Appeared in: *Scientia (Milano)*, **109,** 111-120 (1974).

ON THE IMAGINABLE CONTENT OF DE BROGLIE WAVES

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ABSTRACT. De Broglie's concept of a particle as a singularity in a physical wave of very small amplitude is given imaginable content with the argument that the electromagnetic background used to classically derive the Planck black-body radiation spectrum, provides the wave to which a classical charged particle couples to construct the unit of wave and singularity imagined by de Broglie.

"Mais je ne suis pas sûr que dans un univers où tous les phénomènes seraient régis par un schéma mathématiquement cohérent, mais dépourvu de contneu imagé, l'esprit humain serait pleinement satisfait." —René Thom[1]

1. INTRODUCTION.

In recent times no one has brought deeper doubt and more disciplined criticism to what has become the orthodox understanding of the quantum mechanical concept of duality, than the originator of the concept, de Broglie. He has accomplished this critique by his advocacy of an alternative theory, designated as the "theory of the double solution"[2], which seeks to interpret the wave character of particles in a fashion respecting the integrity of space and time and therefore our classical intuition.

Such a reinterpretation in needed not in order to satisfy human vanity by respecting intuition, but in order to resolve the paradox which perplexes the interpretation of quantum theory. This paradox [3] discussed first by Einstein, can be stated briefly as follows: Suppose a plane wave solution to Schrödinger's Equation, representing a particle in a beam, impinges on a plane detector such that the wave front is parallel to the detector. Eventually one observes that the particle impacts the detector at a distinct location. At the very instant of impact, the entire wave must "collapse" to the point of impact, and must do so faster than the speed of light, which implies that the wave must not be considered a "physical" wave, but instead must be regarded as an "informational" device. On the other hand, particle beams are diffracted at slits, which implies that particles are moved in their individual trajectories apparently through the mediation of the wave. The fact that the wave mediates in physical events means that the wave must be a "physical" entity. Thus, the wave must be both unphysical and physical, paradoxically.

According to the imagery of de Broglie's double solution theory, particle duality is a manifestation of the physical nature of a particle (i.e., something whose classical limit is a particle) as "a very small region of high-energy concentration as a kind of moving singularity."[4] In other words, the wave character of a particle is due to the wave in which the particle resides as a singularity.

On the other hand, wave duality can be accounted for consistently with the classical postulate that there exists fluctuating electromagnetic radiation with a Lorentz invariant energy spectral density. Lorentz invariance insures that no frame is preferred, and implies that the energy spectral density is of the form:

$$E(\omega) = \text{const.} \times \omega, \tag{1.1}$$

where the constant is set equal to $\hbar/2$ phenomenologically. [5]

Now Theimer has shown in a beautifully simple way how this background spectrum leads to the Planck black-body spectrum without a "quantum" hypothesis. [6] He has also derived the following expression for the fluctuations of thermal (blackbody) radiation energy density:

$$\frac{<(\delta\rho_T)^2>}{<\rho_T>^2} = 1 + \frac{2<\rho_B>}{<\rho_T>}, \tag{1.2}$$

where ρ_T and ρ_B are the energy densities of the thermal and background fields respectively, and $\delta\rho$ is the fluctuation magnitude. The significance of this expression is that the first term on the right side is characteristic of classical waves, the second term of a classical system of particles. The sum may be said to characterise a "dualistic" entity; however, Theimer's derivation shows that this dualism need not be predicated on a quantum hypothesis, but can be understood without violating the identity of classical waves. Furthermore, this demonstration is sufficient to completely preclude the need for a "quantum" or photon hypothesis, because it arises from the requirement to explain "photon statistics," which Eq. (1.2) satisfies.

It is the point of this comment to propose the argument that the background hypothesis can also be used to furnish imaginable content of a less ethereal form to the basic idea advocated by de Broglie with his theory of the double solution. [7]. In particular, it is proposed herein, that the background provides the wave to which a charged particle may be said to couple to construct the composite unit of wave and singularity imagined by de Broglie.

2. The Fundamental Ansatz: Energetic Equilibrium with the Background

The fundamental Ansatz upon which the imaginable content for particle duality is built is the claim that any particle with charge structure will obtain, when considered for suitably long periods of time, energetic equilibrium with the mode of the background to which the particle charge structure predisposes it to couple. Alternately, this may be expressed by saying the the particle "tunes" to a particular mode from the background and establishes energetic equilibrium with background signals in this mode.

