“COMPLEMENTARITY” OR SCHIZOPHRENIA: IS PROBABILITY IN QUANTUM MECHANICS INFORMATION OR *ONTA*?

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ABSTRACT. Of the various “complimentarities” or “dualities” evident in Quantum Mechanics (QM), among the most vexing is that afflicting the character of a ‘wave function,’ which at once is to be something ontological because it diffracts at material boundaries, and something epistemological because it carries only probabilistic information. Herein a description of a paradigm, a conceptual model of physical effects, will be presented, that, perhaps, can provide an understanding of this schizophrenic nature of wave functions. It is based on Stochastic Electrodynamics (SED), a candidate theory to elucidate the mysteries of QM. The fundamental assumption underlying SED is the supposed existence of a certain sort of random, electromagnetic background, the nature of which, it is hoped, will ultimately account for the behavior of atomic scale entities as described usually by QM. In addition, the interplay of this paradigm with Bell’s ‘no-go’ theorem for local, realistic extentions of QM will be analyzed.

1. INTRODUCTION

Of the various “complimentarities” or “dualities” evident in Quantum Mechanics (QM), among the most vexing is that afflicting the character of a ‘wave function,’ which at once is to be something ontological because it diffracts at material boundaries, and something epistemological because it carries only probabilistic information. All other diffractable waves, it may be said, carry {momentum, energy}, not conceptual, abstract information, “ideas.” All other probabilities are calculational aids, and like abstractions generally, are utterly unaffected by material boundaries. The literature is replete with resolutions of QM-conundrums selectively ignoring one or the other of these characteristics—in the end, they all fail.

Herein a description of a paradigm, a conceptual model of physical effects, will be presented that perhaps can provide an understanding of this schizophrenic nature of wave functions. It is based on Stochastic Electrodynamics (SED), a candidate theory to elucidate the mysteries of QM.[1] The fundamental concept underlying SED is the supposed existence of a certain sort of random, electromagnetic background, the nature of which, it is hoped, will ultimately account for the behavior of atomic scale entities as described usually by QM. [2] Among the successes of SED, one is a local realistic explanation of the diffraction of particle beams.[3] The core of this explanation is the notion that relative motion through the SED background effectively engenders de Broglie’s pilot wave. Given such a pilot wave associated with a particle’s motion, the statistical distribution of momentum in a density over phase space can be decomposed, in the sense of Fourier analysis, such that the resulting form of Liouville’s Equation, under some conditions, is Schrödinger’s Equation.

From this viewpoint, the 'schizophrenic' character of wave functions can be discussed and understood free of preternatural attributes. These concepts have broad implications for serious philosophical questions such as the "mind-body" dichotomy through teleportation to popular science fiction effects. In addition, the peculiar nature of probability in QM is clarified.

Although much remains to be done to comprehensively interpret all of QM in terms of SED, many of the by now hoary 'paradoxes' can be rationally deconstructed.

A secondary (but intimately related) issue is that of determining the import of Bell's Theorem for the use of the SED paradigm to reconcile fully the interpretation of QM. Arguments will be presented showing that in his proof, Bell (essentially by misconstruing the use of conditional probabilities) called on inappropriate hypothetical presumptions, just as Hermann, de Broglie, Bohm and others found that Von Neumann did before him.[4, 5]

2. De Broglie waves as an SED effect

The foundation of the model or conceptual paradigm for the mechanism of particle diffraction proposed herein is Stochastic Electrodynamics (SED). Most of SED, for which there exists a substantial literature, is not crucial for the issue at hand.[1] The *nux* of SED can be characterized as the logical inversion of QM in the following sense. If QM is taken as a valid theory, then ultimately one concludes that there exists a finite ground state for the free electromagnetic field with energy per mode given by

$$E = \hbar\omega/2. \tag{2.1}$$

SED, on the other hand, inverts this logic and axiomatically posits the existence of a random electromagnetic background field with this same spectral energy distribution, and then endeavors to show that ultimately, a consequence of the existence of such a background is that physical systems exhibit the behavior otherwise codified by QM. The motivation for SED proponents is to find an intuitive local realistic interpretation for QM, hopefully to resolve the well known philosophical and lexical problems as well as to inspire new attacks on other problems.

