Probability and Quantum Paradigms: the Interplay*

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Since the introduction of Born's interpretation of quantum wave functions as yielding the probability density of presence, Quantum Theory and Probability have lived in a troubled symbiosis. Problems arise with this interpretation because quantum probabilities exhibit features alien to usual probabilities, namely non Boolean structure and non positive-definite phase space probability densities. This has inspired research into both elaborate formulations of Probability Theory and alternate interpretations for wave functions. Herein the latter tactic is taken and a suggested variant interpretation of wave functions based on photo detection physics proposed, and some empirical consequences are considered. Although incomplete in a few details, this variant is appealing in its reliance on well tested concepts and technology.

I. QUANTUM PROBABILITY: PEDIGREE AND PATHOLOGIES

Following Born's interpretation of the square of the modulus of quantum wave functions as a probability, the two great logical structures of Quantum Mechanics and Probability Theory have been in a troubled symbiosis. The source of dissidence is that the probabilities arising from this interpretation, although seeming quite reasonably behaved for the most part, still suffer features unknown in ordinary probability theory. Such features include non Boolean algebraic properties, i.e., the underlying wave functions for independent probabilities add, not the resulting probabilities themselves, and some are frequently not positive-definite.

In reaction to this situation, there seems to be two polar reactions as efforts to understand these features. One is, so to speak, to fix the quantum or physics interpretation and vary the probability theory. The other is the complementary tact, that is, to fix the employed probability theory and modify Born's physical interpretation. Over the decades since the discovery of Quantum Theory, many researchers have taken one or the tactic, or even a combination, in efforts to demystify the interpretation of Quantum Mechanics.

Perhaps the most systematic and far reaching study in the spirit of varying or modifying probability as applied to Quantum Mechanics was made by Khrennikov as published in his book: "The Interpretations of Probability."(1) In that work he argues that physicists implicitly, sometimes even quite explicitly, have based their considerations on the basis of Kolmogorov's formulation of probability in terms of positive measures, which has led to even greater abstraction in reasoning than that already customary in treatises on quantum foundations issues. In addition, Khrennikov has considered p-adic analysis to analyze rigorously the complications inherent in using what ideally should be infinite sets as probability spaces for situations where all applications in fact are limited to finite sets. Such efforts must be applauded; rigorous mathematics can only purge obscured errors in reasoning divining Nature's mysteries.

This writer, on the other hand, has taken, for reasons of convenience, not wisdom, the complementary tact of accepting elementary probability theory and sought to vary the Born interpretation, so as to seek resolutions of the many infamous conundrums.(2) This has been quite fruitful seemingly; there appear to be plausible alternate paradigms for the phenomena covered by quantum theory for which direct and simple probability theory is adequate. The main issue then becomes: is there empirical evidence supporting these new (occasionally old actually) alternate paradigms?

The task shall be broken in two. On the one hand, entities which classically are considered particles shall be considered as one phenotype, then on the other, those which are classically thought of as waves will be analyzed separately. The goal then becomes showing, that the particle-type evidences wave-like behavior probabilistically, and the waves likewise evidence particle-like behavior probabilistically.

II. THE PARTICULATE CHARACTER OF WAVE FUNCTIONS

Without going into excessive details, let us recall simply that by-and-large the issue of a particulate nature of something described by a quantum wave function pertains to the electromagnetic interaction, i.e., to photons. The essential empirical fact that underlies the 'photon' paradigm is that electromagnetic radiation at very low intensity seems to be absorbed at specific point-like locations. This fact is interpreted to mean that the energy in radiation itself is contained in similarly small packets and that each detection event results from the absorption of one such packet, i.e., one 'photon.'

While this imagery is plausible, it is not "necessary" in the mathematical sense; alternate notions are equally plausible.(3) For instance, let us take explicit account of the fact that electromagnetic detection within the spectral range of primary concern is accomplished using "photo-detectors." Such detectors are devices that convert incoming electromagnetic radiation into a "photo-current" by lifting electrons from a detector's valence band to its conduction band, which, in turn, are converted in electronic circuitry into countable finite current pulses easily manipulated and, in particular, counted. Now, a photo-current is nothing else but a stream of electrons in wires. Such a current can be reduced in intensity only until there is only one remaining electron—a continuously diminishing or infinitesimal current is not possible. This reality can

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be the fundamental reason for the acceptance enjoyed by the photon paradigm. In other words, it is entirely possible that a fully continuous radiation wave has elicited a discrete photo current without in any way implying that the stimulating radiation itself was discrete or packaged as ‘photons.’

