemble. Together they capture the essence of "photon statistics" and thus, without assuming the existence of discreet quanta, characterize the "quantized electromagnetic field."[3]

Zitterbewegung, in this theory, is interpreted as a manifestation of the interaction of particles with the random, Lorentz-invariant background.

First Quantization, however, pertains to resonance and wave-like phenomena for which physical intuition admits no suggestion of *Zitterbewegung*. However, following SED, all particles with charge structure are considered to be in thermodynamic equilibrium with the background via interaction with signals at frequencies characteristic of their structure. The intuitive paradigm proposed herein to model First Quantization is based on the *Ansatz* that, in frames in which they are moving, particles will be deflected by diffraction patterns in the background signals to which they are tuned. Background waves, being conventional electromagnetic radiation, are diffracted by physical boundaries according to the usual principles of optics.[2] For example, a particle moving towards a slit would equilibrate with a signal that is a standing wave in its own frame but which is a traveling wave in the slit's frame where it diffracts at boundaries such as those of the slit. On passage through the slit, the particle is subject to the lateral energy flux attendant to the diffraction of the background signal to which it is tuned. In other words, it is envisioned that a particle will tend to be jostled into the energy nodes of the diffraction pattern of the "standing wave" to which it is tuned in its own inertial frame but which is a translating wave in the frame of the slit. This effect is similar to the way froth and debris tend to track the nodes of standing waves in rivers or sand tends to settle on the nodes of a vibrating membrane. An ensemble of similar particles in identical circumstances—e.g., a beam of particles impinging on a slit—upon accumulation at the detector discloses the diffraction pattern of the composite wave comprised of components with which the individual particles are in equilibrium.

Consider, for example, a neutral particle or system consisting of a dipole of opposite charges held apart by some internal structure modeled to first order by a simple spring with resonant frequency ω_0. According to the basic SED assumption of thermodynamic equilibrium with the background, the rest energy of this system constituting the particle will equal the energy in the background mode ω_0 which is also the resonant frequency of the system at which it is exchanging energy with the background; that is

$$m_0 c^2 = \hbar\omega_0, \tag{11}$$

where a contribution of $\hbar\omega_0/2$ is made to the right side by both polarization states of the background mode. (In the case of systems with more complex internal structure, ω_0 stands for the sum of the frequencies corresponding to the various possible interactions.)[3] In its rest frame, with respect to each independent spatial direction, a particle will equilibrate with a standing wave having an antinode at its location, which, if the particle is located at $x = 0$, has an intensity proportional to the expression $2\cos(k_0 x)\sin(\omega_0)$.[4] When projected onto a coordinate frame translating at velocity v with respect to the particle, that of a slit for example, this standing wave has the form of the modulated translating wave and is

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[2] Results from experiments in Cavity Quantum Electrodynamics [4] substantiate the concept of the background as ordinary electromagnetic radiation. Were background radiation an immutable ground state or the "quantized vacuum" as conventionally characterized by QM, then, contrary to observation, it might be expected to resemble the vacuum (*viz.*, an absolute space-time void) by being uniformly ubiquitous and by permeating the interstices of materials down to the scale of atoms and even "elementary particles" (essentially) without alteration, in particular at macroscopic cavity boundaries.

[3] Also, m_0 is to be the sum of the masses of the system constituents "renormalized" to account for relativistic mass increases which are due to internal motion. For present purposes, however, the details of the internal structure of the particle are immaterial.

[4] Tautologically, a charged particle does not interact with those signals that interfere destructively so as to have a node at its spacial location.

proportional to

$$2\cos(k_0\gamma(x - c\beta t))\sin(\omega_0\gamma(t - c^{-1}\beta x)); \quad [\beta \equiv v/c] \tag{12}$$

This wave consists of a short wavelength "carrier" modulated at a wavelength $\lambda = (\gamma\beta k_0)^{-1}$ inversely proportional to the relative velocity of the particle with respect to the slit. The modulation on this wave is a relativistic kinematic effect. It arises from the difference in the Lorentz transformations of the oppositely translating components of a standing wave.

The modulated wave, upon propagation through a slit, for example, is diffracted according to Huygens' principle such that the modulation diffraction pattern is imposed on the carrier's diffraction pattern.[5] A particle bathed in this diffracted wave will experience a gross energy flux with a spatial pattern proportional to the square of the modulation intensity imposed on the fine-scale background wave driving the *Zitterbewegung*. In other words, according to this interpretation, boundary conditions on waves in the background modify the stochastic effects of *Zitterbewegung* on the orbits of material particles. The actual detailed motion of a particle, while it reflects the relatively large scale effects of the modulation, is very complex and jitters in consort with spatially modulated *Zitterbewegung*.