As an example, consider a dipole consisting of two oppositely charged particles held apart by a spring such that the resonant frequency of the system is ω_0. The consequence of the above Ansatz is that the time average kinetic energy of the oscillator, written usually as $mA^2\omega_0^2/2$, is equal to the time average energy in the fluctuating electromagnetic background mode ω_0, namely $E(\omega_0)$; i.e.,

$$mA^2\omega_0^2/2 = E(\omega_0). \tag{2.1}$$

This expression is set out here as an hypothesis, with some disregard for the details because the thrust of the analysis presented here is directed toward an understanding of particle duality, and not of the structure of the particle itself. In fact, however, Abraham and Becker [8] have shown that Eq. (2.1) is rigorous to first order; furthermore, Surdin [9] has shown that the second order approximation leads to a "Lamb" type correction. These refinements, however, are not germane to the subsequent basic argument regarding particle duality and the resolution of its concomitant paradox.

It now remains only to interpret Eq. (2.1) in terms of observable or known quantities. To begin, observe that the energy of the oscillating dipole is indistinguishable from the rest energy of the system to an observer who perceives only a massive unchanging system; i.e., an observer unaware of the dipole interaction with the background, who would write:

$$mA^2\omega_0^2/2 = m_0c^2, \tag{2.2}$$

where m_0 is, as it were, a "renormalised" mass greater than the sum of the rest masses of the charged particles comprising the dipole. The difference in mass is due, of course, to the relativistic oscillation of the particles. As it was shown above the energy spectral density which is Lorentz invariant is given by the equation such that the energy per normal mode is:

$$E(\omega_0) = \hbar\omega_0/2. \tag{2.3}$$

Eq. (2.1) can, therefore, be written:

$$m_0c^2 = \hbar\omega_0/2. \tag{2.4}$$

Now, it is of interest and consequence to investigate the composition of the right hand side of Eq. (2.4) in greater detail. Implicit in the above development is the understanding that Eq. (2.4) is valid as written in the rest frame of the dipole, where it is meant to express the fact that the average energy of the system equals the average energy of the mode ω_0. The question becomes, therefore, how to express the concept of energetic equilibrium in an arbitrary frame other that the rest frame of the particle.

In order to resolve this question, a means must be found of transforming the average energy of the background mode to which the particle is tuned. A problem arises in that the time average equilibrium is established with regard to the unit of time of the particle's rest frame. This unit of time is not frame independent so that what has been computed in the particle's rest frame must be recomputed with respect to the appropriate time unit in an equivalent frame. Therefore, at once, it is seen that the averages can not be computed then transformed, rather the transform must be executed first, then the averages computed.

Time average energetic equilibrium between a dipole and isotropic signals in a particular mode in the rest frame of the dipole also implies time average momentum equilibrium since the particle's momentum is zero in this frame and the time average momentum transport of isotropic radiation is also zero. If this statement is physically meaningful, it follows that it must be frame independent; therefore, it follows that time average momentum equilibrium must also hold in each frame when computed with respect to the time unit of that frame.

There now remains only one aspect to the question of how to transform the time average energy equilibrium statement to an equivalent frame and that aspect is: how are the energy and momentum of the signals of the background expressed? It is precisely with respect to this question that the energy spectrum proves most auspicious. Consider the general expressions for the energy and momentum of plane waves in free space, to wit:

$$E = \frac{1}{8\pi}\int |E_0(\omega)|^2 d^3x, \tag{2.5}$$

and

$$P = \frac{1}{8\pi c}\int |E_0(\omega)|^2 d^3x. \tag{2.6}$$

Now, by virtue of the Lorentz invariant energy spectral density, it follows that

$$\frac{1}{8\pi}\int |E_0(\omega)|^2 d^3x = \frac{1}{4}\hbar\omega, \tag{2.7}$$

so that for the average of the background signals, the energy may be expressed as:

$$E = \frac{1}{4}\hbar\omega; \tag{2.8}$$

and momentum as:

$$P = \frac{1}{4}\hbar\vec{k}. \tag{2.9}$$

These expression refer, of course, to average or characteristic signals. In the frame of the particles there are two such signals for each direction in space corresponding to $\pm\vec{k}$. Therefore the total time average of the energy for each dimension in space is in fact:

$$E = < \left(\frac{\hbar\omega_+}{4} + \frac{\hbar\omega_-}{4}\right) >, \tag{2.10}$$

where $<>$ denotes time average of these two signals, so that

$$E = \frac{1}{2}\hbar\omega. \tag{2.11}$$

On the other hand, the total time average of the momentum:

$$P = < \frac{1}{4}\hbar\vec{k} - \frac{1}{4}\hbar\vec{k} >, \tag{2.12}$$

is clearly zero in the rest frame of the dipole. This result is obtained because the fluctuating background signals may be said to be one-half the time represented by a plane wave moving to the left, say, and other half moving to the right, so that on the time average there is no motion.