Further, another reality is that everywhere and always there is noise. It cannot be excluded, in particular in that it might be considered as the contribution of the outside world, which cannot be totally screened off. This in turn, with respect to electromagnetic detection in photo detections, is taken into account in the basic theory of photo detection with the prescription that the intensity of an elicited photo current is in probabilistic proportion to the total electromagnetic wave intensity, i.e., the square of the electric wave amplitude, implicitly, thereby, including noise. This effect is absolutely ‘local.’ That is, the relevant intensity is just that exactly at the point in space and instant in time where and when the single photo electron is elevated into the conduction band of the detector after which it then flows into the counting circuitry.

Since the wave function of a photon is associated directly with electromagnetic fields, this means that the non Boolean aspect of the presumed logic of quantum mechanics is simply a reflection of the fact that photo *detection* is proportional to the local field intensity. The interference terms resulting from squaring wave functions result from an electromagnetic phenomena having no implications for probability theory, which is brought into the story by photo detection theory, but, only *at the point* of photoelectron generation *after* electromagnetic interference has been taken into account.

Below this logic shall be extended to wave functions for particles.

III. POSSIBLE EMPIRICAL CONSEQUENCES

Accepting the paradigm of electromagnetic interaction as comprised of essentially waves propagated as envisioned in non quantum electrodynamics, which then appear as if comprised of parceled ‘photons’ as an artifact of photo detection, implies that all the properties of classical electromagnetic radiation should remain evident. In particular, this includes all phenomena resulting from the transverse nature of electromagnetic radiation, that is, mode polarization.

First it should be noticed that, the structure of polarization effects is governed by the group $SU(2)$. This group is isomorphic to $SO(3)$, the group encoding the structure of rotation on a sphere. What this implies is, that insofar as the non commutivity of $SO(3)$ is geometric, and not a consequence of any fundamental hypothetical input for Quantum Mechanics (namely: that Hamiltonian conjugate variables are non commutative according to Heisenberg’s uncertainty relationship

$$\hat{X}\hat{P} - \hat{P}\hat{X} = i\hbar),$$

the non commutative structure of $SU(2)$ likewise is attributable only to geometry. Therefore, all resultant effects cannot be attributed to quantum structure.(4)

Additionally, by rejecting the photon paradigm, there are direct consequences for what should be expected in terms of correlated photons. Specifically, a parametric down conversion(PDC) crystal is thought to produce anticorrelated pairs of photons. But, if the photon paradigm is inadequate in fact, then the signals produced in a PDC crystal will have to be modeled by another paradigm, and the most likely would seem to be the classical conception of electromagnetic waves. Each “photon” equivalent signal would be a short pulse and the correlation could be expressed as two pulse polarization combinations, one with horizontal-left and vertical-right, the other with the opposite combination.

Such a combination of correlated signals admits at least two types of examinations: one with respect to the timing of the generation of the final photoelectrons, and another with respect to the correlations of the polarization directions.

If now the photon paradigm is faithful to the reality of the generation of the signals, then the individual photons constituting the pair must be generated together in the source crystal and within a time interval restricted by the Heisenberg uncertainty relationship. The random event would be the generation of the pair at the source, and might be best characterized as occurring at an instant within the interval specified by the uncertainty relationship.

In contrast, if the more correct paradigm is that described above in which it is envisioned that essentially classical pulses lift electrons from the valence to the conduction band in the detector, then the randomness is vested in two events, one for each electron. In this case the specific instant of photoelectron generation is determined both by the pulse signal from the source crystal and by a noise contribution at the detector. Insofar as neither the pulse length or the noise is constrained by Heisenberg uncertainty, the timing discrepancy between the instants at which the electrons are elevated can exceed the Heisenberg limit. To this writer’s knowledge, a laboratory study of this issue has not been done.

A similar study of the nature of the polarization states of the signals generated in a PDC crystal can be made to address empirically another feature of quantum states.(5) It is this: for reasons of convenience in spectroscopy it is taken that the ‘state’ of the signal as specified by quantum mechanics is in the form of singlet state which is a linear combination of the two possible outcome states. This combination is rotationally invariant, as can be verified by calculation, and it is considered from this fact that this item as a unit is such that in fact it has no polarization orientation at all, until one or the other of the two signals comprising the pair encounters a measuring device. By authority then of what is called the “projection hypothesis,” it is taken that the process of measurement then “projects” this neutral state onto one or the other of the possible constituents. Now, if this imagery is taken seriously, then rotational invariance means that it should make no difference at all which direction the polarizer has that the first of the pair encounters. If this polarizer ‘projects’ or collapses the wave function of the photon, it would be natural to presume that the polarization direction taken by the photon will be that of the polarizer; in which case, the polarization direction of the other photon in the pair must be deterministically orthogonal. In this way the correlation function for the pair will be ‘rotationally invariant,’ in conformity with the presumed character of the input signal.