Now, a Lorentz transformation into the translating frame applied to both sides of the statement of energy equilibrium, Eq. (11), yields both

$$\gamma m_0 c^2 = \gamma\hbar\omega_0 \tag{13}$$

and

$$p' = \gamma m_0 c = \hbar\gamma\beta k_{0.} \tag{14}$$

From Eq. (14), $\gamma\beta k_0$, can be identified as the De Broglie wave vector from conventional QM. Note that in expression (12), the carrier propagates with the "phase velocity" v while the modulation has the "group velocity" $V_{group} = c^2/v$. In the usual QM interpretation, on the other hand, v is identified as a group velocity for a localized wave packet intended to represent the local character of a particle. In that case the dispersion relation $V_{group} = v = \partial\omega/\partial k$, when integrated, yields the equation $E_{kinetic} = m_0 v^2/2 = \hbar\omega_D$, which relates the particle's kinetic energy E to a frequency ω_D, which is identified as the temporal frequency of the particle's De Broglie wave. A De Broglie wave packet is not required, however, to account for the wave-like behavior of a localized particle, if wave-like properties can just as well be ascribed to modulated *Zitterbewegung*. More importantly, this new interpretation suggests direct resolutions for many of the semantic and epistemological problems vexing other interpretations of QM as discussed below in Section IV.

3. Time Evolution in Quantum Mechanics

Time evolution in QM is governed by the Schrödinger Equation. Being a hyperbolic differential equation, the Schrödinger Equation has solutions which are time reversible, and in this respect, *inter alia*, are structurally distinct from those for the parabolic diffusion equation. This fact, which arises in various and not always obviously related manifestations, precludes interpreting that part of QM connected with the Schrödinger Equation in terms of Stochastic Mechanics, where diffusion processes govern the time evolution of physical phenomena.[5] The conceptual model developed above, *viz.* of the De Broglie wavelength as a manifestation of kinematically derived modulation on *Zitterbewegung* driven by Lorentz

[5] This point is obfuscated by the fact that propagation formulae pertaining to deterministic processes also can be cast to have the apparent structure of Fokker-Planck equations describing the time evolution of stochastic processes. In these cases, however, if fundamental assumptions reintroducing the epistemological problems of

invariant background radiation, however, supports an interpretation for the Schrödinger Equation with a new and coherent intuitive physical motivation based on classical physics. This new physical interpretation respects the fundamental difference between effects attributable to *Zitterbewegung* and described by Second Quantization and effects described by basic QM or First Quantization, including "hyperbolic" time evolution. Consider a particle subject to a force $\mathbf{F}$ and for which the density of trajectories on phase space is $\rho(\mathbf{x}, \mathbf{p}, t)$, where $\rho(\mathbf{x}, \mathbf{p}, t = 0)$ can be regarded either as the distribution of initial conditions for similarly prepared particles, or, equivalently, as the *a priori* probability distribution of initial conditions for a single particle. Time evolution of $\rho(\mathbf{x}, \mathbf{p}, t)$, is governed by the Liouville Equation

$$\frac{\partial\rho}{\partial t} = -\nabla\rho\cdot\frac{\mathbf{p}}{m} + (\nabla_p\rho)\cdot\mathbf{F}, \quad \left[\nabla_p \equiv \sum_{i=x,y,z}\frac{\partial}{\partial p_i}\right]. \tag{15}$$

By virtue of Eq. (14), each value of p in $\rho(\mathbf{x}, \mathbf{p}, t)$ is correlated with a particular wavelength of kinematical modulation. As proposed above, boundaries and geometrical constraints on the waves to which the particle is tuned cause these waves to diffract and interfere so that gradients in their energy densities are induced. These gradients, in turn, result in spatial variations in the magnitude of the *Zitterbewegung,* the averaged effect of which is to systematically modify particle trajectories. Because the energy density of a wave is proportional to the square of its intensity, the wavelength of energy density oscillations caused by the modulation will be half that of the modulation itself. That is, the wavelength of the physical agent modifying trajectories, an energy gradient, is half that of the modulation. Further, an ensemble consisting of multiple particles, either conceptual or extant, will be guided by an ensemble of energy density waves derived from an ensemble of kinematical modulations. The spatial structure of this ensemble wave is found by taking the Fourier transform of $\rho(\mathbf{x}, \mathbf{p}, t)$ with respect to $2\mathbf{p}/\hbar$, the wave vector for the physical agent; i.e.:

$$\widehat{\rho}(\mathbf{x}, \mathbf{x}', t) = \int e^{\frac{i2\mathbf{p}\cdot\mathbf{x}'}{\hbar}}\rho(\mathbf{x}, \mathbf{p}, t)d\mathbf{p}, \tag{16}$$

for which the similarly transformed Liouville Eq. is

$$\frac{\partial\widehat{\rho}}{\partial t} = \left(\frac{\hbar}{i2m}\right)\nabla'\nabla\widehat{\rho} - \left(\frac{i2}{\hbar}\right)(\mathbf{x}'\cdot\mathbf{F})\widehat{\rho}. \tag{17}$$