The question of the origin of this electromagnetic background is, of course, fundamental. In the historical development of SED, its existence has been posited as an operational hypothesis whose justification rests *a posteriori* on results. Nevertheless, lurking on the fringes from the beginning, has been the idea that this background is the result of self-consistent interaction; i.e., the background arises out of interactions from all other electromagnetic charges in the universe.[6]

For present purposes, all that is needed is the hypothesis that particles, as systems with charge structure (not necessarily with a net charge), are in equilibrium with electromagnetic signals in the background. Consider, for example, as a prototype system, a dipole with characteristic frequency ω. Equilibrium for such a system in its rest frame can be expressed as

$$m_0c^2 = \hbar\omega_0. \tag{2.2}$$

This statement is actually tautological, as it just defines ω_0 for which an exact numerical value will turn out to be practically immaterial.

This equilibrium in each degree of freedom is achieved in the particle's rest frame by interaction with counter propagating electromagnetic background signals in both polarization modes separately, which on the average, add to give a standing wave with antinode at the particle's position:

$$2\cos(k_0 x)\sin(\omega_0 t). \tag{2.3}$$

Again, this is essentially a tautological statement as a particle doesn't 'see' signals with nodes at its location, thereby leaving only the others. Of course, everything is to be understood in an on-the-average, statistical sense.

Now consider Eq. (2.3) in a translating frame, in particular the rest frame of a slit through which the particle as a member of a beam ensemble passes. In such a frame the component signals under a Lorentz transform are Doppler shifted and then add together to give what appears as modulated waves:

$$2\cos(k_0\gamma(x - c\beta t))\sin(\omega_0\gamma(t - c^{-1}\beta x)), \tag{2.4}$$

for which the second, the modulation factor, has wave length $\lambda = (\gamma\beta k_0)^{-1}$. From the Lorentz transform of Eq. (2.2), $P = \hbar\gamma\beta k_0$, the factors $\gamma\beta k_0$ can be identified as the de Broglie wave vector from QM as expressed in the slit frame.

In short, it is seen that a particle's de Broglie wave is modulation on what the orthodox theory designates *Zitterbewegung*. The modulation-wave effectively functions as a pilot wave. Unlike de Broglie's original conception in which the pilot wave emanates from the kernel, here this pilot wave is a kinematic effect of the particle interacting with the SED Background. Because this SED Background is classical electromagnetic radiation, it will diffract according to the usual laws of optics and thereafter, modify the trajectory of the particle with which it is in equilibrium.[3] (See Ref. [1], Section 12.3, for a didactical elaboration of these concepts.)

The detailed mechanism for pilot wave steerage is based on observing that the energy pattern of the actual signal that pilot waves are modulating, and to which a particle tunes, comprises a fence or rake-like structure with prongs of varying average heights specified by the pilot wave modulation. These prongs, in turn, can be considered as forming the boundaries of energy wells in which particles are trapped; a series of micro-Paul-traps, as it were. Intuitively, it is clear that where such traps are deepest, particles will tend to be captured and dwell the longest. The exact mechanism moving and restraining particles is radiation pressure, but not as given by the modulation, rather by the carrier signal itself. Of course, because these signals are stochastic, well boundaries are bobbing up and down somewhat so that any given particle with whatever energy it has will tend to migrate back and forth into neighboring cells as boundary fluctuations permit. Where the wells are very shallow, however, particles are laterally (in a diffraction setup, say) unconstrained; they tend to vacate such regions, and therefore have a low probability of being found there.

The observable consequences of the constraints imposed on the motion of particles is a microscopic effect which can be made manifest only in the observation of many similar systems. For illustration, consider an ensemble of similar particles comprising a beam passing through a slit. Let us assume that these particles are very close to equilibrium with the background, that is, that any effects due to the slit can be considered as slight perturbations on the systematic motion of the beam members.

Given this assumption, each member of the ensemble with index, n say, will with a certain probability have a given amount of kinetic energy, E_n, associated with each degree of freedom. Of special interest here is the beam direction perpendicular to both the beam and the slit in which, by virtue of the assumed state of near equilibrium with the background, we can take the distribution, with respect to energy of the members of the ensemble, to be given in the usual way by the Boltzmann Factor:$e^{-\beta E_n}$ where β is the reciprocal product of the Boltzmann Constant k and the temperature, T, in degrees Kelvin. The temperature in