On the other hand, if the classical electromagnetic pulse with photoelectron elicitation paradigm reigns, then the signals emitted by the PDC crystal are one or the other of the possible base signals, not some mystical superposition of mutually exclusive options. In this case, measuring the polarization of this output of the crystal on the axis of the crystal will show deterministic anticorrelation, whereas measuring it off axis will yield only statistical anticorrelation. That is, there will be individual pair events that are not anticorrelated; and, the final result is that the correlation function will not be rotationally invariant. Elsewhere this writer has published a calculation showing that this effect can be used to explain the fact that the correlation signal from PDC crystals in fact exhibits a variation in visibility.(6)

Ideally the visibility will go to zero at an azimuth of $\pi/4$ radians off the crystal axis. In fact the only reported variation known to this writer is just a few percent.(7) While the mere presence of any variation is inexplicable in quantum terms, this low value fails to preclude all doubt that the photon paradigm is invalid. To fully reconcile this matter, however, would require detailed analysis of exactly which events are counted in the data stream analyzed for the reported figure. It could be that the coincidence circuit is so tuned that the offending events are rejected, illegitimately, and so largely not included in the data analysis.

IV. NEGATIVE PROBABILITY DENSITIES

Wigner famously long ago discovered a mapping from wave functions to phase space expressions strongly resembling density functions.(8) He did not claim, however, that these expressions could actually be considered such phase space density functions, because for some solutions of the Schrödinger Equation his mapping yielded non positive-definite expressions. Such cannot ordinarily be interpreted as a probability. But, two possibilities are opened up. One, a non elementary formulation of probability in which such negative expressions arise can be introduced; or, two, a restriction can be put on wave functions to the effect, that only those wave functions—as solutions to Schrödinger's Equation—are ontologically meaningful that do yield positive-definite phase space Wigner functions.

There are several arguments supporting this latter stipulation. One of the most persuasive is, that even in non quantum statistical mechanics the eigen states of the dynamic equation of a Markoff process do not yield positive Wigner densities.(9) Thus, the same circumstance occurring in Quantum Mechanics cannot be ascribed to peculiarities of that theory. In both theories, the remedy is the same; namely accepting as ontological solutions only those that satisfy physically meaningful boundary or initial conditions. Fortunately, it has been found that there do exist solution of Schrödinger's Equation yielding acceptable ontological solutions, namely thermal states, i.e., superpositions of eigenstates weighted with Boltzmann factors. Additionally, for the harmonic oscillator, the coherent states also meet this stipulation, in addition to being the only states that also oscillate back and forth with time as does a real oscillator. (Harmonic oscillator eigenstates oscillate up and down as a unit and otherwise do not do what a physical oscillator does.)

It is not known to this writer if similar variants of coherent (like) states exist for the Kepler problem; i.e., states that rotate around as do electrons around the nucleus. Evidently this is still a matter for research. Without such states, the contention here, that only a very limited selection of states can constitute ontological states, remains an unproven proposal, rather than a solution to this issue. Nevertheless, the historical record so far provides good reason to be optimistic.

V. SUMMARY: OCCAM'S PREFERENCE

Herein a paradigm for the application of probability theory to quantum wave functions has been proposed. It accounts for the non Boolean aspect of quantum probabilities by proposing that probability enters locally at a specific point, so that the fact that quantum probability functions do not add, rather that their underlying wave functions do, is a manifestation of the fact that an underlying wave function is analogous to an electric field for which the square is proportional to the probability of eliciting a photoelectron. The non Boolean character then is a consequence of the fact that electric fields add linearly but not their square or intensities. That photoelectric currents are proportional to the square of electric fields is the basic feature of the standard theory of photo-detection. In sum, thus far this paradigm is straightforward.

The extention to wave functions for particles not normally detected with photodetectors depends on the writer's proposal for understanding "de Broglie waves," which in this conception are taken to be electromagnetic pilot waves derived from interaction with a stochastic background identical to what is otherwise known as the quantized ground state of the free electromagnetic field.(10) With this model of de Broglie waves, the explanation for the point detection of point particles is a direct consequence of the notion that particles are indeed localized entities being guided by pilot waves, and that, at detection, the particle is seen by a particle detector, but not the pilot wave. The probability of entrapment at a particular position is given by the local intensity of the wave function at that position, which explains why it is that non Boolean aspect of wave functions poses no restrictions for the application of simple probability theory. The spacial distribution of these pilot waves can be revealed only by repeating the experiment often to ultimately expose the intensity pattern of the pilot wave in which the particle is as a statistical matter most strongly trapped to be located where the wave is most intense, etc.

Further, we propose that the non positive-definite character of Wigner functions can be remedied by limiting the set of states to be considered as ontological to those which in fact do yield non negative Wigner functions. All the other states are then reduced to the status of mathematical intermediary results without uninterpreted physical meaning. Sophisticated probability theory may not be necessary.

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[1] Preprints of the author’s papers can be downloaded at: `www.nonloco-physics.000freehosting.com`