Solutions for equations of this form are sought by first separating variables using a transformation of the form

$$\mathbf{r} = \mathbf{x} + \mathbf{x}', \quad \mathbf{r}' = \mathbf{x} - \mathbf{x}', \tag{18}$$

which yields

$$\frac{\partial\widehat{\rho}}{\partial t} = \left(\frac{\hbar}{i2m}\right)(\nabla^2 - (\nabla')^2)\widehat{\rho} - \left(\frac{i}{\hbar}\right)(\mathbf{r} - \mathbf{r}')\cdot\mathbf{F}\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\widehat{\rho}. \tag{19}$$

In general, for an arbitrary form of the force $\mathbf{F}(\mathbf{r}, \mathbf{r}')$, this equation still is not separable. However, for the purpose of calculating expectation values, solutions to Eq. (19) are needed only on the line in the $\mathbf{r}, \mathbf{r}'$ plane for which $\mathbf{r} = \mathbf{r}'$. For example, the expectation of $\mathbf{x}$ is

$$< \mathbf{x} > = \int\int \mathbf{x}\,\rho(\mathbf{x}, \mathbf{p})dxdp, \tag{20}$$

QM in new forms are eschewed, the physical "diffusion constant" can be shown on the basis of kinematics alone to equal zero.

which, with the inverse Fourier transform of Eq.(16), becomes

$$<\mathbf{x}>=\int\int\int \mathbf{e}^{\frac{-i2\mathbf{p}\cdot\mathbf{x}'}{\hbar}}\mathbf{x}\widehat{\rho}(\mathbf{x},\mathbf{x}')dxdpdx', \tag{21}$$

so that the transformations of variables, Eqs. (18), give

$$<\mathbf{x}>=\int\int \delta(\mathbf{r}-\mathbf{r}')\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\widehat{\rho}(\mathbf{r},\mathbf{r}')drdr' \tag{22}$$

where $\delta(\mathbf{r}-\mathbf{r}')$ is a Dirac delta function that restricts the solutions of Eq. (19) to contributing only along the line $\mathbf{r}=\mathbf{r}'$ to the integral for calculating $<\mathbf{x}>$. Similar results are obtained for $<\mathbf{p}>$and functions of $\mathbf{x}$ and $\mathbf{p}$; e.g., the expectation of the energy, $<\mathbf{p}^2>/2m$.[6] Thus, techniques yielding separable solutions to Eq. (19) which coincide with solutions to the nonseparable equation on this line, may be used for this limited purpose.

These special solutions on the line $\mathbf{r}=\mathbf{r}'$ can be found by exploiting the symmetry of Eq. (19) in the variables $\mathbf{r}$ and $\mathbf{r}'$ and the fact that $\widehat{\rho}(\mathbf{r},\mathbf{r}')$ is real to write it the form $\psi^*(\mathbf{r}')\psi(\mathbf{r})$. Thus, with the usual manipulations to separate variables, Eq. (19) yields

$$\frac{\partial\psi}{\partial t}-\left(\frac{\hbar}{i2m}\right)\nabla^2\psi(\mathbf{r})+\left(\frac{i}{\hbar}\right)\mathbf{r}\cdot\mathbf{F}\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\psi(\mathbf{r})=\left(\frac{i}{\hbar}\right)\mathcal{F}(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}) \tag{23}$$

and its complex conjugate, where $i\mathcal{F}(\mathbf{r},\mathbf{r}')/\hbar$, which plays the role of what could be called a "function of separation," has a general form determined by symmetry and dimensionality considerations.

If $\psi(\mathbf{x})$ satisfies Eq. (23) when $\mathbf{r}\to\mathbf{r}'$, then $\psi^*(\mathbf{r}')\psi(\mathbf{r})$ will satisfy

$$\left\{\frac{\partial}{\partial t}-\left(\frac{\hbar}{i2m}\right)(\nabla^2-(\nabla')^2)-\left(\frac{i}{\hbar}\right)(\mathbf{r}-\mathbf{r}')\cdot\mathbf{F}\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\right\}\widehat{\rho}\equiv\{\mathcal{O}\}\widehat{\rho}=0 \tag{24}$$

on the line $\mathbf{r}=\mathbf{r}'$. This is seen by writing $\rho(\mathbf{r},\mathbf{r}')$ as $\psi^*(\mathbf{r}')\psi(\mathbf{r})$ in Eq. (24), adding zero in the form $(\mathcal{F}-\mathcal{F})\psi^*(\mathbf{r}')\psi(\mathbf{r})$ and rearranging to obtain

$$\psi^*\left\{\mathcal{O}-\left(\frac{i}{\hbar}\right)\mathcal{F}\right\}\psi+\psi\left\{\mathcal{O}^*+\left(\frac{i}{\hbar}\right)\mathcal{F}\right\}\psi^*, \tag{25}$$