If now, however, the $\omega_\pm$ and $\vec{k}_\pm$ are transformed to another inertial frame and then the averages are computed, the following expressions are obtained for the energy:

$$\begin{gathered} E' = < E'_+ + E'_- >, \\ E' = \tfrac{1}{4}\hbar < \gamma\Big((\omega_0 + c\beta\vec{k}) + (\omega_0 - c\beta\vec{k})\Big), \\ E' = \tfrac{1}{2}\hbar\gamma\omega_0; \end{gathered} \tag{2.13}$$

and for the momentum:

$$P' = \frac{1}{2}\hbar\gamma\beta|\vec{k}_0|, \tag{2.14}$$

where a factor of $1/2$ arises with regard to momentum as an expression of the fact that each sign occurs one-half of the time; i.e., the time average of two equally probable vectors is heir barycentre.

Now, by transforming the energy of the particle and equating momentum and energy parts to the corresponding parts for the background, yields:

$$\gamma m_0 c^2 = \frac{\hbar\gamma\omega_0}{2}; \tag{2.15}$$

and

$$\vec{p} = \frac{\hbar\vec{\beta}|k_0|}{4}. \tag{2.16}$$

In a nonrelativistic approximation, the energy terms expanded give:

$$m_0 c^2(1 + \beta^2/2) = \hbar\omega_0/2(1 + \beta^2/2), \tag{2.17}$$

or

$$E'_v = m_0 \frac{v^2}{2} = \hbar\omega_v, \tag{2.18}$$

where

$$\omega_v = \beta^2 \omega_0 / 4, \tag{2.19}$$

so that

$$\vec{k}_v = \frac{\omega_v}{v} = \frac{\beta k_0}{4}, \tag{2.20}$$

is in agreement with Eq. (2.16) when $\gamma \to 1$. Eqs. (2.18) and (2.20) are recognised as the classical "de Broglie relations," so Eqs (2.15) and (2.16) can be identified as their relativistic generalisations.

Physically, the implication is that, to an observer in a frame translating with respect to the rest frame of a particle with charge structure, the average or effective properties of to the background electromagnetic signals with which the particle is in equilibrium can be characterised as a wave described by the well known de Broglie relations. It is this fact which gives imaginable content to the basic concept of de Broglie's theory of the double solution and which is in complete accord with notions familiar from classical physics. Furthermore, since this "average wave," as it were, is in fact the composition of classical electromagnetic waves, its response to obstacles in the environment is governed by the principles of electromagnetism. As an illustration, let us consider the pedagogical exercise, a particle passing through a double slit apparatus. The results of this experiment can be understood as follows: The particle tunes to an average effective signal, which in the frame of the slit apparatus is described by a wave impinging on the apparatus whose wave vector is that for the "de Broglie wave" of the particle. The effect of the slit on this wave is, according to the principles of wave theory, to establish a diffraction pattern on the back side of the slit apparatus. This diffraction pattern represents a pattern in the energy of the signals in the background to which the particle is tuned, a pattern which gives rise to spatial gradients of energy, of forces, which tend to coax the particle into the troughs in the pattern, much as dust settles on the nodes of a vibrating drum head. The resolution of the philosophical dilemma posed by Einstein, Schrödinger and others, is an equally straightforward application of the understanding afforded by this viewpoint.

Consider, for example, the paradox first proposed by Einstein. If a free particle impinges perpendicularly on a screen punctured by a infinitesimally small hole, then, according to the principles of Quantum Theory, the wave function of the particle beam should emerge from the hole having been refracted into a spherical wave. Furthermore, if a perfectly spherical detector is centred on the hole, then an instant before the particle impacts the detector, the wave function for the particle will be finite over the entire surface of the detector. However, immediately upon impact the wave function must collapse to a zero value everywhere except at the precise location of the impact. This collapse must occur faster than the speed of light, which implies that the wave function cannot be regarded as a physical entity; but on the other hand, the wave must also mediate in the refraction, and must, therefore be physical.

The resolution of this paradox afforded by the background concept is direct and simple. The particles of an ensemble are deflected in passing through the hole by the agency of the fluctuating background so that the informational character of the wave function is freed of the preternatural task to reflect the essentially statistical nature of the fluctuations as they affect the sample paths of the ensemble.

3. Point Particles Exhibit the Mass Renormalisation Divergence of Quantum Theory

The argument presented above appears to be inadequate for the understanding of point charges because they have no preferred mode of interaction with electromagnetic fields. This inadequacy is as much apparent as real. It is only apparent in the following sense. Point particles may be regarded as charge structures which interact with electromagnetic fields in a multiplicity of of modes, in this case every mode. Therefore, Eq. (2.15) may be written:

$$m_0c^2 = \frac{1}{2}\hbar\int_0^\infty \omega f(\omega) d\omega, \tag{3.1}$$

where $f(\omega)$ is an admittance function such that the integration over all modes gives a convergent result which serves as an equivalent ω_e. Following from the fact that all equations regarding de Broglie relations are linear in ω, it is permissable to replace ω_0 with ω_e everywhere. In other words, the linearity of the de Broglie expressions implies that multiple interaction with the background will not lead to different results or conclusions.

