

# What's Wrong with this Rebuttal?\*

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A recent rebuttal to criticism of Bell's analysis is shown to be defective by fault of failure to consider all hypothetical conditions input into the derivation of Bell Inequalities.

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## I. THE DISPUTE

On the pages of *Found. Phys.* [1], under the title: "What's wrong with this Criticism?", N. D. Mermin rebutted criticism of Bell's Theorem and analysis by K. Hess and W. Philipp [2]. The latter authors hold that Bell's analysis of EPR inspired experiments testing the contention that Quantum Mechanics could be 'completed,' i.e., rendered a deterministic theory instead of only a probabilistic one, is fatally flawed. They argue that Bell failed to consider time variable correlations and subsequently failed to find structure permitting a local realistic interpretation of the results of EPR experiments.

While this is the context of the larger dispute, the actual point of contention for Mermin was a much narrower, although equally potent, sub-argument that Hess and Philipp mentioned along the way. It is essentially this: For technical reasons, data taken in feasible experiments cannot meet all requirements in the input into derivations of Bell Inequalities. This is an old observation, although its many renditions are not always easily recognized as being the same issue.

To crystallize the crucial points, recall that Bell's analysis ostensibly proves that for all local realistic theories a certain expression, in Mermin's notation denoted by  $\Gamma$ , satisfies:

$$|\Gamma| \leq 2. \quad (1)$$

Now  $\Gamma$ , as is easily seen from its derivation (which is very well known and will not be reiterated here; see: [3]), for EPR-type experiments is comprised of a particular sum of terms, where each one is the sum of the products of the outcomes in each arm for a given combination of the polarizer settings (or Stern-Gerlach field directions if the experimental objects are electrons). In so far as two different settings are considered for each arm, there are then four combinations so that  $\Gamma$  can be written as:

$$\Gamma = \frac{1}{N} \sum_j [a_a(j)b_b(j) + a_a(j)b_c(j) + a_d(j)b_b(j) - a_d(j)b_c(j)]. \quad (2)$$

It is just here that Hess and Philipp, as have others before them<sup>1</sup>, raise an objection. It is that for the different settings of the polarizers, that is, for the various of the four combinations

or each term alone, one has essentially four different experiments and that it is not legitimate to mix the data and analyze it as if it came from a single experiment.

## II. MERMIN'S CONTENTION

In response, Mermin's rebuttal consists of asserting that Eq. (2) can also be written:

$$\Gamma = (1/N_{ab}) \sum_{j \in X_{ab}} a_a(j)b_b(j) + (1/N_{ac}) \sum_{j \in X_{ac}} a_a(j)b_c(j) + (1/N_{db}) \sum_{j \in X_{db}} a_d(j)b_b(j) - (1/N_{dc}) \sum_{j \in X_{dc}} a_d(j)b_c(j), \quad (3)$$

and that, in a sufficiently long run, that is large enough  $N$ , "each of the four choices for  $xy$  the  $N_{xy}$  indices  $j$  appearing in  $X_{xy}$  constitute a random sample of the full set  $j = 1 \dots N$ , each  $j$  having the probability  $1/4$  of appearing in  $X_{xy}$ . So by standard sampling theory ..."

## III. A LACUNA IN THE REBUTTAL

The point of this rebuttal is necessary but not sufficient, however. The data collected in four equal length sub-runs, one each for each polarizer combination, must satisfy *all* the hypothetical inputs into the derivation of a Bell Inequality, e.g., Eq. (1). That is, the factor sequence  $a_a(j)$  in the first term of Eq. (3) must be identical with the same factor sequence in the second term. Specifically, this means that if in, e.g.,  $a_a(j)$ , '+1' appears  $k$  times, then not only must '+1' appear  $k$  times in  $a_a(j)$  in the second term (that it does indeed, is Mermin's point), but additionally, and neglected in Mermin's rebuttal, the pattern of switches back and forth among the '+1' and '-1's' must occur at the same locations along the sequence. This is, it turns out, necessary because it is an implicit assumption in the derivation of Bell Inequalities (regardless of the detailed logic and discriminator. i.e.,  $\Gamma$ , for any particular derivation of an inequality). It results from steps in the derivation employing factoring among the factor sequences. In random runs of the experiment, it can be expected that the ratio of '+1's' to '-1's' will be equal for sufficiently long subsequences with fixed polarizer settings, just as Mermin surmises; but their pattern of occurrence will not repeat—indeed, this is what is meant by being "random" in this case. Moreover, the correlation existing between the outcomes on both sides of an EPR experiment (one of Bell's hypothetical inputs is that the pairs are correlated) implies that

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<sup>1</sup> Recent studies by Adenier [4] and Sica [5] explicitly make the same point. Apparently, de la Peña, Cetto and Brody [6] were first to recognize the significance of the relevant structure.

the pattern of matches between the two sides in general is not fully random—it follows an extension of Malus’ Law, in fact.