from which the conclusion follows by letting $\mathbf{r}\to\mathbf{r}'$. The specific form of $\mathcal{F}(\mathbf{r})$ necessary to preserve the physical content of the Liouville can be determined by requiring

$$\frac{d<\mathbf{p}>}{dt}=<\mathbf{F}> \tag{26}$$

to hold. This stipulation (which, as physics, adds nothing new but as mathematics extracts structure obscured by the separation technique) leads to

$$\int\psi^*(\mathbf{r}\cdot\nabla\mathbf{F}-\nabla\mathcal{F})\psi d\mathbf{r}=0 \tag{27}$$

which, with the vector identity $\mathbf{r}\cdot\nabla\mathbf{F}-\nabla\mathcal{F}=\nabla(\mathbf{r}\cdot\mathbf{F}-\mathcal{F})-\mathbf{F}$ implies that $\mathcal{F}$ must satisfy

$$\mathbf{r}\cdot\mathbf{F}-\mathcal{F}=-\mathrm{V},\quad [\mathbf{F}\equiv-\nabla\mathrm{V}]. \tag{28}$$

Finally, with this result, on the line $\mathbf{r}=\mathbf{r}'$, Eq. (23) becomes the Schrödinger Equation

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{\hbar^2}{2m}\nabla^2\psi+V\psi. \tag{29}$$

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[6] The manipulations used here to compute expectation values can be shown to lead naturally to unique, symmetric "operator" expressions, thereby obviating *ad hoc* "Hermitezation" rules for the primitive operator equivalents.[6]

Consistency with the assumptions leading to the extraction[7] of the Schrödinger Equation above, requires that its interpretable solutions be those for which the resulting phase space density is everywhere positive and such that $\rho(\mathbf{x}, \mathbf{p}, t = 0)$ is the appropriate initial condition. The relationship between solutions satisfying these physics requirements and the eigenfunctions of the Schrödinger Equation is a complex matter. Although crucial to an ultimate judgment of the coherence of the interpretation espoused herein, it is left for future study. However, it seems highly portent in this regard, that "thermal states;" i.e., mixed states with Boltzmann weighting factors, and coherent states, give physically interpretable, everywhere positive densities among other desirable traits.[7]

For potentials having the form of a quadratic polynomial, the extraction of the Schrödinger Equation given above requires no unusual separation technique. Since such potential include that the harmonic oscillator, a substantial fraction of all analytically soluble problems are covered; but, unfortunately the most important case of all, the Coulomb potential, is not included. The manipulations used above to obtain the Schrödinger Equation are essentially the reverse of those suggested long ago in a study of QM phase space densities.[8] The principle difference with the reasoning herein, is that formerly it was assumed that QM is the fundamental theory and that the solutions of the Schrödinger Equation all represent realizable states although some yield nonsensical phase space densities. (Also, nonquadratic potentials complicate correspondence with the Liouville Equation.) Here, the Liouville Equation is given logical primacy and only those solutions of the Schrödinger Equation are admitted which yield meaningful Liouville densities.

In summary, the considerations above suggest that at the level of "First Quantization," QM can be given a formal or mathematical structure in terms of what might be called a theory of 'Eikonal Mechanics.'[8]The dynamical laws embodied in Eikonal Mechanics are deterministic; statistical aspects of the theory are conceptual in nature and pertain to probabilities implicit in a density on phase space. At its foundation, therefore, Eikonal Mechanics is incomplete because probabilities are an artifact of the theory, not of nature. Underlying Eikonal Mechanics are the stochastic effects of *Zitterbewegung*; at this deeper level probabilities are conceptually the same as those in the study of Brownian motion in statistical physics, so that here also the theory is incomplete. Thus, QM in this rendition is interpreted as a statistical theory with built-in structure to account for alterations to the classical motion of particles that endows their averaged trajectories with ray-like properties. These alterations are a particular manifestation of the influence of all other charged particles in the universe via the mediation of a residue of electromagnetic background radiation.

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[7]The word "extraction" is used here deliberately in the belief that expository physics should eschew mathematical terminology (e. g., THEOREM, PROOF, even DERIVATION, *etc.*) as it invites the inference of a degree of logical rigor, which in the face of the vast complexity of nature is often unobtainable or yields results idealized into irrelevancy. Moreover, it is probably wise not to read physical significance into these or similar symbolic manipulations that has not been invested deliberately and explicitly; indeed, conceiving mathematical models for natural phenomena may be less an exploration of Nature (the privilege of the experimentalist) than an exploration of just one's own imagination.

[8]'Wave Mechanics' would be as good a name were it not already virtually synonymous with all of QM, including Second Quantization. The use here of either term is meant to emphasize the fact that although the Schrödinger Equation contains essentially the same dynamical laws as the Liouville Equation, it is formulated especially to account for the ray-like nature of particle trajectories induced by relativistic, kinematical modulation.

