# WAVE PHENOMENA AND QUANTUM MECHANICS

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ABSTRACT. This note attempts to show, that Quantum Mechanics is always in accord with the wave formulas induced by the discontinuity hypothesis.

The pervasive physical significance of wave phenomena is based on the fact, that they are among the most intuitive and productive for describing discontinuous processes in small volumes and short times. According to EINSTEIN, for example, the effects of BROWNian motion can be attributed to the consequences of the atomic structure of matter; Energy and momentum oscillations within cavities lead directly to EINSTEIN's conception of light (quanta); and, energy oscillations in a crystal are in close relationship to the existence of discreet stationary states in mechanical structures. In that the analysis of mechanical systems has become feasible by virtue of Quantum Mechanics, there must be a close relationship between the above named effects and oscillations. This note is, therefore, an extention of work the author, together with BORN and JORDAN[1, Chapter 4, §3], have done on the development of quantum mechanics[1]. A deeper investigation of such phenomena seemed to me to be in order, as doubt has been voiced from many sides on the very existence of discontinuities.

§1. The mathematical portion of the following considerations extends a recent paper of the author[3] on quantum mechanical resonance, familiarity with which is assumed below. To begin, consider the following problem: Two identical atoms, $a$ and $b$, are found in the states $n$ and $m$ (ignoring for the moment all radiative interactions), i.e., $W^a = E_n$ and $W^b = E_m$; and, let them be coupled together only by a very weak interaction. Then, there will occur a certain resonance between them; it can be described from two different viewpoints:

1.1. In the course of time leaps of energy occur with a particular frequency, such that spontaneously and discontinuously $W^a$ jumps from $E_n$ to $E_m$, or visa versa. In other words, in the course of time a "quantum of light"[2], $(E_n - E_m)$ repeatedly is transfered from $a$ to $b$ or back. On the average, from symmetry considerations, we expect an atom, $a$ say, half the time to have this quantum of energy, while $b$ has it the other half.

1.2. This resonance can be seen as an analogue to the classical exchange between two oscillators. In this view the energy is sloshing back and forth with a relatively low [secular] frequency between the atoms. Thus, the energy of each atom is a harmonically periodic function of time. This description fully contradicts the former. Via quantum mechanics the

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[1]The criticism from SMEKEL[2], is not related to the case considered here, namely mechanical systems; whether this criticism could be extended to radiation in cavities, remains an open question; I prefer to assume that cavity and crystals are fully analogue.

[2]Here, perhaps, one should speak of a "quantum of sound," in so far as we are considering a mechanical system.

description of the correspondence is substantially modified thereby enabling a practical treatment: there is, for a particular state of the total system, little sense in talking about the energy of one atom. Only the time average of any time dependant quantity has physical significance for a particular state of the total system. The energy of one atom is, in this circumstance, representable by a matrix that corresponds to a harmonic function of time. The harmonic elements of this matrix are attributed to two states of the total system. So long as no transformations of the whole system are under consideration, only the time averages are observable in principle.

We seek to show now, that for all effects observable in principle, both descriptions are equivalent. The first issue is, whether it is possible, using scattering to determine the energy level of either atom, to fix it at $E_n$ or $E_m$, or something in between. From the point of view of either description, the answer is: no. In the second view, one need only apply the definitions from quantum mechanics to the whole system (both atoms $a$ and $b$) to see, that (up to quantities the order of the interaction) only the same energy increments: $E_n - E_m$, as it is also true for the non-interaction case, can be transfered by scattering processes. In order to proceed, let us calculate the quantum theoretical matrix for the energy of a single atom ($a$, say): all matrices for the interacting system are derived from those of the undisturbed system via a canonical transformation, $\mathbf{S}$, in matrix form (See: Eqs. (8) and (9) in [3].):

$$(1) \qquad \begin{aligned} \mathbf{W}' &= \mathbf{S}^{-1}\mathbf{W}\mathbf{S}, \\ \mathbf{q}' &= \mathbf{S}^{-1}\mathbf{q}\mathbf{S}, \end{aligned}$$

where $\mathbf{S}$ is ( Eq. (12) in [3]):

$$(2) \qquad S_{nm,nm} = \frac{1}{\sqrt{2}}, \ \ S_{nm,mn} = \frac{1}{\sqrt{2}}, \ \ S_{mn,nm} = \frac{1}{\sqrt{2}}, \ \ S_{mn,mn} = -\frac{1}{\sqrt{2}}.$$

If one denotes the energy of atom $a$ in the unperturbed state as $\mathbf{W}^a$, and in the perturbed state as $\mathbf{W}'^a$, then

$$(3) \qquad \mathbf{W}'^a = \mathbf{S}^{-1}\mathbf{W}^a\mathbf{S}.$$

Given Eq. (2), this is a matrix with elements:

$$(4) \qquad \begin{aligned} W'^a_{nm,nm} &= 1/2(E_n + E_m), \quad W'^a_{nm,mn} = 1/2(E_n - E_m), \\ W'^a_{mn,nm} &= 1/2(E_n - E_m), \quad W'^a_{mn,mn} = 1/2(E_n + E_m). \end{aligned}$$