We shall not go into all the details here as this matter has been explicated in detail elsewhere [7; 8]. But in conclusion, we see that the “very simple way past [Hess’ and Philipp’s] objection” does not lead to the conclusion that Mermin wishes to support.

#### IV. THE STRUCTURE ILLUSTRATED

To further illustrate the logical impossibility of relating data taken from feasible experiments with sequences required for Eq. (3), consider the following.

First, let us simplify notation as follows:

$$(1/N_{ab}) \sum_{j \in X_{ab}} a_a(j)b_b(j) = \langle a_1 b_1 \rangle, \quad (4)$$

so that Eq. (3) becomes

$$\Gamma = \langle a_1 b_1 \rangle + \langle a_2 c_2 \rangle + \langle d_3 b_3 \rangle - \langle d_4 c_4 \rangle. \quad (5)$$

Now, if  $a_1 \equiv a_2$ ,  $b_1 \equiv b_3$ ,  $c_2 \equiv c_4$  and  $d_3 \equiv d_4$  (here identity means both that the sequences have the same quantity of +1’s and the same pattern of switches between +1’s and –1’s), then Eq. (5) can be written:

$$\Gamma = \langle a(b+c) \rangle + \langle d(b-c) \rangle, \quad (6)$$

so that in the sum over all  $j$  the absolute value of either one term or the other is 2 while its partner is 0 alternatively, such that the average satisfies Eq. (1).<sup>2</sup> This in turn leads one to imagine that if  $a_2$  is re-sorted to have the identical pattern of switches as  $a_1$ , then perhaps the data taken from a feasible experiment can be re-sorted so as to satisfy *all* the hypothetical inputs into the derivations of Bell Inequalities.

This turns out not to be possible, however. Suppose the second term in Eq. (5) is re-sorted so that  $\tilde{a}_2$  (the tilde indicates the re-sorted variant) is essentially identical with  $a_1$ , then the second re-sorted term becomes  $\langle a_1 \tilde{c}_2 \rangle$ . This new ordering must be cascaded to the fourth term to become  $\langle \tilde{d}_4 \tilde{c}_2 \rangle$ , and finally to the third term:  $\langle \tilde{d}_4 \tilde{b}_3 \rangle$ . The final result is that Eq. (6) becomes:

$$\Gamma = \langle a_1(\tilde{b}_1 + \tilde{c}_2) \rangle + \langle \tilde{d}_4(\tilde{b}_3 - \tilde{c}_2) \rangle. \quad (7)$$

In order for  $|\Gamma|$  in this form always to be less than or equal to 2, it is necessary that  $\tilde{b}_1 \equiv \tilde{b}_3$ ; but here, separate independent re-sortings are involved; they would be essentially identical

<sup>2</sup> The termwise factorizations leading to Eq. (6) are parallel to those in the derivation of a Bell Inequality; in both cases factorization requires identically ordered sequences in order to be executable.

<sup>3</sup> The arguments made by Hess and Philipp and by Jaynes are to be distinguished from others usually designated ‘loopholes.’ The latter concern

only under the most improbable circumstances. The circuit cannot be closed; there is no reason from the physics of the situation, classical by explicit supposition, why these separate re-sortings must be the same. As a consequence, we see, data from feasible experiments cannot be re-sorted to comply with all the hypothetical inputs into derivations of Bell inequalities, and therefore, they cannot be used in Eq. (1) to discriminate between local-realistic and other theories.

#### V. QUO VADIMUS?

These ‘re-sorting’ considerations expose exactly the structure that torpedoes Mermin’s rebuttal of Hess’ & Philipp’s criticism of Bell’s analysis. At the same time, however, they cannot be turned around to support Hess’ & Philipp’s particular analysis critical of Bell’s arguments. Indeed, this writer argues elsewhere [7], that time variable correlations are not necessary to invalidate the logic of Bell’s approach. A much more incisive line of fatal critical analysis was founded by Edwin Jaynes [9]<sup>3</sup>. He, apparently, was first to observe that the probabilistic nature of EPR and GHZ experiments requires Bayes’ formula for *conditional* probabilities, which, if introduced into Bell-type analysis, prevents the derivation altogether of inequalities for the very same structural reasons reviewed above and thus far overlooked by proponents of Bell’s “Theorem.” (For details see, e.g., [10].)

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only features rendering experiments ambiguous or invalid; the former concern fundamental defects that pertain even when detectors are 100% efficient, etc.