4. COMMENTS ON INTERPRETATIONAL PARADOXES

Kinematical modulation as embodied in Eikonal Mechanics provides a physical model for the phenomena described by First Quantization which offers new perspectives on philosophical problems intrinsic to most widely considered interpretations of QM. Many of the seemingly novel features of this new interpretation already have been found heuristically to be implicit in the structure of QM if the same degree of logical consistency is enforced for the semantics of the terminology applied to its symbols as is required mathematically of the relationships among the symbols themselves. These features, therefore, are explicit elements in certain nonstandard interpretations, and *ipso facto*, implicit in the most widely endorsed, “Copenhagen,” interpretation.

Although much has been said concerning paradoxes and ambiguities in the orthodox interpretation of QM and the attendant measurement process, perhaps one of Einstein’s Gedanken experiments still captures the kernel of the problem in the most concise and revealing fashion.[9] Following Einstein, consider a beam of particles impinging on a virtually infinitesimal hole in a barrier. According to the principles of QM, upon passage through the hole the (plane wave) beam diffracts to become a hemispherical wave. Now, if a hemispherical detector, centered on the hole, were used to detect the diffracted beam, in time the observed result would be a uniform distribution of distinct, point-impacts over the surface of the detector. If, as Einstein observed, a QM wave function constitutes the ontological essence of individual particles, then, although immediately preceding each impact the wave function for each particle is finite over the whole surface of the detector, at the instant of the impact it must “collapse” to a delta function at the impact point. A collapse or focusing of wave functions, occurring even instantaneously, is unobjectionable if wave functions are only mathematical artifacts for calculating expectation values and not the embodiment of the material essence of particles. However, purely formal, nonphysical or “mathematical” interpretations of wave functions seem to be precluded by the fact that QM wave functions, like electromagnetic radiation and waves in physical media, interact with material boundaries. In Einstein’s Gedanken experiment, for example, the wave function diffracts upon passage through the hole, implying that it has substantive existence.

Eikonal Mechanics resolves this conflict by identifying wave functions as indeed just mathematical expressions for computing expectation values on phase space; but, with the difference that, these expectation values pertain to special trajectories, namely those which are ray-like in response to spatial patterns in the energy density of kinematically modulated background waves. The physical work of diffracting the trajectories of localized material particles is accomplished by the external agent of modulated *Zitterbewegung* acting on them like a guiding hand. The whole effect is embedded in the mathematical structure of QM, specifically the Schrödinger Equation, for which the solutions give phase space densities of the physically feasible set of ray-like trajectories available to particles or systems. Thus, measurement is not an ontological issue for Eikonal Mechanics because it is only a “recalibration” of phase space densities, essentially revising the initial conditions. Measurement has the same role here as is has in classical statistical physics, where it only reduces the degree of ignorance but does not purport to revamp realities instantaneously.

The imagery of Eikonal Mechanics has a certain resemblance to propositions independently derived in other studies which strive to render the interpretation of QM semantically consistent. In particular, the concept of modulation waves was anticipated by the De Broglie-Bohm theory of the “pilot wave.”[10, 11] The conceptual difference between the pilot wave theory and Eikonal Mechanics is that, whereas the pilot wave is envisioned as part of the physical essence of a particle—emanating from it in the manner of an ethereal scout, or conversely as the carrier of the particle as a confined high density region, modulation waves are considered to be in existence in the particle's environment *a priori* and emanate, so to speak, from all other particles in the universe.

Another aspect of modulation waves was anticipated in the "many worlds" interpretation of QM.[12] In this interpretation wave function collapse is obviated by positing[9]that each possible trajectory for the universe (and individual particles in particular) exists such that at each juncture where there are diverse potential outcomes, the universe (including every particle individually) replicates with one "daughter" fulfilling each potentiality.[10]The interpretation based on Eikonal Mechanics suggests that while the geometry of a particular situation accommodates the existence of a bundle of possible ray-like orbits available to particulate systems, individual systems in effect are bumped stochastically by background radiation from orbit to orbit while maintaining their individual identity. In other words, the multiplicity implicit in QM is located in the potentiality implicit in the background and not the actuality of material particles.

In the contemporary literature on the interpretation of QM, the focus is often on Bell's Theorem which has been credited with showing that there is

> "...a contradiction between several principles we had wished could all be true, including the validity of quantum theory, Einstein's relativity principle, causality acting forward in time, local objective reality and the structure of space[; but,] experiments support some of these principles to a certain degree of accuracy only. The contradiction can be resolved by abandoning one or several of these principles."[13]

In view of this statement, the question immediately arrises: within the model based on Eikonal Mechanics, which of these principles has been abandoned?