Those quantities of a particular state that can be taken as observable in principle are: the time average of $\mathbf{W}^a$ itself, mean square energy, and any oscillation averages. In general, all these oscillations can be attributed to a time average of some function of $\mathbf{W}$, e.g., $\mathbf{f}(\mathbf{W})$ (for example the average of $\mathbf{W}^2, \mathbf{W}^4$, etc.). Thus, we shall compute the time average of such a function: it would be:

$$(5) \qquad \mathbf{f}' = \mathbf{S}^{-1}\mathbf{f}\mathbf{S},$$

that is:

$$f'_{nm,nm} = 1/2\left(f(E_n) + f(E_m)\right);$$

likewise:

$$f'_{mn,mn} = 1/2\left(f(E_n) + f(E_m)\right).$$

One sees directly, that these values correspond exactly with those from the first viewpoint. One could, in fact, pose the question the other way around: is there a function, $E(t)$, for

which the time average of a function $f(E(t))$ satisfies:

$$\overline{f(E(t))} = \frac{1}{2}\left(f(E_n) + f(E_m)\right)?$$

In fact one will find, that only functions of the type for which the total dwell time in each sub-state is equal, will have this quality, which is precisely the conditions presumed for the first viewpoint.

The result is, therefore, that in this application Quantum Mechanics comes to exactly the same conclusions as those based on the discontinuity hypothesis; in other words, it is apparent, that the discontinuity hypothesis fits naturally in quantum theory. It seems, that Quantum Mechanics has as much or little to say about discontinuity as is empirically available. The exact time of a transition, even the transitions themselves, do not directly manifest themselves. It is thus, our opinion that only the relative total time spent in each state is physically significant.

Is should be noted, that all calculations are subject to approximations in which the interaction is taken to be vanishingly small. This is the only approximation in which the problem can be sensibly posed. The very concept of "energy of an atom" makes sense only when the interaction energy can be neglected.

§2. Mathematical support for the agreement between mean quantum oscillations and discontinuity theory derives from the form of Eqs. (1) and (5). This form has such a extensive application in Quantum Mechanics, that the notions outlined above on oscillations beg generalization. To begin, the assumption in §1., that $a$ and $b$ are two atoms, is not crucial; it could be any two similar mechanical systems. Further, the arguments remain the same if multiple interacting systems are under consideration. The unperturbed system exhibits then greater degeneration. The precise form of Eqs. (1) and (5) remains unaltered, however, only the matrix $\mathbf{S}$ itself is different. With respect to the relationship between Quantum Mechanics and discontinuity, the elements of the $\mathbf{S}$ matrix are unimportant, only the form of the canonical transformation is critical. Let us denote the perturbed state of a mechanical system with "$\alpha$" and the unperturbed state with "$\beta$." Then a canonical transformation, Eqs. (1) or (5), within the discontinuity viewpoint means the following: *If the system is in the state $\alpha$, then $|S_{\alpha\beta}|^2$ gives the probability, that (by collisions or sudden removal of perturbations, etc.) the system will be found in state $\beta$.* For any function of $\mathbf{W}^a$, according to Eq. (5):

$$(6) \qquad\qquad f_a(W^a) = \sum_{\beta} |S_{\alpha\beta}|^2 f(W_{\beta}^{\alpha}),$$

where $W_{\beta}^{\alpha}$ signifies those values of $\mathbf{W}^a$, that the system $a$ in state $\beta$ takes on. Following general principles, it is true that

$$(7) \qquad\qquad\qquad \sum_{\beta} |S_{\alpha\beta}|^2 = 1.$$

As a second generalization, we can relax the stipulation that similar systems are under consideration. The only essential assumption is that the same energy increments, $E_n - E_m$, arise in all the subsystems, otherwise (even in systems in which it does not occur) there would be no resonance. In terms of the discontinuity viewpoint, the energy decrement $E_n - E_m$ must fit into each subsystem. The calculations from §1 can be applied unchanged in this case.

The third extention that should be considered, pertains to the type and size of the investigated oscillations. The above considerations remain unchanged if instead of the energy of

subsystems, we consider some other quantity, which for the unperturbed case can be represented by a matrix. Such quantities include the total momentum, angular momentum about a fixed axis, etc. The calculations of the mean oscillation of all these quantities agree from both viewpoints. *In so far as, within Quantum Mechanics, energy, angular momentum etc. of a mechanical system as functions of time are to be considered, only those functions of such quantities of the type for which the average dwell time in each sub-state is equal, are suitable.* The probability coefficients, $|s_{\alpha\beta}|^2$, are identical in either viewpoint.

Finally, it can be taken that even for aperiodic oscillations, the techniques of §1 retain their validity, as the form of the canonical transformations, Eqs, (1) and (5) remain appropriate. As an example, finally, let us consider how the calculations of BORN, JORDAN and this author, on oscillations in a crystal are captured by the considerations herein.[1] They comprise energy oscillations in a small portion of the crystal. In the unperturbed system, it is to be imagined, that the small portion should is cut out. Then in the perturbed system there arises a resonance in those frequencies that are common to both the crystal and small portion. This is approximately the case for all oscillations for which the wave length is small compared to the linear dimensions of the small portion. Only for such eigen oscillations does the eigen oscillation concept made sense. From the considerations in §1, even without calculation, we can see that mean square oscillation and all higher harmonics from Quantum Mechanics must yield the same result as obtained with BOSE-EINSTEIN light statistics. (Specifically these statistics are relevant, because even in Quantum Mechanics, the issue is always the energy values of stationary states. See: [3].) For the case of mean square oscillations, the calculation is given therein.

The calculations herein seem to me to be an argument, that a continuum interpretation of the quantum formalism, that is one involving DE BROGLIE-SCHRÖDINGER waves, does not correspond to known facts. Rather, the reality of the discontinuity hypothesis in quantum mechanics is contained in our calculations.

## REFERENCES

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