The inadequacy is real, however, in that the admittance function, $f(\omega)$ has no rationalisation within the context of these considerations. This fault is, however, faithful to quantum theory where precisely this problem arises in mass renormalisation calculations and is resolved only through the ad-hoc imposition of cut-offs. [10] With regard to this difficulty, this author finds two possible resolutions suggested by the concept of background radiation. One, the radiation reaction to accelerations caused by interactions with the background may lead to a suitable acceptance function, $f(\omega)$. Two, the background may be Lorentz invariant only to first order, while in fact being convergent. In any case, any means whatsoever that would lead to an acceptance function is adequate for the conclusions obtained regarding duality.

4. Spin is a manifestation of Polarisation of the Background

A fundamental aspect of electromagnetic radiation is its two state character manifested as polarisation phenomena. Since the background signals with which a charged particle are in equilibrium are electromagnetic, the consequences of polarisation must be included in the fundamental Ansatz employed above. This can be most effectively accomplished by elaborating the Ansatz with the stipulation that the helicity of the particle and the 'effective' de Broglie wave be the same. Symbolically, in terms of four-vectors:

$$[\sigma\cdot\vec{p}, \Pi\gamma m_0c^2] = \hbar[\sigma\cdot\vec{\beta}\gamma|k_0|/4, \Pi\gamma\omega_0/2], \tag{4.1}$$

where σ in this equation represents a Pauli spin vector and Π is the 2×2 identity matrix, which in this context, are nothing more than the formalistic devices through which the two states of polarisation are taken into account. This "contenu imagé" of this stipulation is the following: a point charged particle can tune to either of two waves, which may be thought of as clock- or counter-clock-wise polarised in an arbitrary frame. To an observer in this arbitrary frame, the particle will appear to be driven in either right or left hand helical motion of the same sense as the effective de Broglie wave to which the particle is tuned. Of course, this naive imagery is overstated and in fact unnecessary. A more realistic image would be that of an ensemble of identical particles in interaction with a randomly polarised signal of the background whose statistical properties (expectations) are identical to the ideal situation in which particle execute perfect helical motion.

This model of electron spin is by no means unique to this author. Smit [11] in his book on ferrites uses the model to comprehend certain phenomena in these metals. By employing a novel formulation of tensor analysis, called “space-time algebra,” Hestenes [12] has shown that what is here presented as an hypothesis, is in fact, a consistent interpretation for the Dirac equation.

5. CONCLUSION

There appears to be no limit to the imaginary constructions that result from compounding considerations of the above sort. For example, the Pauli Exclusion Principle might be rendered as the statement that two point charges in proximity will tend to equilibrate with oppositely polarised background waves, as each particle being driven in circular motion is an effective magnetic dipole and magnetic dipoles energetically prefer to antialign. As a second example, a massive Boson can be though of as a bound combination of fermions, which, as a unit, equilibrate with the background scalar wave composed of the sum of two polarised background waves.

To be sure, the plausibility of the viewpoint stated herein is damaged by the observation that the energy of the background diverges. Although others have suggested possible remedies, we confine our remarks to the point that the value of this or any hypothesis must in the end be judged both by its internal consistency and by is usefulness in comprehending the patterns of nature; no one is compelled to accept it, but those who are troubled by the paradoxes in the interpretation of quantum theory may be able to find merit in this alternative, especially since it has imaginable content.

REFERENCES

[1] R. THOM, *Stabilité structurelle et morphogénèse,* W. A. Benjamine, 1972
[2] L. DEBROGLIE, Found. Phys. **1**, 5 (1970).
[3] L. BALLENTINE, Am. J. Phys. **40**, 1763 (1972).
[4] Ref. 2.
[5] T. H. BOYER, Phys. Rev. **174,** 1631 (1968); **174,** 1764 (1968).
[6] O. THEIMER, PHYS. REV. D, **4**, 1597 (1971).
[7] A. F. KRACKLAUER and R. E. COLLINS, Texas J. Sci. **25,** 193 (1974).
[8] A. ABRAHAM and B. BECKER, *Theorie der Elektrizität,* B. G. Teubner Verlag, 1923.
[9] P. BRAFFORD, M. SURDIN and A. TARONI, Compte Rendue **261**, 4339 (1965).
[10] E. G. HARRIS, *A Pedestrian Approach to Quantum Field Theory,* Wiley, 1972.
[11] J. SMIT, *Ferrites,* Wiley, 1959.
[12] D. HESTENES, J. Math. Phys. **14**, 893 (1973).

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