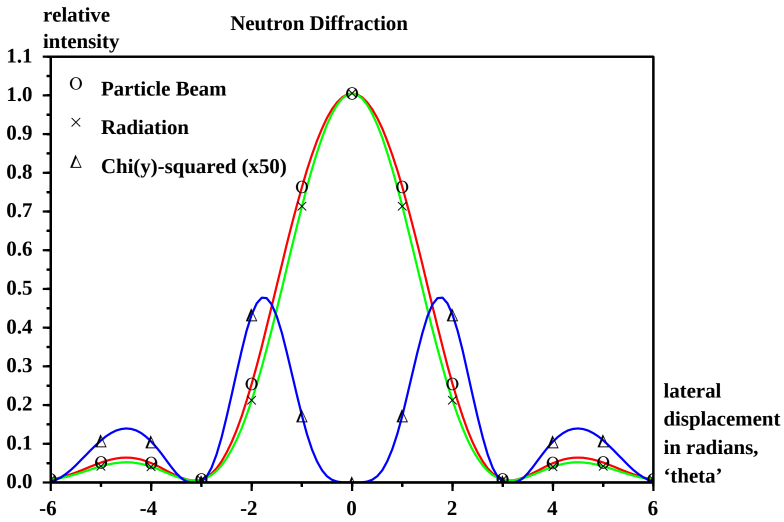

FIGURE 2.1. A simulated single slit neutron diffraction pattern showing the closeness of the fit of Eq. (6) to the pure wave diffraction patten. See Ref. [3] for details. wave diffraction

this case is that of the electromagnetic background serving as a thermal bath for the beam particles with which it is in near equilibrium.

Now, the relative probability of finding any given particle; i.e., with energy $E_{\{n,j\}}$ or $E_{\{n,k\}}$ or ... , trapped in a particular well will be, according to elementary probability, proportional to the sum of the probabilities of finding particles with energy less than the well depth,

$$\sum_{\{l|E_{n,l}\le d\}} e^{-\beta E_{n,l}} \approx \int_0^D d(\frac{E_n}{E_0})e^{-\beta E_n} = \frac{1}{\beta E_0}(1-e^{-\beta D}), \tag{2.5}$$

where approximating the sum with an integral is tantamount to the recognition that the number of energy levels, if not *a priori* continuous, is large with respect to the well depth.

If now d in Eq. (2.5) is expressed as a function of position, we get the probability density as a function of position. For example, for a diffraction pattern from a single slit of width a at distance D, the intensity (essentially the energy density) as a function of lateral position is: $E_0 \sin^2(\theta)/\theta^2$ where $\theta = k_{pilot\,wave}(2a/D)y$, and the probability of occurrence, $P(\theta(y))$, as a function of position, would be

$$P(y) \propto (1-e^{-\beta E_0 \sin^2(\theta)/\theta^2}). \tag{2.6}$$

Whenever the exponent in Eq. (2.6) is significantly less than one, its r.h.s. is very accurately approximated by the exponent itself; so that one obtains the standard and verified result that the probability of occurrence, $P(y) = \psi^*\psi$ in conventional QM, is proportional to the intensity of a particle's de Broglie (pilot) wave.

3. Schrödinger Equation

A consequence of the attachment of a De Broglie pilot wave to each particle is that there exists a Fourier kernel of the following form:

$$e^{\frac{i2\mathbf{p}\cdot\mathbf{x}'}{\hbar}}, \tag{3.1}$$

which can be used to decompose the density function of an ensemble of similar particles. Consider an ensemble governed by the Liouville Equation:

$$\frac{\partial\rho}{\partial t} = -\nabla\rho\cdot\frac{\mathbf{p}}{m} + (\nabla_p\rho)\cdot\mathbf{F}, \quad \left[\nabla_p \equiv \sum_{i=x,y,z}\frac{\partial}{\partial p_i}\right]. \tag{3.2}$$

Now, decompose $\rho(x, p)$with respect to p using the De Broglie-Fourier Kernel:

$$\widehat{\rho}(\mathbf{x}, \mathbf{x}', t) = \int e^{\frac{i2\mathbf{p}\cdot\mathbf{x}'}{\hbar}}\rho(\mathbf{x}, \mathbf{p}, t)d\mathbf{p}, \tag{3.3}$$

to transform the Liouville Equation into:

$$\frac{\partial\widehat{\rho}}{\partial t} = \left(\frac{\hbar}{i2m}\right)\nabla'\nabla\widehat{\rho} - \left(\frac{i2}{\hbar}\right)(\mathbf{x}'\cdot\mathbf{F})\widehat{\rho}. \tag{3.4}$$