The answer provided by Eikonal Mechanics is: none at a fundamental level, although the mathematical or formal structure of QM abandons local objective reality; i.e., the local identity of particles in favor of extended densities conditioned instantaneously by distant boundaries. This can be understood as follows. Consider a particle in a box. According to Eikonal Mechanics, a particle is a localized entity which is in equilibrium with background waves standing in the box. This means that the behavior of a particle at a particular space-time point in a box is affected by waves whose structure is determined, in part, by boundary conditions; i.e., the sides of the box, located at space-like separations from the particle. Here is the source of the nonlocal aspect of QM which seems to violate local reality or time-like causality. In the imagery of Eikonal Mechanics, however, the standing waves in the box result from background radiation emitted in the past by other sources at null or light-like separations. The nonlocal feature in QM, according to this interpretation, is the same as that in any steady state solution to wave equations where boundary conditions determine the solution in a region of interest. Therefore, according to this interpretation, although particle densities on phase space do manifest nonlocal effects, such effects do not constitute an ontological problem because the radiation inducing these effects propagated along light-like separations in Minkowski Space from the past. Phenomena peculiar to QM can be considered to be "multibody effects" caused by classical interactions from all other

[9]Actually, the many-worlds interpretation is mandated by logic if QM as formulated is assumed to be fundamental and complete, because then, in principle, there must be a wave function for the whole universe for which no external agent exists to cause collapse.

[10]Specifically, in this interpretation each 'measurement act' has to be a juncture at which replication occurs; but, presumably then only an insignificant minority of such junctures actually could involve conscience observers, i.e., all interaction is 'measurement.'

charged particles in the universe, rather than manifestations of the fundamental nature of particle interactions exclusively.

Underlying the relatively gross behavior of particles as described by Eikonal Mechanics is the fine-grained *Zitterbewegung*. All phenomena described by Eikonal Mechanics, that is "First Quantization," invite deeper analysis to understand the "anomalies" and "shifts" caused by *Zitterbewegung*. And, at least in concept, this can be accomplished using SED for which there is no problem with interpretation.

5. Spin and External Fields

To include external electromagnetic fields in this formalism, let the total energy of the radiation to which the particle is exposed include that of the background plus the potential energy from additional fields; i.e., let

$$m_0 c^2 = \hbar\omega + e\phi \tag{30}$$

where e is the charge on the particle and $\phi(\mathbf{x})$ is the potential function for the additional field. The gauge for the external field has been chosen so that the four-vector potential in the rest frame of the particle is purely scalar or time-like. This is tantamount to assuming that the potential energy of the additional fields contribute to the rest mass of the particle such that $(m_0)_{total} = m_0 + e\phi/c^2$.[14]

While clear as formalism, these considerations depend on the underlying theory of electromagnetic interaction, in particular the use of potential functions to take full relativistic account of interaction. Reference 13 provides reason to suspect that there is considerable room for deeper understanding of the use of potential functions consistent with special relativity. Nevertheless, the role of Eq. (30) as an axiomatic proposition preceding the logic and manipulations used to extract the Schrödinger Equation, elucidates the structure of QM by indicating that obscurities in the physical meaning of electromagnetic potentials in QM are not wholly intrinsic to QM *per se*, but substantially to relativistic kinematics and dynamics.

Intuitive motivation for spin consistent with concepts of Eikonal Mechanics can be found as a manifestation of the existence of two independent polarization states for electromagnetic radiation. This can be seen by conceptually decomposing each background mode into its two polarization states and considering that a given density on phase space $\rho(\mathbf{x}, \mathbf{p}, t)$ consists of two parts, ρ_1 and ρ_2, each pertaining to that portion of an ensemble in equilibrium with one polarization state. Time evolution of a tandem set of densities is governed by a tandem form of the Liouville Equation

$$\vec{1}\begin{bmatrix} \partial\rho_1/\partial t \\ \partial\rho_2/\partial t \end{bmatrix} = \left[(\sigma\cdot\frac{d\mathbf{x}}{dt})(\sigma\cdot\nabla) + (\sigma\cdot\mathbf{F})\right]\begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} \tag{31}$$

where σ and $\vec{1}$ are the Pauli spin matrices and the 2×2 identity matrix respectively. By using the 3-vector identity $(\sigma\cdot\mathbf{V})((\sigma\cdot\mathbf{W}) = \mathbf{V}\cdot\mathbf{W} + i\sigma\cdot(\mathbf{V}\times\mathbf{W})$ together with the equation for energy equilibrium in the presence of additional fields, Eq. (30), and the methods of Fourier decomposition of modulation waves as presented in Section III above, straightforward but somewhat tedious manipulations yield the Pauli version of the Schrödinger Equation for Fermions

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(\sigma\cdot(i\hbar\nabla - \frac{e\mathbf{A}}{\mathbf{c}})\right)^2\psi + \phi\psi, \tag{32}$$

where $\mathbf{A}$ is the electromagnetic vector potential associated with ϕ Eq. (30).