To solve, separate variables using:

$$\mathbf{r} = \mathbf{x}+\mathbf{x}', \quad \mathbf{r}' = \mathbf{x}-\mathbf{x}', \tag{3.5}$$

to get

$$\frac{\partial\widehat{\rho}}{\partial t} = \left(\frac{\hbar}{i2m}\right)(\nabla^2-(\nabla')^2)\widehat{\rho} - \left(\frac{i}{\hbar}\right)(\mathbf{r}-\mathbf{r}')\cdot\mathbf{F}\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\widehat{\rho}, \tag{3.6}$$

which can (sometimes) be separated by writing:

$$\widehat{\rho}(\mathbf{r}, \mathbf{r}') = \psi^*(\mathbf{r}')\psi(\mathbf{r}), \tag{3.7}$$

to get Schrödinger's Equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar^2}{2m}\nabla^2\psi + V\psi. \tag{3.8}$$

4. Conclusions

Within this paradigm, Quantum Mechanics ***is*** incomplete as surmised by Einstein, Padolsky and Rosen.[7] It is built on the basis of the Liouville Equation while taking a *particular* stochastic background into account. The conceptual function of Probability in QM is just as in Statistical Mechanics. Measurement reduces ignorance; it does not precipitate "reality." Of course, measurement also disturbs the measured system, but this presents no more fundamental problems that it does in classical physics. 'Heisenberg uncertainty,' on the other hand, is seen to be caused simply by the incessant dynamical perturbation from background signals. In so far as the source of background signals can not be isolated, this source of uncertainty is intrinsic, but not fundamentally novel. For these reasons, "duality" is superfluous. Particles have the same ontological status as in classical physics. Individual particles in a beam pass through one or the other slit in a Young double slit experiment, for example, while their De Broglie piloting waves pass through both slits. Beyond the slit, the particles are induced stochastically to track the nodes of their pilot waves so that a diffraction pattern is built up mimicking the intensity of the pilot wave.

From within this paradigm, the now infamously paradoxical situations illustrating various problems with the interpretation of QM never arise or are resolved with elementary reasoning. In particular, wave functions are not vested with an ambiguous nature.

The SED Paradigm also clarifies the appearance of interference among "probabilities." Numerous analysts from various view points have (re)discovered that fact that Probability Theory admits structure (used by QM) that goes unexploited in traditional applications. (E.g., see Gudder, Summhammar, this volume) While each of these approaches provides deep and surprising insights, none really offers any explanation of why and how nature exploits this structure. Just as a certain second order hyperbolic partial differential equation becomes *the* "wave equation," as a physics statement only with the introduction; e.g., of Hooke's Law, so this extra probability structure can be made into physics only with an analogue to Hooke's Law.

SED provides that analogue for particle behavior with its model of pilot wave guidance. In this model, radiation pressure is responsible for particle guidance.[3] Radiation pressure is proportional to the *square* of EM fields; i.e., the intensity (in this case of the the background field as modified by objects in the environment) which is not additive. Rather, the field amplitudes are additive and interference arrises in the way well understood in classical EM. In other words, QM interference is a manifestation of EM interference. The relevant Hook's Law analogue is the phenomenon of radiation pressure. For radiation, this is all intimately related, of course, to classical coherence theory as applied to "*square* law" photoelectron detectors, which, when properly applied, resolves many QM conundrums, including those instigated by Bell's Theorem surrounding EPR correlations.

APPENDIX: BELL'S THEOREM

The interpretation or paradigm described herein conflicts with the conclusions of Bell's "no-go" theorem, according to which a local, realistic extention of QM should conform with certain restraints that have been shown empirically to be false. To be sure, this paradigm does not deliver the hidden variables for exploitation in calculations, but it does indicate to which features in the universe they pertain—namely, all other charges. The character of these hidden variables is dictated by the fact that they are distinguished only in that they pertain to particles distant from the system of particular interest; thus, internal consistency requires that they be local and realistic.[8]

The basic proof. Bell's Theorem purports to establish certain limitations on coincidence probabilities of spin or polarization measurements as calculated using QM if they are to have an underlying deterministic but still local and realistic basis describable by extra, as yet, 'hidden variables,' λ, distributed with a density $\rho(\lambda)$. These limitations take the form of inequalities which measurable coincidences must respect. The extraction of one of these inequalities, where the input assumptions are enumerated as Bell made them, proceeds as follows:

Bell's fundamental *Ansatz* consists of the following equation:

$$P(a,b) = \int d\lambda \rho(\lambda) A(a,\lambda) B(b,\lambda), \tag{4.1}$$

where, per *explicit assumption:* A is not a function of b; nor B of a. This he motivated on the grounds that a measurement at station A, if it respects 'locality,' can not depend on remote conditions, such as the settings of a distant measuring device, i.,e., b. In addition, each, by definition, satisfies

$$|A| \leq 1; \quad |B| \leq 1. \tag{4.2}$$

Eq. (4.1) expresses the fact that when the hidden variables are integrated out, the usual results from QM are recovered.

The extraction proceeds by considering the difference of two such coincidence probabilities where the parameters of one measuring station differ:

$$P(a,b) - P(a,b') = \int d\lambda \rho(\lambda)[A(a,\lambda)B(b,\lambda) - A(a,\lambda)B(b',\lambda)], \tag{4.3}$$

to which zero in the form

$$A(a,\lambda)B(b,\lambda)A(a',\lambda)B(b',\lambda) - A(a,\lambda)B(b',\lambda)A(a',\lambda)B(b,\lambda), \tag{4.4}$$

is added to get:

$$\begin{aligned} P(a,b) - P(a,b') = \int d\lambda \rho(\lambda)(A(a,\lambda)B(b,\lambda))(1 \pm A(a',\lambda)B(b',\lambda) + \\ \int d\lambda \rho(\lambda)(A(a,\lambda)B(b',\lambda))(1 \pm A(a',\lambda)B(b,\lambda), \end{aligned} \tag{4.5}$$

which, upon taking absolute values, Bell wrote as:

$$\begin{aligned} |P(a,b) - P(a,b')| \leq \int d\lambda \rho(\lambda)(1 \pm A(a',\lambda)B(b',\lambda) + \\ \int d\lambda \rho(\lambda)(1 \pm A(a',\lambda)B(b,\lambda). \end{aligned} \tag{4.6}$$

Then, using Eq. (4.1), "*Ansatz*, " and normalization $\int d\lambda \rho(\lambda) = 1$, one gets

$$|P(a,b) - P(a,b')| + |P(a',b') + P(a',b)| \leq 2, \tag{4.7}$$

a Bell inequality.[9]

Now if the QM result for these coincidences, namely $P(a,b) = -\cos(2\theta)$, is put in Eq. (4.7), it will be found that for $\theta = \pi/8$, the r.h.s. of Eq. (4.7) becomes $2\sqrt{2}$. Experiments verify this result.[10] Why the discrepancy? According to Bell: it must have been induced by demanding "locality," as all else he took to be harmless.

Critiques. Although Bell's analysis is denoted a 'theorem,' in fact there can be no such thing in Physics; the axiomatic base on which to base a theorem consists of those fundamental theories which the whole enterprise is endeavoring to reveal. Moreover, buried in all mathematics pertaining to the physical world are numerous unarticulated assumptions, some of which are exposed below.

The analytical character of dichotomic functions. In motivating his discussion of the extraction of inequalities, Bell considered the measurement of spin using Stern-Gerlach magnets or polarization measurements of 'photons.' In both cases, single measurements can be seen as individual terms in a symmetric dichotomic series; i.e., having the values ± 1. It is therfore natural to ask if the correlation computed using QM, $P(a,b) = -\cos(2\theta)$, and verified empirically, can be the correlation of dichotomic functions. It is easy to show that they *can not* so be; consider:

$$-\cos(2\theta) = k \int P(x-\theta)P(x)dx, \tag{4.8}$$

where $\rho(\lambda)$ is $k/2\pi$ and where the $P's$ are dichotomic functions. Now, take the derivative w.r.t. θ, to get:

$$2\sin(2\theta) = \int \delta(x-\theta_j)P(x)dx = \sum_j P(\theta_j) = k, \tag{4.9}$$

and again

$$4\cos(2\theta) \equiv 0, \tag{4.10}$$

which is false. QED

Some authors (see, e.g., Aerts, this volume) employ a parameterized dichotomic function to represent measurements. Such a function can be dichotomic in the argument but continuous in the parameter, e.g., of the form $P(\sin(t)-x))$, for which then the correlation is taken to be of the form

$$Corr(t) = \int_{-\pi}^{\pi} D(x-\sin(2t))D(x)dx. \tag{4.11}$$

However, this approach seems misguided. First it assumes that the the argument of $Corr$, t, can be identical to the parameter of the dichotomic function $P_t(x)$ rather than the 'off-set' in the argument, here x, as befitting a correlation. Moreover, the same sort of consistency test applied above also results in contradictions; therefore, such parameterized functions do not constitute counterexamples invalidating the claim that discontinuous functions can not have an harmonic correlation. At best, this tactic implicitly results in the correlation of the measurement functions w.r.t. the continuous parameter, t, which is interpreted as the "weight" or frequency of the the dichotomic value. This tactic, however, does not conform with Bell's analysis in which the dichotomic values are to correlated, rather it corresponds with the type of model proposed below, without, however, recognizing Malus' Law as the source of the 'weights.'

Conclusion: There is a fundamental error in Bell's analysis; the QM result is at irreconcilable odds with the conventional understanding of his arguments.[11]

This can be revealed alternately, following Sica, by considering four dichotomic sequences (with values ± 1 and length N) a, a', b and b' and the following two quantities $a_ib_i + a_ib'_i = a_i(b_i + b'_i)$ and $d'_ib_i - a'_ib'_i = a'_i(b_i - b'_i)$. Sum these expressions over i, divide by N, and take absolute values before adding together to get

$$|\frac{1}{N}\sum_i^N a_ib_i + \frac{1}{N}\sum_i^N a_ib'_i| + |\frac{1}{N}\sum_i^N a'_ib_i - \frac{1}{N}\sum_i^N a'_ib'_i| \leq$$

$$\frac{1}{N}\sum_i^N |a_i||b_i + b'_i| + \frac{1}{N}\sum_i^N |a'_i||b_i - b'_i|. \tag{4.12}$$

The r.h.s. equals 2; so this is a Bell Inequality. Conclusion: this Bell Inequality is an arithmetic identity for dichotomic sequences; there is *no* need to postulate "locality" in order to extract it.[12]

Discrete vice continuous variables. By implication Bell considered discrete variables for which the correlation would be

$$Cor(a,b) := \frac{1}{N}\sum_i^N X_i(a)Y_i(b), \tag{4.13}$$

But: experiments measure the number of hits per *unit time* given a, b; and then compute the correlation, each event is a density, not a single pair. The data taken in experiments corresponds to the read-out for Malus' Law, not the generation of dichotomic sequences for which each term represents an event consisting of a pair of photons with anticorrelated polarization or a particle pair with anticorrelated spins. This discrepancy is ignored in the standard renditions of Bell's analysis. It is, however, serious and suggests a different tack.

Consider, following Barut, a model for which the spin axis of pairs of particles have random, but totally anticorrelated instantaneous orientation: $\mathbf{S}_1 = -\mathbf{S}_2$.[13] Each particle then is directed through a Stern-Gerlach magnetic field with orientation a and b. The observable in each case then would be $A := \mathbf{S}_1 \cdot a$ and $B := \mathbf{S}_2 \cdot b$. Now by standard theory,

$$Cor(A,B) = \frac{<|AB|> - <A><B>}{\sqrt{<A^2><B^2>}}, \tag{4.14}$$

where the angle brackets indicate averages over the range of the variables. This becomes

$$Cor(A,B) = \frac{\int d\gamma \sin(\gamma) d\varphi \cos(\gamma - \theta)\cos(\gamma)}{\sqrt{(\int d\gamma \sin(\gamma)\cos^2(\gamma))^2}}, \tag{4.15}$$

which evaluates to $-\cos(\theta)$; i.e., the QM result for spin state correlation. Conclusion: this model, essentially a counter example to Bell's analysis, shows that continuous functions (vice dichotomic) work. It is more than just natural to ask where do the 'gremlins' reside in Bell's analysis? There are at least two.

One has to do with the following covert hypothesis: Bell's 'proof' seems to pertain to continuous variables in that the demand is only that $|A|\,(|B|) \leq 1$. This argument, however, silently also assumes that the averages, $<A> = <B> = 0$. It enters in the derivation of a Bell inequality where the second term above is ignored as if it is always zero. When it is not zero, Bell inequalities become; e.g.,

$$|P(a,b) - P(a,b')| + |P(a',b') - P(a',b)| \leq 2 + \frac{2<A><B>}{\sqrt{<A^2><B^2>}}, \tag{4.16}$$

which opens up a broader category of non quantum models.