Except for anomalies to be taken into account by Second Quantization or SED and caused by *Zitterbewegung*, as is well known, Eq. (32) gives the correct magnetic moment for the electron. (In spite of prejudicial nomenclature, herein spin matrices are actually just the vehicles for attaching the algebraic structure implicit in what, in the language of Differential Geometry, is a two dimensional bundle attached to each independent tangent fiber at every point of a three dimensional manifold. A two-dimensional structure is needed to take account of the two polarization states.) Bosons, as usual, are comprised of multiple Fermions. The imagery suggested by these manipulations implies that spin magnetic moments result from currents attendant to *Zitterbewegung* motion. In the absence of magnetic fields, particles equilibrate randomly with both polarization modes in the background. An external magnetic field, however, will lift the degeneracy by causing those particles whose instantaneous random motion is, when projected on the oriented plane perpendicular to the axis of the magnetic field, positive rotation, to precess so as to tend to couple preferentially to, and thereafter maintain energetic equilibrium with a particular circular polarization state of that background mode propagating parallel to the axis of the magnetic field. Those particles with the opposite sense of rotation will likewise couple with the opposite polarization state. Thus, the binary result of Stern Gerlach experiments with respect to an arbitrary axis can be understood as a consequence of the fact that the magnetic field itself is responsible for inducing the axis of precession with respect to which the background mode is decomposable as the sum of two circular polarization states, each of which is then independently available for energetic equilibrium.

6. BELL'S THEOREM

In analysis of the foundations of QM, Bell's Theorem has taken a central position because it provides an experimentally testable distinction between QM and possible alternate formulations involving hidden variables. Thus, it is essential to analyze its significance from the vantage of Eikonal Mechanics.

Recall that the proof of Bell's Theorem is formulated in terms of the disintegration of a spin free boson into two fermions of opposite spin. This event is particularly significant because it is also a realizable demonstration of a situation originally analyzed by Einstein, Padolski and Rosen (EPR) to demonstrate the incomplete character of QM. They noted, in effect, that according to conventional QM, prior to observation, the wave function for each derivative particle or "daughter" comprises an ambiguous mixed state which is collapsed upon measurement to a specific, observed value. Thus, when one daughter is measured, by virtue of spin conservation, the wave function for her sibling is fixed instantly. If QM is complete and fundamental, then the objective state of each daughter must also be ambiguous prior to a measurement of either daughter and this, in turn, implies that the objective states of both daughters, at space-like separation from each other, are altered; i.e., rendered specific, by a spin measurement made on only one.

EPR then argued, as is well known, that QM must be incomplete because, by admitting ambiguous mixed states that require instantaneous transmittal of information from one particle to its twin when a measurement is made on only one, it violates Special Relativity. In concept, the problem with the EPR event is exactly the same as that illustrated by a particle beam diffracting through an orifice as discussed above in Section IV.

The proof of Bell's Theorem consists in showing that QM expectation values for the correlation of spin measurements, P, made with respect to arbitrary axis denoted by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in the realization of the decay version of the EPR event can not satisfy the inequality

$$1 + P(\mathbf{b}, \mathbf{c}) \geq |P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c})|, \tag{33}$$

which, as Bell found, should obtain for all such correlations of local, dichotomic variables.[1] The connection to the physics of the EPR event in the proof of this Theorem is established by constraints imposed on the explicit form of the correlations P, to wit:

$$P(\mathbf{a},\mathbf{b}) \equiv \int d\lambda\, \rho(\lambda) A(\mathbf{a},\lambda) B(\mathbf{b},\lambda), \tag{34}$$

where λ represents possible hidden variables, $\rho(\lambda)$ a probability distribution over the variables λ, and A and B are the results of measurements of the spin of the daughters. Now, to duplicate QM results, A and B must be dichotomic; i.e., $A = \pm 1$, $B = \pm 1$. As the measuring instruments; e.g., Stern-Gerlach magnets, are capable of registering a continuum of values, this constraint fixes A and B as expressions of the properties of the derivative particles. To be "local" requires that A does not depend on $\mathbf{b}$ and B does not depend on $\mathbf{a}$. Given these constraints, Eq. (33) follows directly. The fact that the QM expectations calculated for the EPR event do not satisfy Eq. (33) is taken to mean that theories with hidden variables can not duplicate the structure of QM. Furthermore, experiments seem to verify QM rather than Eq. (33).

However, the relevance of the assumption that a spin measurement of either daughter is independent of the other can be questioned. This assumption is reasonable if it is assumed that each daughter departs the scene of the disintegration in an ambiguous state so that either dichotomic outcome is possible before the act of measurement itself breaks the ambiguity and precipitates a distinct value. If, on the other hand, each daughter in fact departs the scene of the disintegration with properties that unambiguously determine the value of the spin which will be subsequently measured with respect to an arbitrary axis, then instantaneous information transmission across space-like separations is not required to account for the result. This is exactly the situation as envisioned from the vantage of Eikonal Mechanics for which wave functions do not constitute the essence of material existence, rather they are just symbolic expressions for computing densities in the phase space of ray-like trajectories for what are distinct material particles in the classical sense.