A second covert gremlin having broader significance is discussed below.

Are 'nonlocal' correlations essential? The demand that in spite of the introduction of hidden variables, λ, that a probability, $P(a,b)$, averaged over these extra variables reduce to currently used QM expressions, implies that:

$$P(a,b) = \int P(a,b,\lambda)d\lambda. \tag{4.17}$$

By basic probability theory, the integrand in this equation is to be decomposed in terms of individual detections in each arm according to Bayes' formula

$$P(a,b,\lambda) = P(\lambda)P(a|\lambda)P(b|a,\lambda), \tag{4.18}$$

where $P(a|\lambda)$ is a conditional probability. In turn, the integrand above can be converted to the integrand of Bell's *Ansatz*: $P(a,b) = \int A(a,\lambda)B(b,\lambda)\rho(\lambda)d\lambda$, *iff*

$$P(b|a,\lambda) \equiv P(b|\lambda), \ \forall a. \tag{4.19}$$

This equation admits, it seems, two interpretations:

(1) When this equation is true, the ratio of occurrence of outcomes at station B must be statistically independent of the outcomes at A. Therefore, as the hidden variables λ are 'extra' and do not duplicate a and b, even if the correlation is considered to be encoded by a λ , it will not be available to an observer. But, the correlation by hypothesis does exist and is to be detectable *via* the a's and b's; therefore,

this equation can not hold. Thus, within this interpretation, Bell's *Ansatz* is not internally consistent.

(2) Alternately, if the a on the l.h.s. is superfluous, so is b; so that $P = P(\lambda) = 0$ except at one value of λ, where it equals 1, or is a Dirac-delta function . That is, the correlation is totally encoded by the hidden variables, as follows if a sufficient number of new variables are introduced to render everything deterministic—as often assumed. Consequently, individual products of probabilities at the separate stations, i.e., AB's, in Bell's notation, become Dirac delta-functions of the λ. If everything is deterministic, then there can be no overlap of the of the non-zero values of pairs of probabilities for a given value of λ, and therefore, in the extraction of a Bell inequality, all quadruple products of P's with pair-wise different values of λ in Eq. (4.5) are identically zero so that the final form of a Bell inequality is the trivial identity:

$$|P(a,b) - P(a,b')| \leq 2. \tag{4.20}$$

In either case, "locality" is not to be so employed so as to exclude correlations generated at the conception of the spin-particles or photon pairs, i.e., "common causes." The non existence of instantaneous communication can not impose a restraint here; it must bear no relationship to the validity of Eq. (4.19).

In addition, Eq. (4.20) reconciles Barut's continuous variable model with Bell's analysis.

Bell-Kochen-Specker 'Theorem'. Besides Bell's original theorem there is another set of no-go theorems ostensibly prohibiting a local realistic extention for QM. In contrast to the theorem analyzed above, they do not make explicit use of 'locality,' rather they use certain properties (falsely, it turns out) of angular momentum (spin). In general, the 'proof' of these theorems proceeds as follows: The system of interest is described as being in a 'state' $|\psi\rangle$ specified by observables $A, B, C \ldots$. A hidden variable theory is then taken to be a mapping v of observables to numerical values: $v(A), v(B), v(C) \ldots$ Use is then made of the fact that if a set of operators all commute, then any function of these operators $f(A, B, C \ldots) = 0$ will also be satisfied by their eigenvalues: $f(v(A), v(B), v(C) \ldots) = 0$.

The proof of a Kochen-Specker Theorem proceeds by displaying a contradiction; consider, e.g., two 'spin-$1/2$' particles for which the nine separate mutually commuting operators can be arranged in the following 3 by 3 matrix:

$$\begin{array}{ccc} \sigma_x^1 & \sigma_x^2 & \sigma_x^1\sigma_x^2 \\ \sigma_y^2 & \sigma_y^1 & \sigma_y^1\sigma_y^2 \\ \sigma_x^1\sigma_y^2 & \sigma_x^2\sigma_y^1 & \sigma_z^1\sigma_z^2 \end{array} \tag{4.21}$$

It is then a little exercise in bookkeeping to verify that any assignment of plus and minus ones for each of the factors in each element of this matrix results in a contradiction, namely, the product of all these operators formed row-wise is plus one and the same product formed column-wise is minus one.[14]

Now, recall that given a uniform static magnetic field B in the z-direction, the Hamiltonian is: $H = \frac{e}{mc} B\sigma_z$ for which the time-dependent solution of the Schrödinger equation is: $\psi(t) = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{-i\omega t} \\ e^{+i\omega t} \end{bmatrix}$, and this in turn gives time-dependent expectation values for spin values in the x, y directions:[15]

$$< \sigma_x >= \frac{\hbar}{2}\cos(\omega t); \; < \sigma_y >= \frac{\hbar}{2}\sin(\omega t), \tag{4.22}$$

where $\omega = eB/mc$.

Proof of a Bell-Kochen-Specker theorem depends on simultaneously assigning the [eigen]values ± 1 to σ_x, σ_y and σ_z as measurables for each particle. (With some effort, for all other proofs of this theorem one can find an equivalent assumption.) However, as Barut[13] observed and can be seen in Eq. (4.22), if the eigenvalues ± 1 are realizable measurement results in the "B-field" direction, then in the other two directions the expectation values oscillate out of phase and therefore, can not be simultaneously equal to ± 1. Thus, this variation of a Bell theorem also is defective physics.

A local model for EPR (polarization Correlations. The following model incorporates the features of polarization correlations without preternatural aspects or the concept of 'photon.' The basic assumption is that the source emits oppositely directed, anticorrelated classical electromagnetic signals:

$$E_A = \hat{\mathbf{x}}\cos(\nu) + \hat{\mathbf{y}}\sin(\nu); \;\; E_B = -\hat{\mathbf{x}}\sin(\nu+\theta) + \hat{\mathbf{y}}\cos(\nu+\theta), \tag{4.23}$$

where factors of the form $\exp(i(\omega t + \mathbf{k}\cdot x + \xi(t))$, where $\xi(t)$, is a random variable, are dropped, as they are suppressed by averaging.[16] Now, the random variables with *physical* significance, emerging in the detectors per Malus' Law, are $E^2_{A,B}$. It is the detectors that digitize the data and create the illusion of 'photons.' But, because Maxwell's Equations are not linear in intensities, rather in the fields, a fourth order field correlation is required to calculate the cross correlation of the intensity:

$$P(a,b) = \kappa < (A\cdot B)(B\cdot A) >, \tag{4.24}$$

where brackets indicate averages over space-time. (This appears to be the source of "entanglement" in QM, which is seen to have no basis beyond that found in classical physics.) Here, Eq. (4.24) turns out to be:

$$P(+,+) \propto \kappa \int_0^\pi (\cos(\nu)\sin(\nu+\theta) - \sin(\nu)\cos(\nu+\theta))^2 d\nu, \tag{4.25}$$

which gives $P(+,+) = P(-,-) \propto \kappa \sin^2(\theta)$ and $P(-,+) = P(-,-) \propto k\cos^2(\theta)$. The constant, κ, can be eliminated by computing the ratio of particular events to the total sample space, which here includes coincident detections in all four combinations of detectors averaged over all possible displacement angles θ; thus, the denominator is:

$$\frac{2k}{\pi}\int_0^\pi (\sin^2(\theta) + \cos^2(\theta))d\theta = 2\kappa, \tag{4.26}$$

so that the ratio; becomes:

$$P(+,+) = \frac{1}{2}\sin^2(\theta), \tag{4.27}$$

the QM result. This in turn yields the correlation

$$Cor(a,b) := \frac{P(+,+) + P(-,-) - P(+,-) - P(-,+)}{P(+,+) + P(-,-) + P(+,-) + P(-,+)},$$

$$Cor(a,b) = -\cos(2\theta). \tag{4.28}$$

If the fundamental assumptions involved in this local, realistic model are valid, then there would be observable consequences. For example, if radiation on the "other side" of a photodetector is continuous and not comprised of "photons," then,photoelectrons are evoked independently in each detector by continuous but (anti)correlated radiation. Thus, the density of photoelectron pairs should be linearly proportional (baring effects caused by limited coherence) to the coincidence window width. On the other hand, if photons are in fact generated in matched pairs at the source, then at very low intensities, the detection rate should be relatively insensitive to the coincidence window width once it is wide enough to capture both electrons.

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