Moreover, the proof of Bell's Theorem explicitly exploits the assumption that spin correlations are to be computed with the dichotomic data resulting from measurements. While seemingly reasonable, this appears not to be consistent with the calculation carried out using QM rules. The QM result,

$$< \sigma_1 \cdot \mathbf{a}, \sigma_2 \cdot \mathbf{b} >= -\mathbf{a} \cdot \mathbf{b}, \tag{35}$$

is obtained if A and B are taken as projections on the axes of the measuring devices and dichotomicity imposed, so to speak, to interpret results only *after* (if at all) the correlation is computed. This may reflect the fact that in orthodox QM, measurement is subordinated to an appendix of the formalism.[11]

7. Conclusion

Besides their obvious utility for philosophical analysis and pedagogical aids, interpretations for physics theories ideally serve to provide coherent mental images and mutually

[11] A recent publication, [16], criticized the interpretation of experiments exploiting atomic cascade events to generate oppositely polarized photons to test Bell's Theorem. The essence of the criticism is that properly accounting for the spherical symmetry of photons so generated invalidates the conclusion that such experiments support QM vice possible hidden variable theories. Even without this criticism, however, a substantiation of Bell's result derived from experiments on "photons" would be less satisfying that an experiment with entities classically regarded as particles. Because of the existence of phenomena such as wave packets, needles, and solitons for waves, particle-like behavior by waves (especially interacting with *particulate* detectors) is less counterintuitive than wave-like behavior, in particular diffraction, by Gibbsian ensembles of individual particles.

consistent vocabulary in order to facilitate thinking about and describing physical phenomena. Although they are not unequivocally required to formulate or use operationally correct, quantitative physics theories—as the history of QM has demonstrated abundantly—images and evocative vocabulary nevertheless often provide guidance for modeling particular phenomena and inspiration for new experimental, theoretical or calculation techniques. The acceptance of a particular interpretation, therefore, is determined in part by the success of specific models and calculations which it prompted. Nevertheless, more is usually desired; a physical model should have universality and render all phenomena within its presumed domain intuitively comprehensible.

Perhaps the central challenge in this regard for an interpretation for QM based on Eikonal Mechanics is to provide visualization or intuitive justification for the Pauli exclusion principle, in particular as applied to atomic structure. For a start, consider a pair of electrons in orbit about a nucleus where each electron assumes a spin magnetic moment by virtue of exposure to the apparent magnetic field generated by its orbital motion. The image of such an interacting couplet seeking an energy minimum in antialignment, seems natural. Hence the rule that electrons tend to form pairs with spin "up and down" for each set of orbital quantum numbers. For increasing numbers of electrons, which repel each other and shield the nucleus, the increase in total energy for the electron collection and decrease of individual binding energies also seem intuitively natural.

The image of electrons segregated and stacked by energy eigenvalue as neatly as implied by the exclusion principle, even if only into zones, is, however, less natural. But, literal compliance with this principle may not be necessary. All that is required to account for observed spectra, is that emission and absorption occur as if electron orbits were on the average neatly ordered and singly populated by eigenvalue. This could well occur in a macroscopic sample if the combined effect of the background and boundary conditions is that the only radiation to escape from the sample (or to fail to penetrate through it in the case of absorption) is that which does not fit the equilibrium interplay between background and atom. For electrons orbiting a point charge, spherical symmetry is the boundary condition which determines the eigenfunction space and eigenvalues; i.e., it structures the energy levels of nonequilibrium emission and adsorption. Thus, in the steady state, individual electrons might be envisioned to be constantly migrating through various eigenzones or exchanging orbits among themselves such that on the average, energy is exchanged only with the background. Thus, in this imagery, statistical characteristics are vested in the motion of individual particles rather than in the population distribution among states each of which is envisioned to comprise ensembles of fixed energy level systems locked into specific orbits. This imagery, in fact, seems uniquely consistent with the previous observation that physically interpretable densities seem to result only from mixed states with all eigenfunctions present. Still, the question of just how to determine specific weighting factors for eigenfunctions comprising physically realizable mixed states is open but not obviously unresolvable.

The problems with fundamental QM addressed herein are overwhelmingly verbal. QM as a quantitative codification of physical phenomena has been verified virtually beyond question. What is *said* about reality based on QM, however, is at least semantically inconsistent; one consequence of which is that QM defies visualization. Eikonal Mechanics, as outlined above, proposes vocabulary with intuitive images to portray reality in a new way on the basis of QM without changes in its current mathematical (symbolic) formulation. New or reformulated calculations simply to replace customary QM techniques, are in general not required; the primary value of any new paradigm at this level is philosophical and pedagogical. Hopefully, however, the intuitive concepts underlying Eikonal Mechanics, whether or not they ultimately withstand critical analysis of their use to interpret QM, can be exploited or extended in order to contribute to the explanation of still ill understood physical phenomena or the discovery of utterly new ones.

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